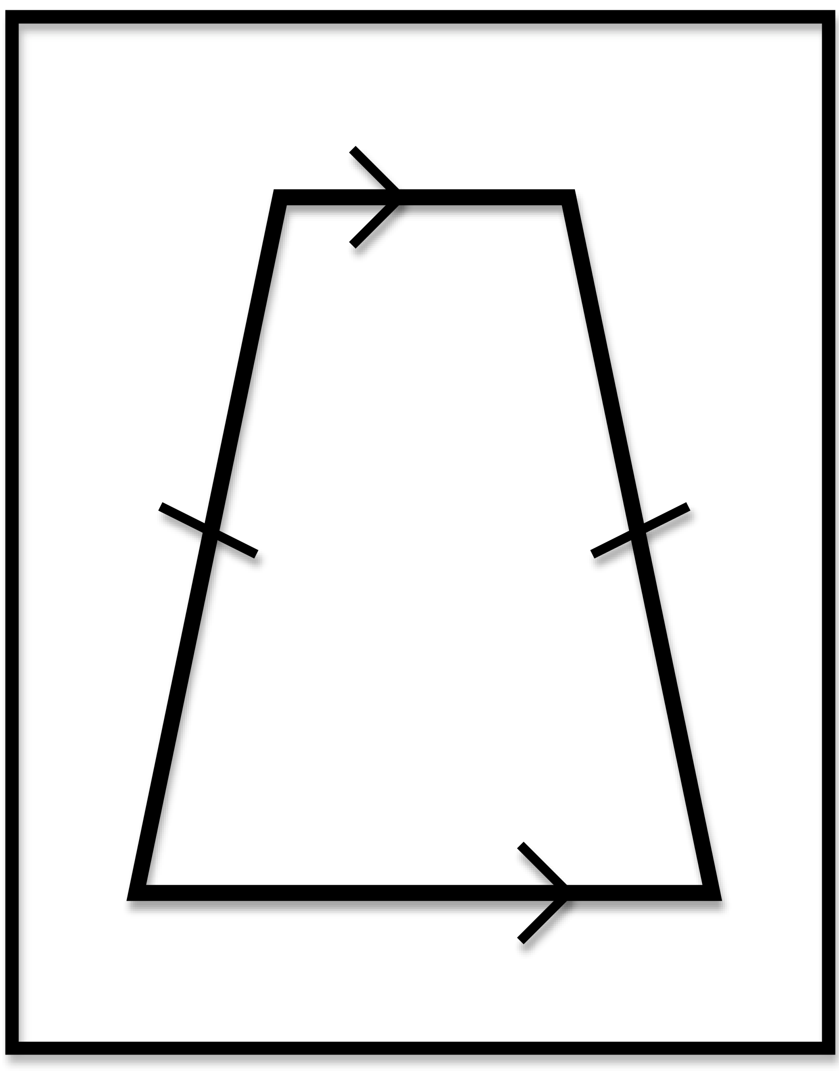
vinculum

Journal of the Saskatchewan Mathematics Teachers' Society
Volume 3, Number 1 (April 2011)

PROBLEMS AND REFLECTIONS



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SMTS objectives – as outlined in the January 1979 SMTS Newsletter – include:

1. To improve practice in mathematics by increasing members' knowledge and understanding.
2. To act as a clearinghouse for ideas and as a source of information of trends and new ideas.
3. To furnish recommendations and advice to the STF executive and to its committees on matters affecting mathematics.

vinculum's main objective is to provide a venue for SMTS objectives, as mentioned above, to be met. Given the wide range of parties interested in the teaching and learning of mathematics, we invite submissions for consideration from *any persons interested in the teaching and learning of mathematics*. However, and as always, we encourage Saskatchewan's teachers of mathematics as our main contributors. *vinculum*, which is published twice a year (in April and October) by the Saskatchewan Teachers' Federation, accepts both full-length **Articles** and (a wide range of) shorter **Conversations**. Contributions must be submitted to egan.chernoff@usask.ca by March 1 and September 1 for inclusion in the April and October issues, respectively.

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Problems

EDITORIAL: PROBLEMS AND REFLECTIONS

Egan Chernoff

The Saskatchewan Mathematics Teachers' Society (SMTS) is – and actually has been for quite some time – an affiliate of the National Council of Teachers of Mathematics (NCTM), which is the world's largest professional organization dedicated to the teaching and learning of mathematics. According to www.nctm.org, *affiliates* "are independent organizations whose missions and goals are similar to those of the National Council of Teachers of Mathematics".

I am telling you about our affiliate status with the NCTM for two reasons. First, if you have not already, I urge you to head over to www.nctm.org and check out their wealth of resources. I have spent many an evening going through the multitude of different sections (e.g., professional development, lessons and resources, conferences, etc.) and always come away with something new to try or think about. Second, in November 2010, I submitted the first three issues of *vinculum: Journal of the Saskatchewan Mathematics Teachers' Society* for consideration for the 2010-2011 NCTM Affiliate Recognition for Outstanding Publication. Before you get too excited, we did not win. We did, however, get some constructive criticism, which is evidence of the symbiotic relationship between the NCTM and its affiliates. According to the NCTM, on the one hand, the strength of our journal is the inclusion of high-quality research articles; on the other hand, we were criticized for not having a particularly wide variety of articles for our membership, which we have taken to heart with this latest issue of *vinculum*.

This current issue is the product of the tireless efforts of thirteen practicing elementary and secondary mathematics teachers who, in the fall of 2010, were enrolled in a graduate mathematics education class at the University of Saskatchewan. During the course, one of their main tasks was to take the mathematics problems we discussed during our class and to implement them in their own classrooms during the following week – no excuses! In other words, yes, they knew there was not enough time and, yes, they knew that some of the problems had nothing to do with their grade level or the topic or the curriculum they were currently teaching and, yes, [insert other flimsy, frequently used excuses used here]. Funny thing is, and despite all the well-known excuses, introducing the problems into their classrooms was a resounding success – for both the teachers and the students.

Given the infectious nature of what happened, each of the thirteen teachers has submitted their favourite problem, which you can implement in your classroom tomorrow. Further, each of the problems is followed by insightful reflections from individuals who have already implemented the problem in their classroom. It is now your turn to take these problems into your own classroom and give them a try – no excuses!

Switching gears, the SMTS turned 50 this year! As you all know, the teaching and learning of mathematics in Saskatchewan has a long and storied history and an integral part of the past 50 years (1961-2011) of history has been *vinculum: Journal of the Saskatchewan Mathematics Teachers' Society* (in its many different renditions). As such, the next few issues will present ten memorable articles from each of the past five decades (i.e., 50 articles from the past 50 years of the journal), which will provide an opportunity to share this rich history with the members of the SMTS. In doing so, we will provide a historical account of many of the trends and issues associated with the teaching and learning of mathematics. These next few issues are meant to serve as a celebratory retrospective on the work of the Saskatchewan Mathematics Teachers' Society.

PRESIDENT'S POINT

Evan Cole

I have been thinking about identity lately – mainly about our identity as a Society, somewhat about my own identity as a math teacher, and to a much lesser extent $sin^2\theta+cos^2\theta=1$.

The past three years have brought about much change for the Saskatchewan Mathematics Teachers' Society. As a Special Subject Council of the Saskatchewan Teachers' Federation, part of our role is to help teachers adapt to curricular changes. Our mathematics curricula have seen a major overhaul in the past few years, as we have made a shift to the Western and Northern Canadian Protocol. This substantial change has brought about a fervor of activity and a re-investment of time and energy into developing new resources and the advent of new professional development opportunities like our Saskatchewan Understands Math (SUM) Conference.

A few months after Sciematics 2008 a group of us met in a coffee shop to talk about an opportunity that came our way. As we sat there planning what would turn into the first SUM Conference, our group knew we were onto something. There was excitement at the table as we envisioned a conference that would encounter new curricula head-on and inspire teachers. I do not think that any of us were looking to replace Sciematics – we just wanted something different, where we felt at home being math teachers.

Since this meeting, a lot has changed. SUM Conference has turned into a largely successful annual conference. This caused us to question our participation in Sciematics, which was co-hosted by us and the Saskatchewan Science Teachers' Society (SSTS). For many years we had trouble meeting our obligations to co-host Sciematics, especially so in 2008 and 2010, as we could not find people who were willing to step into the Conference's leadership roles or host sessions. This is not the case for us with SUM. The leadership is in place and the energy is there to make sessions happen. I think part of this goes back to identity. Sciematics has an identity of its own – I discovered this in my conversations with participants at Sciematics 2010 as many did not know that the SSTS and SMTS existed, let alone hosted the conference.

We as math teachers are passionate about math and our *mathy-ness*. We draw our identity from our collective, our problem-solving nature, and our shared love of terrible, terrible puns. So, I announce, here, that we have chosen to end our partnership with the SSTS to host Sciematics.

2011 marks the 50th anniversary of our Society. We have some big ideas for celebrating; a birthday party at SUM in the spring, a special 50th anniversary collection of journal articles, guest speakers, and so on. It is our hope that you engage in one, or many, of these celebrations of being a math teacher in our province. Keep an eye on our site, www.stms.ca (or your RSS reader), starting in the fall for more details.

PRODUCTS WITHIN THREE CONSECUTIVE NUMBERS
Gale Russell

Problem statement: Choose three consecutive numbers. Investigate the square of the middle number and multiply the outer two numbers. Is there a relationship between the two products and why? Report on your findings. (Boaler, 2002, p. 57).

Problem description: Depending upon the participant's past experiences and knowledge, some time might need to be spent on understanding consecutive numbers. Explanation for the findings can be made through different types of representations.

Possible extensions questions:
1. What types of numbers do the findings extend to?
2. How would consecutive rational numbers, consecutive radical numbers, and so on be defined?
3. So what?

References
Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning. Revised and expanded edition.* New York, NY: Routledge.

I tried this problem with two different groups: a group of pre-service teachers and with two friends. Details of my observations in both group settings will be discussed below, but first there are three important generalizations of my observations that I would like to share.

First, both groups found the context, although rather abstract, to be engaging and there was much mathematical discussion about what was being seen, where it could be extended to, and why. Second, even for those who chose algebraic solutions to the problem, or parts of it, it was the visual representations that were created and discussed that ultimately made the problem make sense for everyone. Finally, those participants who did not automatically go to an algebraic solution strategy felt that their non-algebraic solutions were inferior to the algebraic attempts made by other participants, even if those attempts lead to no conclusion.

Procedures and observations

Pre-service teachers. The group of elementary/middle level pre-service teachers had just been exploring how to represent and understand multi-digit multiplication. After having the pre-service teachers provide examples of three consecutive numbers and explain the relationship between the numbers, I had them each select a different two-digit number, which became the middle number of their three consecutive numbers. They were then given the problem, were asked to determine the two products for their set of consecutive numbers, and to record the resulting product in a chart on the board. As they headed to the board, there was a great deal of discussion and conjecturing happening. One of the pre-service teachers said "I'm starting to think there might be a pattern," as he wrote his products on the board, while another said "Wait – I need to go check this – I've obviously got it wrong". When I asked the second pre-service teacher why he felt he had something wrong, he told me that the difference between his two products was not 1. This was the first time I had seen evidence of this particular individual reflecting upon the reasonableness of his answers. As they returned to their desks, I saw lots of calculations, both paper and pencil and on the calculator happening, so I inquired as to what they were calculating. Some of them were trying larger numbers (and in some cases, much larger numbers), while others were checking with single digit numbers. One tried 0, 1, and 2, and when that "worked" she moved on to negative integers.

We talked about what they were thinking and many of the pre-service teachers were asking questions like "will this work every time" and "why does it work"? These two questions became the focus for the remainder of the class period. Some of them considered the multiplication tables that they had constructed earlier to look for the pattern. When they found it, the students then tried creating multiplication tables for larger products, but this was quickly given up because it was time consuming and "boring".

Some of the pre-service teachers also considered representations using groups and arrays. They could see that when they represented the outside product that it could be rearranged by removing a group or row and starting one new group or column, but that they were always short one in completing that group arrangement/array for the middle product. This was the closest that any of them got to a proof of the pattern that they had observed.

Very few of the pre-service teachers attempted to play with the products algebraically, but those who did use algebra used a separate variable for each of the three consecutive numbers (e.g., a, b, and c), which did not prove beneficial. However, one week later, one of them came and told me that she had decided to check if it worked for fractions. In order to do this, the individual had assumed a definition of consecutive numbers to mean any pair of numbers that have a difference of 1. This person was amazed that it still worked, and despite being a self-declared math-hater, continued to pursue trying different sets of numbers to see if the pattern held.

Friends. I also invited two friends to work on this problem, this time starting with the question of "can you find three consecutive numbers such that the product of the first and third number is one less than the square of the second". One friend, who I will call Lynn, is a lover of puzzles and enjoys playing with mathematical ideas. The second friend, who I will call Susan, believes that she is not capable of real math because she does not like, or immediately go to, working with symbolic notation, but rather likes to consider specific examples and numeric patterns.

Before the question was fully asked, Lynn had already written down x, $x+1$, and $x+2$, and upon completion of the problem statement, Lynn started solving the equation $(x)(x+2) = (x+1)^2-1$. She quickly carried out the algebraic computations and got the following statement: $x^2+ 2x = x^2+2x$, at which point she said "well, that does not help". When I asked why it did not help, she said "all I did was show that they are equal". Thus, although she had in fact proved that the requested relationship existed for all sets of three consecutive numbers, Lynn did not understand that was what her algebraic manipulations were showing her. She had been expecting a specific value for x to "fall out" and when it did not, she assumed that there was some type of trick involved in the wording of the problem.

Susan, on the other hand, did not want to engage with the problem at first. When I inquired why, she told me that she knew I was looking for something involving "x". I assured Susan that I was not looking for "x", but rather three consecutive numbers that had the desired property, however they could be found. Susan then talked about how she would like to solve the problem – by just trying some numbers. When I asked why she was not doing that, she said that she would not be "doing math" that way. After encouragement that her strategy was mathematics, Susan quickly generated 5 examples that worked, asking for a calculator as she moved into two and three digit numbers. She also decided that it was likely that any three consecutive numbers would work.

Neither Susan nor Lynn had had any background in working with concrete or pictorial representations in mathematics in school, so I introduced the array model to them. Both Susan and Lynn began creating arrays for some sample sets and were able to explain why the difference would be 1. I then returned to Lynn's first attempt, an actual proof of the conclusion that the two had just reached, and we discussed the meaning of the final expression that Lynn had attained. Susan removed herself from this discussion completely, and Lynn commented that she seemed to recall something like this from systems of equations, but did not know that it could be applied elsewhere.

Conclusion

My experiences with this problem in the two settings showed me that the problem in itself is engaging and can lead to rich mathematical discussion and learning. It also reinforced my belief that we if we emphasize the abstract and the routines too soon and too much, we are missing out on a great deal of the richness and joy found in mathematical learning and thinking. This problem is definitely accessible to anyone who has been introduced to multiplication meaning it can definitely be used with students in grade 4 and up.

MY TURN: Leanne Lomax-Forden

I tried this problem with a group of 14 grade 7 students. I began by asking them if they knew what consecutive numbers were. Responses varied, but they knew that it had to do with a series of numbers. We defined consecutive numbers, wrote down several examples, and even generalized by showing that x, $x+1$, $x+2$, $x+3$... represented any series of 4 consecutive numbers.

Initially, I planned to ask them to find a series of three consecutive numbers which, when the first and last numbers were multiplied by each other were one less than the middle number squared. Asking students to discover this relationship for themselves would likely work from grade 8 up. Due to the fact that the group had just learned the term "square", I opted to provide them with an example: If our numbers are 6,7,8, then $6 \times 8 = 48$ and $7^2 = 49$

One student commented it was "cool" that the result was two numbers that were consecutive integers. The class was then encouraged to see if this would work for any other series of consecutive numbers. I allowed the use of calculators for this activity, as I wanted students to explore large and small numbers. Many students began by exploring single digit numbers and advanced to multi-digit numbers. Several also experimented with negative integers and fractional numbers. Because many grade 7s have not been exposed to integer multiplication, calculators facilitated their ability to accomplish the task.

After students determined the situations for which this worked, I had them look at a standard multiplication table and to circle the results and look for patterns to appeared. My reasoning was to show them how the groups of numbers corresponded. If I were to teach this again, I would have students model the results in an area grid to show the different groups that occur.

This was a great activity, I found it interesting that students were not limited by what they knew, they asked questions, and worked together to gain information that was above them in grade level. From a teaching perspective, the beauty of this question is that it is open-ended and allows students to enter the question with the knowledge base that they have.

MY TURN: Jacquie Johnson

This problem is a good example of how to get students to engage in a mathematical problem just for the sake of exploring number patterns. I gave the problem to a group of grade 6 students. They were very engaged in the fact that when you have three consecutive integers the product of the outside numbers was one less than the square of the middle number. I found this quite interesting because I knew that they had not been formally introduced to squaring a number, but had little trouble dealing with it. Most of the pairs of students started with one digit whole numbers and moved on to trying large two digit and three digit numbers. One group of three students wanted to see if it would work with negative numbers which was a little more difficult for them, not having worked with negative numbers in school but, armed with calculators with a change sign key, they were able to make some interesting discoveries around integer operations as they found out that the pattern remains for negative numbers as well.

To sort out what was happening, many of the students tried to explain it with words such as "if we multiply a number that is smaller by one than the middle number by the number that is larger by one than the middle number then the product will be one less than multiplying the middle number by itself". While that is true, there is nothing in the explanation that convinces me why this should happen. If I follow that logic I think that it should be the same. When I talked to the students about that they agreed. With a little prompting one girl attempted to show me why with the use of arrays. I am convinced that she had it worked out in her mind, but could not explain it in words. Lunch arrived and we ran out of time.

The next time I do this problem I would have some sort of tiles or counters available for the students to use so that they could physically move objects in the arrays to see what was happening. I predict that many students would then been able to experience an ah-ha moment that we all want our students to experience.

THE LEAP FROG ACTIVITY

Hilary Hinds

Problem statement: In this activity, use a line of coloured counters or act it out getting some friends to play the roles of frogs and toads. Place 3 frogs (let's say red counters) in a line on one side and 3 toads (let's say green counters) on the other side with one space between the two sets. In how many moves can the frogs take to completely change position with the toads? They can hop one space forward over or around the opposing player at a time.

Basic rules of leap frog: The frogs and toads have to change places taking as few moves as possible. They can slide to an adjacent empty space or hop over one other frog or toad to an empty space on the other side. Frogs and toads cannot move backwards and can only hop over one frog or toad at a time.

Extensions and questions:

1. Have students complete the task within a specific time.
2. Change the conditions of the movements using (one or two movements or a combination).
3. Have students set out a row of chairs and sit on either side. Boys can be placed on one side and girls on the other.
4. Does it always take the same number of moves?
5. Can you find a rule for the number of moves, no matter how many frogs or toads?
6. What happens when you have one less coloured counter or add one more coloured counter to either side?

The leap frog activity is an interactive activity, which uses a combination of movements. It can be used to teach a wide range of mathematical concepts, and this is done while individuals are having fun. It can also be used to develop individuals' mathematical thinking skills, problem solving, and reasoning. This activity also includes a mental component to it where individuals get to solve the activity mentally. It is important to note that for the quick solution, have each coloured marker alone so that no two counters of the same colour are ever together. Several other extensions of leap frog are possible, where individuals can add or remove one position. This can be done to increase the difficulty level. Other extensions include changing the combination of the movement, where one or two movements or a combination of both can be made.

Feedback from observations

Two key concepts stood out while individuals attempted to solve the leap frog activity: individuals thinking process and object permanence. For individuals to accomplish the task, they talked and thought out aloud, while conducting a mental calculation in their head. It would appear that the thinking process of both adults and children were the same as they took a similar approach in solving the problem. Although I am unable to say for what kind of thinking process these individuals were engaged in, as more observation was needed before arriving at a conclusion, other researchers could offer a better explanation of these kinds of thinking.

Piaget's notion of object permanence was observed while individuals tried to solve the problem. Several individuals, after switching positions, thought the object still remained. Separate from that one, individual thought that by changing the positions of frogs and toads would get a quicker and faster result. This individual believed that by changing their strategy and approach to the movements taken would produce a quicker result.

From my observation, I noticed that individuals needed further clarifications on the rules of the activity. Here the language is important, for individuals to have a clearer idea of what is meant by a hop and backward movement. This activity is age appropriate, given that individuals from different age groups tried to find the solution. Despite the difference in ages, all individuals produced and made similar attempts. Individuals could make a number of generalizations, while forming different range of patterns. From my observations, individuals had to use manipulatives to solve the problem. It was interesting to see that the individuals did not become frustrated after three attempts. In fact, they were all excited about finding the solution. In the beginning, all individuals said it was impossible for them to switch position based on the movements that were given. After they could find the solution, I added two other components to the activity to increase the difficulty level. Here I gave them a specific time by which to complete the activity, and then I added one other frog and toad (coloured counter) to either side. What I noticed though is that the more frogs and toads (coloured counters) were added the more time was required. Individuals had difficulty in explaining what they were doing wrong. However, after the third attempt they said what was happening. One person noted that she had to move one at a time without having any two behind each other. She said as long as she could separate each, she could solve it with ease. Both groups of individuals were unable to produce a specific formula for finding the number of movements it would take to complete the changes in the position.

MY TURN: Michelle Naidu

I tried this activity with a mixed ability group in their resource room period. Students were in grades 9 to 11, but the majority were in alternative math programming and function below grade level. I was very pleased with how well the students took to this activity. I provided the students with two different colours of linking cubes to use as their "frogs". The nice feature of the cubes is that the linking piece of the cube could be used to remind the students which way forward was. Shortly after starting the activity I also had the students draw 7 blanks on scrap paper to keep track of where their empty spot was as they were confusing themselves and quickly getting frustrated. The ability for the students to just "play" with the cubes until they discovered the pattern allowed all the students an entry point to the problem. Another nice feature was that, even after solving the problem, students often forgot how they solved it, so it prevented the early solvers from simply giving their friends the answer instantly. Due to time constraints, students were not immediately able to extend past the initial swapping sides of the frogs; however, I look forward to how the early success of the simple problem will allow for me to continue returning to this problem while holding student interest. I think this success will allow the students to continue persevering in smaller installments towards the generalization or the problem.

MY TURN: Lisa Eberharter

After I introduced students to the problem, they used coins, dot markers, and square tiles to help understand the moves it took to have the objects switch positions. Students of all levels managed to start this problem and spend time trying to find a solution. When students found a solution they were excited and called me over to show me their solution. Some groups could not immediately repeat the process they used, so they asked me to come back in five minutes. As the class found the procedure, they were asked to generalize a solution for what was happening. I asked for observations about the pattern and I was surprised by the variety of answers around the room. One group found an increasing, decreasing pattern involving the number of jumps and one step moves: they wrote 1, 2, 3, 2, 1. Another group found that as they changed the number of markers the number of moves went from 8 to 15 to 24 to 35, leading them to notice it was going up by odd numbers. I asked a couple of groups to put what they had on the board. When the other students saw the equation $x(2+x) = \#\ of\ moves$ they started to see the connections to their own work. Some asked for clarification and the students at the board had no problem explaining how they knew that their answer worked or how they came up with the formula. I have played this game with students in previous years without asking them to generalize a formula, and I was extremely impressed with how well students articulated their reasoning for the rest of the class. I also found that this particular problem allowed students to see the variety of solutions that can be present in one classroom and how there is more than one way to approach a problem. The great part of this problem is that all students had some level of success with either finding a way to switch positions or finding a way to generalize a solution. I will definitely use this problem in upcoming years.

SQUARES GAME

Julie Helps

Question: You have a square game board with dimensions *n x n*. The game board has a penny in every square except for two. One corner of the board is left empty, and the corner furthest from this empty square has a die in it. What is the smallest number of moves required to get the die into the initially empty cell given that the only valid move is to move a marker into an adjacent empty cell (not diagonally)?

Extensions and questions:

1. Try the game with a 3 x 3 game board.
2. Try the game with a 4 x 4 game board.
3. Repeat the game with a 5 x 5 game board, then a 6 x 6 game board.
4. Fill in the following table using your values:

Side Length	# of Moves

5. What pattern do you notice? Predict the result of a 7 x 7 game board.

This problem makes an excellent introduction to linear relations, or an equally successful comprehensive review after linear relations have been taught. The concepts covered in this problem can be extended to cover multiple curricular objectives, as well as applying successful problem solving strategies to a real-life situation.

This problem can be left quite general, allowing students to approach the question in their own way, and allowing teachers to modify the problem to suit their needs. With the students, I decided to focus on a problem solving strategy that I believe can help them with other problems in mathematics, which is to solve a simpler problem. I broke the above question down, and had the students begin by looking at a 3 x 3 game board. This led to an examination of larger game boards, which in turn led to an identification of a pattern. This proved to be an extremely successful strategy for the students. They were able to work with a simplified form of the problem at the beginning, but the complexity of the problem was not compromised. They were still expected to find the overall pattern and make general conclusions, but they were able to find a successful starting point to begin their investigation.

Possible extensions

This question would be a great introduction to the problem solving strategy of solving a simpler problem, but the curricular ties extend far beyond problem solving strategies. Possible extensions would be to have the students create a table of values for the results of specific trials, and then graph that data. Once the graph has been created, the students can be asked to use their graph to interpolate/extrapolate values, which they can then check concretely. Students could also be asked to describe the relation in words, determine the equation of the relation, determine if the relation is a function, and if that function is linear. This could also lead into a discussion of independent and dependent values, continuous or discrete variables, and rate of change.

This problem could also be extended by asking students to look at how the problem changes if the initial parameters change, such as not requiring the board to be square, allowing diagonal moves, having three blank boxes, etc.

Saskatchewan curriculum ties

This activity would be a great addition to any Grade 9 or Grade 10 class where students learn about linear relations, and in Saskatchewan. Specifically, many objectives from Math 9 and Foundations of Mathematics and Pre-calculus 10 can be demonstrated in this activity.

Lesson implementation

This activity was given to a Foundations of Mathematics and Pre-Calculus 10 class, first thing in the morning. They are usually quiet and focused, most likely due to the fact that they just woke up. These students had just written a test the previous day, and were ready to learn about linear relations. This activity served as an introduction to linear relations, as well as a review of their work with linear relations in Math 9.

Students worked in pairs, and they were allowed to pick their partner. I introduced the problem by telling the class that I had been presented with this same problem in my class the night before, and I did not know the answer, so I needed their help. They were more than willing to help me with this task. Each student was given the required materials, and they quickly started reading the problem and getting to work. The worksheet I gave the students was quite structured, and broke the problem into smaller pieces in hopes they could identify a pattern. The students quickly finished the task for a 3x3 square, and after comparing their answers with others in the class, moved on to larger squares.

The students then created a table of values for their observations, and used those values to graph the data. They were asked to use their graph to predict the minimum number of moves required for a 2x2 square. This reinforced their understanding of interpolation from Math 9. Although the activity sheet did not ask students to check this result, every group drew a 2x2 square and replicated the results. I also asked them to use their graph to determine how many moves were required for an 8x8 square, which reinforced their understanding of extrapolation from Math 9. The final question on my activity sheet asked the students to determine an equation that represented the data. The majority of groups were able to determine a valid equation.

The students were engaged for the entire 40 minute class period, and most were able to complete the activity in that time period. Very few students had questions as they were going through the worksheet, and the only question that caused difficulty for some students was determining the equation of the relation.

Possible pitfalls

Overall this activity went smoothly, especially for my first time through. Students rarely asked for clarification, and they enjoyed the activity so much that there was not one behaviour issue to speak of (which is rare for this class). The only error I had to watch for was students who got the wrong values for the minimum number of moves, since their data would no longer be linear.

MY TURN: Randi-Lee Loschack

I introduced this problem to a Math C30 class expecting them to see the relationship to linear functions quite easily since they have worked with linear functions since grade 10. I supplied the pennies and students used a small piece of paper as the object they were trying to move from one corner to the other. I was very surprised with the outcome.

All students got involved with the activity and were able to find the pattern quite quickly. However, when I asked them if they could find the general equation for the function, they were stumped. A couple of groups, after some prompting were able to tell me that it was linear, but could not remember the general form of a linear function. I then asked students if they could tell me what a function was, and not a single student offered an explanation. Needless to say I was shocked. Perhaps introducing such an activity before introducing linear functions in grade 9 would be a great activity for students to relate back to when they are thinking about what a linear function looks like. When they hold it in their hands and are able to extrapolate the data themselves, hopefully they would be able to draw more meaning from the mathematics.

This is a great activity to test the knowledge base of students who have already learned about linear functions or as an introductory activity for linear functions. All students enjoyed trying to find the pattern, and it led to an interesting discussion on what they remembered about functions thus far.

MY TURN: Lindsay Shaw

The array problem was introduced to a Foundations of Mathematics and Precalculus 10 class after they completed factoring; at this point they had not seen linear functions. With no background in linear functions, I adapted the problem to make it more concrete. I made a series of arrays (2 x 2, 3 x 3 boxes etc) and I gave the students paper clips and an eraser. When some students were done their current assignment I gave them the problem to move the eraser to the empty block in the least amount of moves. Soon the entire class was doing the problem and writing their answers on the board, some worked in groups others worked alone. Soon many of the students saw a pattern arising and some of the students were not moving the actual paperclips anymore, instead using the pattern to extrapolate their finding to figure out what an 8x8 or a 12x12 would be. The key is that I was able to take the idea of arrays and adapt it to the students' ability and make it more concrete for the students. I will pick up the same problem when we get to linear functions and take it to the general equation that Julie had intended. The student's enjoyed the problem and found a lot of success with it.

EULER'S FORMULA
Jacquie Johnson

Is there a relationship (pattern) between the number of faces, vertices and edges in polyhedrons? What is it? And does it hold true for composite objects (Euler's formula)? With very little investment in time and energy you can have students engaging in this classic problem. All you need are some regular polyhedrons and perhaps some composite objects in order to let the students do some counting and recording and soon they will be on their way to discovering some mathematical beauty that exists in our world.

There are patterns and relationships all around us. One of the jobs a mathematician does is seek out those patterns and expose them. The human brain is a pattern seeking organ and it seems to be engaging for students to participate in pattern seeking activities. In this case we imitate the mathematician Leonhard Euler. Is there a pattern or relationship between the number of faces, edges and vertices of three dimensional objects?

I did this problem with a class of grade eight students. The students in this particular class are just getting used to a student-centred style of teaching and learning mathematics. The majority of the class is very transient and attendance is not great, although the administration team is very strong and is doing many innovative things to improve the state of affairs for the students and their families. However, at this time many of the students are uncomfortable taking risks and their persistence for working on any type of problem is very short. The classroom teacher teaches the majority of the subjects to the students. He admits to being a bit uncomfortable teaching in an open-ended problem solving manner. He is much more comfortable explaining to the students how to do mathematics and have them practice; however, he is giving this a chance and has told me he is beginning to see the benefits of reform.

This activity is very versatile in terms of different grade levels and outcomes. Solid objects are part of geometry at every grade. Patterns and Relations is an entire strand from K to grade 9. Equations and formulas are part of the mathematics curriculum from grades five and up. I think high school students would be interested exploring composite and curved objects. I think this activity could be done at any grade level, with different expectations. I think we might be pleasantly surprised to see what very young children would discover about the relationship between the faces, edges, and vertices.

I wanted to give the students exposure to lots of different three dimensional objects, some regular and some irregular. What I mean by that is I wanted the students to be able to work with objects that are easy to name and organize as well as some objects that are composite, or made up of more than one object. In the school that I was working in I could only find one set of foam objects, and some had curves, like a cone, sphere and so on. I put those aside. Since there was only one set I also used pattern blocks because of they were so readily available. Besides, I thought it might be good for the students to use the pattern blocks because of the fact that some of the faces are narrow and I wanted them to be comfortable with that. It was not an issue. The students had no problem recognizing the “skinny” faces.

I handed out the objects, a handful to each group, and tried to give each group a large foam object as well. On this particular day there was a substitute teacher in the class as well as myself and an EA. I let the students choose there own groups/partners. I asked the students to count the faces, edges, and vertices and see if there was some sort of pattern or relationship. The substitute teacher wanted me to review those terms, but I did not. I let the students work with it for a while with their partner or group of three. In the end there was only one group that was shutting down because they did not know the vocabulary terms of edges, vertices, and faces so I showed them. As they counted I asked them to record their findings. The teacher in the classroom was adamant that I show them an example of a table to use on the board, so I did and found then that all groups used the exact same table. I'm quite sure the students could have come up with a way to record their data, but sometimes you need to pick your battles.

After the students had counted everything I put on their desks and had recorded the information, I asked them to look for anything peculiar with the data. One pair of boys found Euler's Formula in about one minute. I gave them a composite object and asked them to find out if it held true for that object as well. This is when the two boys were genuinely engaged. The rest of the students continued to search for a pattern, especially after they knew that one group had found it, so it did in fact exist, although I did hear some grumbling under the other students breath about the group that had found it being "brainiacs" and so on in a derogatory way. I chose to ignore the rumblings in the name of mathematics. Soon there were a pair of girls that had given up completely and one girl was acting up. In retrospect, I'm quite sure that they did not have any idea what we were trying to do and I did not get to them in time to give them any feedback on what they were attempting. If the groups had been chosen by a teacher that knew the students, this likely would not have been an issue. Other groups were close, but I could tell that they were giving up, and in all fairness it was right before lunch.

I called the group back together and had the two boys explain to the rest of the class what they had found. The rest of the class seemed to find it quite fascinating and started to go through the data they collected to see if it worked. We found that almost all students made some mistakes in counting so it was virtually impossible for them to find the formula. I then gave a short biography on Euler and talked about mathematicians in general and what they do. I left the formula on the board and asked the students to share it with their regular teacher when he returned the next day. The two boys that found the formula had not finished counting the composite object and insisted on staying in at lunch to find the answer they were seeking.

The lesson fit in very well as an introduction to the unit on equations they were just starting.

MY TURN: Hillary Hinds

Is there a relationship (pattern) between the number of faces, vertices, and edges in polyhedrons? What is it? And does it hold true for composite objects?

For this activity I used two different groups of individuals, both adults and children. The adults, after looking at the question said they were not familiar with the laws of polyhedrons and composite objects and needed to do some research before they could attempt the problem. The children on the other hand, needed reminders of the difference among the faces, vertices, and edges – although they knew what they were they had them mixed up. To approach this question, individuals used (manipulative) solids

They generalized by saying that there were relations among the faces, vertices, and edges of polyhedrons; therefore the same can be said about composite objects. Composite objects consist of two or more polyhedrons. They did this by moving different objects around and placing, two or more together. To arrive at the relationship they wrote the composed objects and numbers down as they moved objects around.

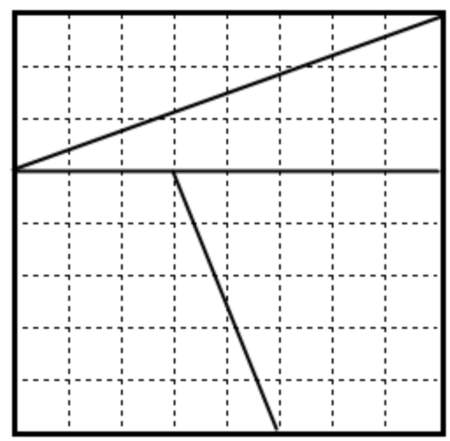
e.g., Cube + Triangular Prism
faces_____
of edges_____
# of vertices_____
From their observations they were able to tell me that *V+F-E=2*.

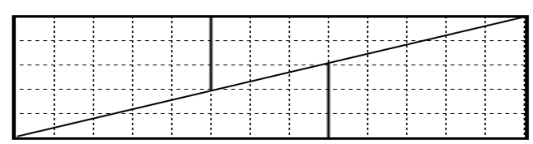
THE STAIRCASE PROBLEM

Lisa Eberharter

Liam's house has a staircase with 12 steps. He can go down the steps one at a time or two at a time. For Example: He could go down 1 step, then 1 step, then 2 steps, then 2,2,1,1,1,1. In how many different ways can Liam go down the 12 steps, taking one or two steps at a time?

Extension: Is it possible that the area in an 8×8 square can be changed into the 5×13 rectangle below?





It is the teachers' role to make decisions everyday that influence student learning. Whether this is demonstrating rules and procedures for students or asking them to participate in doing mathematics, the classroom environment plays a pivotal role and the teacher is the one that creates that atmosphere. When giving students a task such as finding the number of ways that Liam can go down a staircase, taking one or two steps at a time, it is important to facilitate continued engagement of students with the task. There are many factors that a teacher balances in order to achieve student success within problem solving. The questions posed by the teacher, in order to allow students to reach their own solution without abandoning the task due to the belief that it is too difficult for them, is essential to the success of the problem.

Students may run into difficulty when trying to write out the sample space for the number of ways that Liam can go down all twelve stairs. If students are stuck and about to give up on writing down all the options, it is the role of the teacher to guide them to try the problem from a different, more manageable perspective. The questioning skills of the teacher are important at this point so that students continue to work on the problem and form their own solutions, not the one the teacher would use. The NCTM (2000) states:

> Worthwhile tasks alone are not sufficient for effective teaching. Teachers must also decide … what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge. (p. 19)

The variety of levels of students within a classroom makes the goal of keeping the entire group engaged in one mathematical problem a balancing act. Teachers want to help their students when they are struggling with a problem, however giving the students a method needed to solve it does not keep the student actively engaged in problem solving. When students are actively engaging in mathematics, they make sense of their ideas, communicate the ideas to other students, question the viewpoints of others and collectively try to come up with a solution. Questions for students like, "if twelve steps is difficult to write out, is there another way to approach it that is not so overwhelming" may guide them to try writing out the sample space for the first few steps and start looking for a pattern that would assist them in finding the twelfth.

Students who continue to work through this problem also need a teacher who can interpret their answer and encourage them to communicate it to their peers. It is the type of pedagogical knowledge of how to structure a classroom environment where students feel comfortable sharing and working with mathematics, as well as asking questions that guide students through their own thinking instead of imposing a certain method, that are essential tools of the mathematics educator. Ball & Bass (2000) state that teachers "must be ready to hear students' ideas, and to hypothesize about their origin, status and direction" (p. 98). Teachers gain content knowledge through the classes taken at the University level; however, it has primarily been up to the individual teacher to increase pedagogical content knowledge that assists in interpreting student responses within the classroom environment. It can be a difficult task to try to understand the reasoning of students that are at various levels of mathematics and one that teachers have been working through without strong levels of support.

This problem was taken from a Trevor Brown workshop in 2008 entitled "Lighting Mathematical Fires". Trevor Brown was also a presenter at the Saskatchewan Understands Mathematics (SUM) 2009 conference at the University of Saskatchewan. Finding appropriate mathematical tasks that challenge and engage students is a problem that I have encountered over the last few years. Trevor Brown has been a source for many tasks that I have taken into the classroom.

Observations

I gave this problem to three different levels of students. The first group was one that was working on a counting unit involving permutations and combinations. This group was well versed with the notion of sample space and had been working on it for a couple of weeks. Students in this class came up with two different approaches. A couple of groups started writing out the number of ways that Liam could go down the first few stairs and then started to notice the pattern. The other students were determined to use some of the methods that had been discussed in class throughout the counting unit and they made a connection to identical single or double steps. Students broke down the different ways that Liam could go down the entire set of twelve stairs. They saw that he could go down using 12 single steps so that was 12!, however there was no difference between the manner in which Liam took each single step so they divided that by 12!, yielding one possibility. They continued with 10 single steps and one double step, writing that it was 11!/10!, and then 8 single steps and two double steps writing 10!/8!2!. This pattern continued and they added up all the possible outcomes to get a final answer. When students had to write their solutions on the board and explain their logic, comments like, "well that seems easier than what I did" were heard around the room.

Students who had already taken the counting unit and were in a grade twelve class recognized the problem and the phrase, "number of ways" and were having difficulty remembering the procedure and how to apply it to this situation. They spent some time trying to ask for help on the procedure; however, many of them could not come up with it. Thinking they had to solve this problem with prior knowledge appeared to take away some interest in the problem and students were less engaged. Other students were struggling with writing out the sample space for twelve stairs and with some questioning, attempted to try smaller amounts of stairs. After the focus was shifted to more manageable numbers, students quickly found the pattern. One student made the comment, "it feels so good when I get these problems; I feel so smart".

The youngest group to attempt this problem was a Workplace and Apprenticeship 10 class. They were given the choice to work on the original staircase problem, or the extension problem. Students either started drawing out the sample space, not yet knowing what that was, or they got scissors and cut out the shapes in the 8 x 8 square. Initially I told them to try to manipulate the square into a 5 x 13 area (not having shown them how), and only after they found it did I ask them about the discrepancy in the $64u^2$ compared to the $65u^2$. At the end of the hour no student had completely solved either problem, however some were close. As they left I told them that the following day each group would have to make one statement to the class about something they think works or explain why a method did not work. The following day as students talked, I wrote their statements on the board. The class discussion lead to a discovery about Fibonacci number in the staircase 1, 2, 3, 5, 8, 13, 21….as well as recognizing the 5, 8, 13 correlation with the area of the square and rectangle in the other problem. I asked students if they could make more problems involving a square and rectangle of other areas, and students started trying combinations of three numbers from Fibonacci. The discrepancy in area can also be explained using slope, so I could use this problem in a Foundations of Mathematics and Pre-Calculus 10 class.

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MY TURN: Randi-Lee Loshack

This question was given to a group of Math A30 students. I placed the problem on the board, but I changed it slightly. Instead of giving them the twelve stairs, I broke it up showing 1 stair, 2 stairs, 3 stairs, 4 stairs,….then 12 stairs. I then proposed the question to them, and they were able to come up with the solution after about 10 minutes with the problem.

All students who obtained the solution did it by seeing the pattern and extending it to 12 stairs. Once they found the pattern it opened the door to a discussion on Fibonacci numbers and what they were. Students were really interested in information I shared with them and I encouraged them to search for some other areas where Fibonacci numbers existed. They are just beginning the counting unit in Math A30 and I am curious to see if they will be able to answer the same question using permutations. This is a great question because all students can enter at some level. Many schools have stairs, and students could even try experimenting on the stairs if they wanted to use a visual.

I also did a little experimenting of my own and tried the question by going up double and triple stairs. It led to an interesting pattern as well. Try it and see how you do!

MY TURN: Hillary Hinds

Students worked in groups of four as they tried to come up with the solution to the problem. Some groups tried to use cut-outs or drawings to represent the actual staircase movement. Despite this, they did not arrive at a solution. I was concerned as none of them got it, after 40 minutes. I gave them no help whatsoever. I think I should have given them more time. I also think it was a bit older than them though. I realized that a few were on to it, but lacked the discipline to pull through to the end, and verify their answers. Some just tried to use a formula of some sort...$2^n = 4096$; they were not able to explain how they arrived at this formula. One person said the result was 19 ways, but was unable to say how he arrived at the result.

I approached one other adult with the problem and they used $n!/(n-2)!$ to solve the problem, where n = *the number of steps*, in this case 12. Then $12!/(12-2)!=12!/10!=12(11)=132$ *ways*. He also said it would be easier to solve the problem by breaking it into parts. He also noticed a pattern to the problem, but was unable to say what it was.

HOW MANY BOOKS ARE THERE IN OUR SCHOOL DIVISION

Leanne Lomax-Forden

Problem description: Book your school's library and bring students to the library. You may wish to discuss the importance of estimation in math and in "real life" before you begin the problem. Ask your students the question: how many books are there in our school division's elementary or high schools? Direct your students to record the steps/thinking that they go through in order to solve the problem. Their goal is to provide you, the teacher, with an estimated number of books and the process that they used to come up with their estimation. Keep in mind, your students will need to determine how many schools there are in the school division. I found it helpful to provide them with internet access. In true Fermi style, you may ask them to estimate how many schools there are in the division. Keep in mind that, many school librarians will be able to tell you exactly how many books are in your library, so you can see how close your students' estimates were to the actual answer.

Student direction: Students will begin in a number of different places. Some may begin by figuring out how many elementary schools there were in your division via access to the internet. Others will immediately go to the shelves and begin counting books. Still others may sit down and begin discussing a plan to solve the problem. The order that the students follow is not important.

A *Fermi question* is one that seeks a quick, estimated answer to a question that is difficult or impossible to answer. Enrico Fermi, a Nobel Prize winner, inspired Fermi questions. He had an amazing ability to estimate the answer to complex questions with little or no background information. While Fermi questions were originally used in Physics, they are extremely useful and engaging in math courses.

Using Fermi questions in mathematics classes is a terrific way to bridge the gap between school math and the real world. Due to the fact the questions are open-ended, they allow students to enter the problems with the knowledge that they currently have and move forward. In addition, there are no exact answers for these questions, as they rely on student's abilities to make assumptions and employ their estimation skills. The goal for students should be to estimate an answer to the question with in a power of 10. Fermi questions are a fun way of developing problem solving and logical thinking.

Sample Fermi question

How many golf balls will fit in a suitcase?

How much water is there in Lake Superior (Newtons)?

How many litres of gasoline are used in your hometown in one week (litres)?

How many hairs are there on a human head?

Library problem

The library problem is one of many examples of Fermi questions. I designed the question as a way to work on students' ability to estimate and deal with large numbers. Many valuable discussions came out of this experience and a large majority of students reported that they enjoyed the change of pace and location of the class. While this question could be assigned to individuals or larger groups, both teachers that tested the problem felt that working in partners gave students the greatest chance to participate in problem solving.

Extension/Adaptation

During our debriefing session, one of the students wondered out loud how much it would cost to purchase all of the books in our school division's library. Another wondered how the number of books in their library would compare with a high school library. Fermi questions are easy to adapt for the differing ability levels within your classroom. For example, a weaker student may be asked to estimate the number of books in one section of the library. A more advanced student may want to dig into an estimated cost of the books in the library or school division.

Considerations

Your students may be required to make assumptions about elements within a Fermi question. For example, the students made the assumption that all of the elementary schools in the division had libraries that were of similar sizes. In reality, there are schools that have larger and smaller facilities than ours. However, in order to rapidly solve the question, the students used our school as a rough estimate. Taking the time to discuss assumptions made by your students may lead to enlightening conversations about the strategies that they are utilizing.

References

Whippey, Patrick. Physics Olyimpics. *The University of Western Ontario, Department of Physics and Astronomy, 2001*. [online: http://www.physics.uwo.ca/science_olympics/events/puzzles/fermi_questions.html]

MY TURN: Ron Georget

This question was great to get the students engaged. I chose to have the students in groups of three and they had the opportunity to make their own groups. We went into the library for approximately 25 minutes after having gone through the questions as a class. I explained that I wanted an estimation of the number of library books in our library (including those that are currently signed out by students and staff). The extension problem was to find out how many books were in all the Catholic school libraries in Saskatoon. The librarians were able to get print outs of the actual numbers for our school and every other school in the division. It was great to see the variety of solution methods that were used. Some groups counted the books on a shelf then counted the shelves and so on. One group researched online to find out an average number of books in an elementary school. This was less effective for our own library but great for the extension problem. Most groups thought of the books signed out by students and had to make estimates for those numbers as well. Many groups came to ask me if they were close. When they did this, I asked them to describe how they came to their answer. From there I would question their methods and ask questions regarding how full shelves were or the different sizes of shelves. Some groups took those things into consideration and others did not. One assumption that skewed some results was the belief that a high school library contains more books than an elementary school. The school had the second largest library (2nd to another elementary school) with 19 239 books. The closest group was able to estimate 18 870 books, which is only a difference of 369. I look forward to trying this question again. I think I would have students work in groups of two, because there is a lot of time where one of three group members is not working. I also had to give the students time to finish their calculations when we returned to the classroom. I would say that 60 minutes would be plenty of time for the students to work on this problem and describe their work.

HOW MANY CARAMELS FIT INTO THE HIGH SCHOOL GYM

Lindsay Shaw

A Fermi question requires estimation of physical quantities to arrive at an answer. A Fermi question is posed with limited information given and requires students to ask many questions. This type of question demands communication and emphasizes process rather than "the" answer.

Your Fermi question: How many caramels will fit into the gymnasium?

Outcomes and indicators (Foundations of Mathematics and Pre-Calculus 10):

1. Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems thatinvolve right triangles.
2. Demonstrate understanding of SI and imperial units of measurement including
   - relationships between and within measurement systems.
   - volume of spheres, and right cones, cylinders, prisms, and pyramids
   - linear measurement

You may work in groups or by yourself, but each person must hand in an answer. Your answer and write up must include:

- Your conclusion
- All of the work that lead you through to your solution, should be clear
- All justifications for your answers

You will be marked (out of 30) on:

- Your mathematical thought
- Your write up
- And how logical your answer is

Feel free to use the materials provided (clinometers, tape measure, ruler, caramel).
Adapted from http://www.elemath.ca/Challenges/fermi.htm

What

That was the question posed to grade 10 students, and were told run with it.

How

The question posed is classified under the category of Fermi questions. Fermi questions are estimation questions that are designed to help students, establish and understand a set of assumptions and parameters relating to the question. Estimations and assumptions are essential to Fermi questions; students must develop their own set of estimations that will form the basis for their assumptions, and both must be justifiable. Fermi questions are excellent teaching tools, since the questions can be as simple or as complex as one would like, while also providing an opportunity for the teacher to push some students beyond the scope of the original question.

Why

A Fermi question challenges math students to think beyond regular problem solving. These questions require students to draw on various aspects of their learning. In this question, the grade 10 students had to measure a large area of the floor using an imperial measurement, as imperial tape measures were all that was available. The students then used their metric rulers to measure the caramel, therefore requiring the students to make conversions. When dealing with the height of the gym, the grade 10 students used their knowledge of trigonometry and their use of clinometers to determine the inaccessible height. Finally, the students had to seize their knowledge of volume in order to relate the volume of a caramel to the volume of the gymnasium.

The wonderful thing about this question, and other Fermi questions, is the openness that it presents to the students and the teacher. The direction and the level of complexity are entirely up to you. You are able to challenge your students and introduce them to the concepts of mathematical thought, writing mathematical processes and using logical reasoning. You may want to keep some students at a more basic level so that they feel success and build confidence with math in anticipation that they will try new and more difficult concepts in the future.

What went well

The students successfully applied their previous knowledge and used it in a coherent and efficient manner. I was impressed: the students did not ask for assistance but simply ran with the question. In short, they were responsible for their own learning. The students have become accustomed to problem solving for marks and just for the sake of problem solving. In a few short months, as I have started to change my teaching practice, I have noticed a change in my students. My students are more confident and willing to try an assortment of new tasks and the change is welcome.

What I would change

The next time I try a Fermi question with grade 10 students, I would have some of them examine the impact of changing the dimensions of the caramel. I would also push to have students explain their process better; my students are still learning that they need to explain their thinking in a clear and logical way.

Extensions

I will use this section to speak about how this same problem can be applied to younger grades as well as how you may extend it to older grades. In terms of younger grades you could have the students figure out how many caramels fit into a small box or shoe box. This could be performed physically, by placing that many caramels into a box. This could also been done theoretically by having student measure the dimensions of a shoe box, determining the volume of the box and the caramel and solving the problem from there. The key is that this problem can be adapted to be very concrete.

For older grades this problem becomes more abstract. You could choose to have the students look at how many Smarties® or M&M's® fit into the gym. This question is more complex because students are trying to fit an object that is not a rectangular solid into a rectangular solid (gym). The students have to then think about the volume of the actual item, how they fit together, the volume of the absent space and more. Then one could deal with a non-measureable object to put the caramels or Smarties® into. For example how many Smarties® would fit into Mosaic® Stadium?

It is the power of this and other Fermi questions that allow students to expand their skills of estimation, logical reasoning, rationalization, and accountability. I hope you find similar or enhanced success in your classroom.

MY TURN: Tamara Schwab

I used the caramel Fermi question with a group of adult mathematics students who have been working together for several months on a wide variety of group mathematics problems. The focus of the group is on having the adults work through challenging mathematical problems together, emphasizing group communication and ensuring that all members of the group continually understand what the group is doing as they work through the problem. I gave them the caramel question and they decided together how they were going to approach the problem. Without a gym to measure, they decided to use a random gym size based on their analysis that gyms come in many different dimensions. From there they began using estimation, stating that the dimensions of a single caramel are roughly ½" x ½" x ½", so four caramels were in a cubic inch. The group then converted the dimensions of the gym from feet to inches and roughly calculated the total number of caramels. They then began to discuss all of the items in gyms (e.g., benches, lights, and gym apparatus) that would reduce volume and subtracted an estimated 1% of the caramels from their original total. When they were done, I reminded them that they have to demonstrate their findings and conclusions. This led them to draw a diagram of a cubic inch, discovering that there would actually be eight caramels in a cubic inch, and they adjusted their work. The session ended with a discussion of how physics relates to the caramel problem. I found the session very interesting because after an initial discussion of the problem, both the self-identified strong and weak math individuals in the group felt comfortable. Also, rather than trying to be exact in their calculations, the group automatically used estimation without needing to be prompted and applied their more abstract understanding of what the problem was asking. Both I and the group of adult students enjoyed the caramel problem and I would use it again.

MY TURN: Julie Helps

This problem was given to a Foundations of Mathematics and Pre-Calculus 10 class at 9:00 a.m. The students were incredibly excited to see food that early in the morning, but every group ensured they had completed their measurements before eating the caramels.

The two units of this course that I had not yet taught were measurement and trigonometry, both of which would have allowed for more extensions in this particular problem. Because my students were not familiar with the tangent ratio, I modified the problem to "how many caramels fit in my classroom?" They were then able to measure all three dimensions of the room. The students also were not familiar with converting from imperial to metric, so they were given rulers and meter sticks that had both units. Some groups measured in inches, some in centimetres, but every group was consistent in their unit of choice.

The students really enjoyed this activity, and were amazed at how large the number was. Some groups finished earlier than others, so I had them do some extension questions relating to my purchase of the caramels that morning. Since 25 caramels came in each bag, I had them calculate how many bags would be necessary to fill the room. I also asked how much it would cost to fill the room, at $2.49 per bag. The final question was how much money I would have saved if the bags were on sale for $2.00 each.

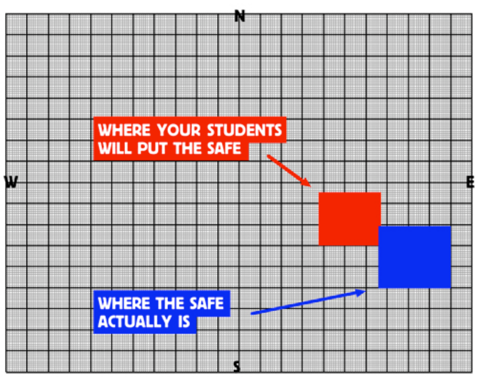
I will definitely use this activity again, but next time I will wait until the students have experienced trigonometry and unit conversions, to make this a more comprehensive learning experience.

THE ITALIAN JOB
Michelle Naidu

Problem: View clip here: http://www.mrmeyer.com/wcydwt/italianjob.mp4

Using the grid paper provided, locate where Edward Norton should paint his square of explosive paint in order to blow up the floor and secure the safe.

Problem description:
Play the clip for students as many times as required for them to have the information they need. By providing the students with grid paper where every minor line represents one inch and major grid marks occur every 10 lines, you are inviting the majority of your students to make a common error. This problem appears simple enough for students to confidently do alone (neither painter had access to any help). Once completed, invite a student to draw their safe on the board, and let conversation go from there.



More information and the problem can be found on Dan Meyer's blog at: http://blog.mrmeyer.com/?p=6339

The new Saskatchewan Mathematics Curriculum is based on the idea that for a deep understanding of mathematics, students must learn by constructing knowledge. The curriculum places emphasis on teachers explicitly teaching things like symbols and vocabulary that students would not naturally have access to, while allowing students to discover other mathematics through engaging in strategic play with mathematical ideas and perspectives in order to, as the curriculum mentions, make strong meaningful, easily accessible connections necessary for learning.

The curriculum also identifies seven processes inherent in the teaching, learning, and doing of mathematics. This activity directly targets communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing. It also addresses a common misconception held by students; that in the imperial system decimals are inches.

The set-up
Begin by showing your students the clip from the move *The Italian Job*. Pass your students a piece of grid paper where one grid line is equal to an inch and major grid lines occur every 10 lines. Explain to your students that they are now Edward Norton and need to locate the safe in the room in order to be able to steal the millions of dollars in gold bricks. You can replay the clip as many times as they need, but encourage students to work alone – Edward did not have any help, after all.

By marking major grid lines every 10 lines instead of every 12, you are inviting your students to make the error that decimals are inches. After giving students a reasonable amount of time to draw in their safe, invite a confident student up to the board to share their solution. Remember, millions of dollars in gold depends on their right answer! A lively, student led discussion will follow as students realize they have placed their safes in different locations.

Why it is successful
The key to this problem is that it appears to be really easy. It borders on insultingly easy depending on the age of your students, which is great. Whether your students recognize that there are 12 inches in a foot

and place the safe correctly or simply count off 14.8 on the grid, everyone in your classroom can feel confident that they are doing the problem "correctly" with little to no help. This confidence that they have the correct answer to a simple problem will allow your students to debate amongst themselves when they realize there are different "correct" answers in the room. Before you know it, they will be communicating and debating about math *without you*.

Similar problems to encourage communication, reasoning and problem solving in your classroom

This problem belongs to a larger set of problems initially created by blogger Dan Meyer. The "What Can You Do With This" (WCYDWT) problems are problems that aim to:

- recreate mathematical reasoning for students as [it is found] in the world around [us].
- involve students in both the solution to *and* the formulation of meaningful questions.
- exploit students' intuition and prior knowledge in the solution of those questions.

Dan's blog is an excellent resource for more WCYDWT problems. However, many other teachers have started creating and sharing their own problems, so a Google or twitter search will also yield lots of tried and critiqued problems.

Reflections on the problem

I did this activity with a group of grade 10 Workplace and Apprenticeship (W & A) students who were in an adapted classroom environment. The students belonged to an extended grade 9 program that transitioned into the W & A class. Historically, they have had little success with math and are reluctant to try things without a lot of teacher direction and encouragement.

Even though the cardinal directions were provided for them on the grid paper, many students were not sure how to orient themselves. A student very briefly explained this to the class. There were also many students who asked how many inches were in a foot. I discouraged students from sharing the answer, but did allow them to share how they could find out (look at a ruler, look in their text, etc.).

I had more technical difficulties than anticipated with this problem. Using Adobe software was very unintuitive for the students to be able to trace their answer for the class. Next time I would use Microsoft's Word or PowerPoint software and have the rectangle tool pre-selected (in a colour and shaded) for the students.

Instead of jumping straight to the classroom discussion of the correct answer, I had students compare with others around them. I think this was helpful to get conversation going with this group as, in general, they are not confident. I did circulate quite a bit to help fuel debate because when two students had different answers weaker students were quick to default to "s/he must be right". To help with this, I strategically pointed students towards others who I knew they would be more comfortable conversing with. By the time we arrived at a large class discussion, two camps had clearly formed and each had a spokesperson that was comfortable explaining the groups' reasoning to the rest of the class. From here we were able to arrive at class consensus as to the correct answer. Student engagement throughout the question was high, and the large group discussion was almost unnecessary as the smaller discussions were far more animated and involved more students in the conversation.

MY TURN: Ron Georget

I posed this problem to a grade 8 classroom. I played the clip from start to finish twice and repeated the sections where measurements or directions were given. While the students started to work on the problem, I played the portion of the clip that gave the dimensions over to ensure that the students had the proper dimensions. I had students work individually to keep the solutions unique to each individual student.

The students began to question the dimensions of width and depth. To clarify the issue we decided as a group that the width was east-west and the depth was north-south. Once the students had their solutions, I put the same grid paper on the Smart Board and asked one student to sketch his solution. I asked him to explain how he came about his solution. I found that most students knew that they could not count the major grid lines as feet. One of these students made the point that the major grid lines should have been 12 x 12, made his calculations by considering the major grid lines to be feet anyway. It was interesting that even a confident and capable student was prone to the error he was simultaneously warning classmates about.

After our discussions involving the solutions, we looked at the clip a few more times. I paused the clip at the scene where the measuring instrument reads 14.8 feet. In the classroom, asking the students if this was correct was where the students erupted in an engaging conversation. It was great to see the students so involved and out of their desks when they realized the explosive paint was not painted in the correct location, but the thieves plan still worked without a problem. This is a great activity which I will use every year.

MY TURN: Lisa Eberharter

I used a slight modification of this problem to correspond with the outcomes of Workplace and Apprenticeship 10. Instead of giving the students a grid, I handed out a blank piece of white paper and indicated that the longest side of the paper should be North and South. After playing the part of the video that included the measurements two or three times, I asked the class if they could find the location of the safe. We discussed how precision of measurement was essential in this particular situation. Initially, students chose any scale they wanted. To hype up the problem I gave out "gold" to all students that managed to draw the exact location of the safe. I used my answer key held up to the light with their answer sheet to verify the accuracy of their drawing.

After working for approximately 10-15 minutes, the class realized that, depending on the scale they chose, it may not match up precisely to my drawing. I indicated that I used *1mm = 1inch*, and students flipped their paper over and used the other side if they had been using a scale different from my own. Students were quite enthusiastic about checking their safe locations.

We followed up by converting their measurements from feet and inches, like the ones used in the movie clip, into meters. Having students convert units lead into a discussion about where the different systems originated, why certain countries do not use the same system and the difficulties going between systems. I would do this activity again. Students enjoyed trying to find the exact location of the safe and it reinforced the importance of accurate measurements and the difficulties that arise when converting units between systems.

ALARM CLOCK COUNTING

Nathan Banting

This problem appears basic; it is this simplicity that encourages every student to begin. It can be introduced in a variety of ways, all of which should include an illustration.

The students are presented with a narrative situation where they glance at their alarm clock to see if they are running behind schedule. Unfortunately, they had left a book across the bedside table, and it has completely blocked the lower half of the familiar neon patterns that make up the clock's digits. After sufficiently explaining the graphic, the teacher tells the class that they needed to be up by a certain time in order to make an appointment. The teacher can modify the pattern on the clock face and the time the students must awake. Using this information, the class is asked to answer one remarkably complex question, "Are you late?"

It is important that the teacher avoids using mathematically rich words before students make the connections. Asking the students, "What is the *probability* that you are late?" will narrow their course towards theoretical probability, and the connections with fractions and ratios may be dismissed by the student as subsidiary – and unnecessary – to the problem.

When the task begins this broad, students often begin too narrow. The most common progression of student thought follows a four level process. It becomes the teacher's job to support the student throughout the thought process, while taking important stops along the way.

Student levels of thought

At the first level, students simply choose a single time that they believe the illumination pattern represents; they then eliminate the variability in the problem, and move back into a concrete mindset. If the teacher acts quickly, and has a digital clock display in the classroom, they can cover up the bottom half of the display and ask the student what time it is. In this case, the student will use situational knowledge to estimate what the pattern represents. Usually, the student picks a time that makes sense within the narrative. For example, if they are told it is early in the morning, they are more likely to choose digits that represent, to them, a reasonable morning hour. The first level of thought eliminates the variability in the problem by selecting an absolute time that the pattern must represent. Although this could be an interesting extension at a higher level of thought – asking students how many illumination patterns could only represent one time – most students should be pushed beyond this first level of thought.

At the second level of student thought, students begin to compile lists of times that will fit the given illumination pattern. This usually begins with a trial and error method, but is soon perfected to a permutation of possible digits at their respective positions. Students will begin to understand which digits fit the individual neon patterns, and then begin to mix and match them to create plausible times. The students at this level have moved past the initial urge to classify the pattern specifically, but are still very much within the concrete realm. They are experimenting, describing, and systematizing each trial, and eventually making Boolean decisions. At this level, the teacher will hear a lot of heated conversation over the classification of certain times. Students will present various arguments for why the time in question could or could not be represented by the pattern of lights. "You cannot have a zero O'clock"; this fact quickly changes students opinions on certain times. Also, certain digits are permissible as the last number in the time, but impossible to appear in the second last slot. The illumination pattern chosen at the beginning should be specifically engineered to create these conversations between students at this level of thought. Here they begin to compartmentalize the problem, and compile a list of acceptable solutions.

At the third level, students begin to see the relationship between the trials from the last level. With each discovered time that fits the pattern, the chances of being late are effected. The students begin searching for a way to represent their new found knowledge; this is a difficult process for them to scribe, but it is substantially easier to complete verbally. Phrases like, "5 out of 12" and "5 for every 12" begin to emerge at this level. Of course, the numbers that students verbalize will depend on the pattern chosen by the teacher as well as the correct classifications at the second level of thought. At the third level of thought students begin to understand the trials as connected pieces of data. They will present a fraction, ratio or percent based upon a series of trial and error calculations. This solution is a valuable inroad to the study of fraction, ratio, and percent, but is not the deepest level of thinking the problem affords.

At the fourth level of thought, the student begins to understand how the number of possibilities for each individual digit affects the total number of possible times. Few students reach this level without intelligent teacher questioning, but some will decipher and classify certain light patterns as "good" because they could represent several digits, and other patterns as "bad" because they represented very few. Asking students to create the illumination pattern that represents the largest number of times may further this level of thought. They will begin to piece together those digit patterns that are "good" in order to create the most abundant pattern on the clock face. This level of thought can be directed in many different directions; once students see the parts as interrelated, the teacher may choose to forget the initial problem of being late, and dive into a sophisticated discovery of combinatorics and the fundamental counting principle.

I do not want to create the illusion that the levels of thought truncate after the fourth. There are certainly several extensions and alterations to this problem that can unlock continued, fruitful thought. It was my attempt to use my personal history with the prompt to classify the most common pathway of student thought.

Important learning outcomes from each level

It is the tendency of teachers to move students along until they meet a desired outcome. In this case, if the teacher is intending to cover fractions, they may attempt to move students along the thought path to level three as soon as possible. If this urge can be resisted, there are valuable learnings that can occur at each level of student thought.

Students at the first level are struggling with the variability of the problem. They do not see the task as complex, because they are not equipped with tools to handle uncertainty. The teacher can use this mindset to discuss acceptable ways to handle situations that are not one-hundred percent certain. In what situations have they encountered uncertainty? How have they handled these types of situations? The teacher can lead the discussion into mathematical ways of representing uncertainty, namely the use of variables. At this level of thought, the student's ability to solve questions with variability may be unsteady, but a conversation that helps them begin to understand the difference between constant and variable situations is a valuable one to have.

Students at the second level of thought are beginning to accumulate trials and judge if they are acceptable solutions to the problem. This thought process lends itself to another algebraic learning. The teacher can begin a conversation about solutions to problems. The students will have accepted certain times because they fit the illumination pattern. Ask them why one is accepted and another is denied. Common responses include, “because they fit the pattern” and, “because it works out”. Extend the idea of being a clock face solution to being a solution of an equation. Why are some numbers considered solutions? How can we find the ones that “work out” or “fit the pattern”? A simple example may begin to bridge the mathematical ideas of solutions to the clock and solutions to equations. When is something considered a solution to a problem? Students struggle with the idea of variability at level one, and the concept of finding solutions to that variability at the second. Both of these learnings have rich algebraic connections.

The third level lends itself to more specific skills, while the previous two have addressed broad concepts. At the third level, students begin to compare the solutions and judge how often they would be late. They begin to search for a representation of their work; the fraction, percent and ratio are all valuable learnings from this level. When students are trying to find that fraction that represents the amount of times they are late, they begin to understand that each trial that fits the pattern adds to the total number in the denominator, but only those that make them tardy are included in the numerator. The teacher can then encourage the students to label each. The denominator becomes the number of possible times, and the numerator becomes the number of times you would be late. Using these labels, the students can explain what fractions mean. Once they understand what fractions stand for, they can begin to make sense of subsequent ratios or percents.

The fourth level allows for numerous inroads into the world of counting and probability. The students’ ability to see the relationship between the number of solutions for the individual digits and the total number of possible times can be used to uncover the fundamental counting principle. As mentioned earlier, the students at this level can also be challenged to find every illumination pattern that could only stand for one time, or to find every impossible illumination pattern. Once students begin to become more comfortable with these higher order skills, the teacher can begin to invent interesting tweaks like the implementation of army time. How does a 24-hour clock effect the impossible times, and the original question of the lateness? Can you have an illumination pattern that only represents one time in an AM/PM system? If the class is very responsive, extensions into conditional probability can also be explored.

Summary and conclusions

The alarm clock counting problem is designed from a familiar context. It is this familiarity, paired with the presentation of the question, which creates the initial intrigue. The flexibility of the question allows students to begin at a low level of mathematics, and extend their knowledge as the question’s challenges adapt to their learning. It accompanies the gamut of concrete to abstract thought, and encompasses both theoretical and computational mathematical skills. The use of this problem can begin to inspire mathematical thought from a most unassuming question – “are you late”?

DID PYTHAGORAS PLAY BASEBALL?
Ron Georget

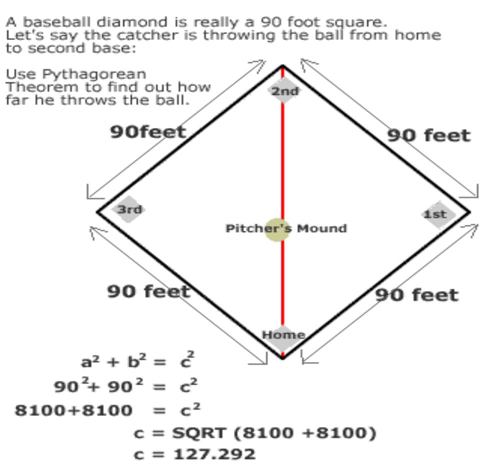
Introduction: I wanted an activity that would introduce students to the Pythagorean Theorem. I also wanted the activity to show students a real life application of the theorem. This activity allows the students to be creative in exploring manipulatives and tools to find a solution to the problem.

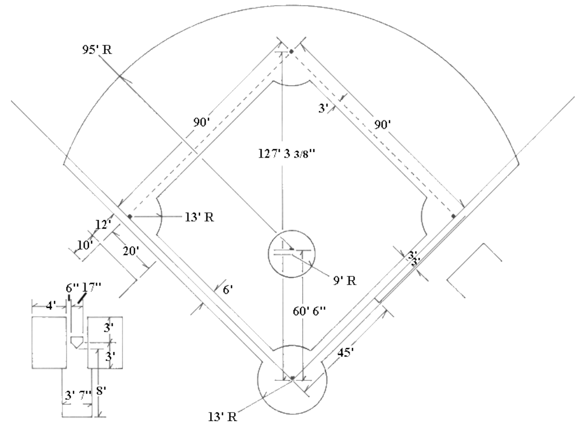
Problem: With the manipulatives and tools available to students, recreate a scaled model of a baseball diamond. A desired scale can be assigned in order to assess other outcomes as well.

Grade level: This activity can be used to introduce/assess outcomes involving squares, square roots, Pythagorean Theorem, ratios and scale, right angle triangles, and many more.

Things to know:

- The Pythagorean Theorem states that the sum of the squares of the length of the legs of a right angle triangle is equal to the square of the length of the hypotenuse.
- Formula: $c^2 = a^2 + b^2$
- The following are two images that show the necessary dimensions of a baseball diamond:





Thomas H. Palmer once said, "If at first you don't succeed, try, try again". This quote is something all teachers need to consider. If an activity is unsuccessful, it does not always mean that there is no merit in the activity, but simply that it was unsuccessful that time. This question can give confidence to baseball, softball or fastball players in your classroom. The activity can be altered to the targeted grade level and the sky is the limit on where their imagination can go. I did this activity twice and below you will find an explanation of the differences as to how I delivered the activity. I am greatly pleased with the results of the second attempt and hope that everyone can learn from my initial mistake. Believe it or not, Pythagoras has influenced one of the most popular sports on earth.

I first wanted to try this activity to introduce a grade eight unit on square roots and the Pythagorean Theorem. I wanted to experiment with the idea of project-based math and this was one step in that direction. A portion of the grade eight class did not take part in the band program offered at the school. These students were my trial group for this activity. I set up the problem by asking the students who played baseball (including softball or fastball). Next I asked the group to draw a baseball diamond. I had a variation of results from a diamond to the entire baseball field, including dugouts, batter boxes and on deck circles. I asked if anyone knew the dimensions of the diamond and was pleased to find that multiple students were able to educate the rest of the group that it is 90 feet from one base to the next (in baseball). I hoped that the knowledge of baseball that students had would translate into engagement to complete the challenge I would set upon them.

I gave the students the task to try and create (draw) an exact baseball diamond with the aid of the Pythagorean Theorem. Quickly students realized that it could not be the exact size and I proceeded to show them a scaled model of the diamond so that it would fit their page. Many students had trouble getting started so I helped them with the idea of creating two congruent triangles that we could apply the theorem to. This helped the students figure out the distance from home plate to second base. However, some students were still not able to piece the diamond together. I gave those students pieces of string cut to the appropriate lengths and they were able to create the diamond on their desks. When I returned the students to the task at hand, they were then able to recreate their solution on paper. The method of recreating was taken through similar approaches using the idea of a compass to measure the length of the string and recreate its possible points with an arc. Creating three more arcs would lead the students to finding the location of all four bases.

It was only once I returned to my graduate studies class that I realized how badly I failed at reaching my goal. I ended up using this idea as an application to the group since I "helped" by teaching them everything that they needed to know. When someone was having trouble, I helped them by giving them the scale, showing them the two triangles and giving them the correct length strings to recreate it. I did not give the students the opportunity to discover their capabilities in the activity. In response to this failure, I needed to try it again. This time I wanted the students to learn more from the activity.

I decided to try the exact same activity, but this time I was going to use it on the whole grade eight class. I introduced the problem in the same fashion as I had previously done, because I think this portion of the activity was successful. When I introduced the task that the students were going to complete, I left them with the idea of recreating the diamond with the aid of the Pythagorean Theorem. After a few moments I was walking along the math manipulatives and simply said 'if anyone was interested, there were many tools and manipulatives that the students could use if they so chose.' I do not know why, but keeping out of the activity and not helping anyone, seemed to engage the students. I tried to make it clear that I would not guide those looking for assistance, and soon no one was asking for help. This was what I wanted, the students were working on a variety of outcomes without understanding the situation as math. They were simply trying to figure out the challenge that was put to them.

When I walked around, I saw many different things happening. Some students were using manipulatives such as string, rulers and pieces of paper. Other students were using tools such as calculators, rulers and compasses. I allowed the students to complete their baseball diamond and also asked that they describe to me how they were able to recreate the image. The group that had seen the activity previously had the chance to create their own solutions and some did, while the others followed the steps that I had unfortunately shown them. From the other students, I had solutions that I would never have considered, but that were absolutely correct. Most students were able to use the Pythagorean Theorem to find the distance from home plate to second base. From here they were able to use rulers, strings or a compass to figure out where first and third base were found. One student even used a string and pencil as a compass, which shows the capabilities of their imagination.

One of the most unique solutions submitted began with a circle. At first glance, I thought the student created a random square and then made a circle to include all four corners on the circumference. Only after reading the explanation, I realized that the circle was the key point to her solution. For this student, one centimeter represented ten feet (1cm: 10ft). She showed her work with the Pythagorean Theorem to find the distance from home plate to second, which was approximately 12.7cm. She viewed this as the diameter of a circle and proceeded to find what the radius would be and to draw the appropriate circle. Next she had to decide where she wanted home plate to be situated. From this point she drew a 9cm chord to the edge of the circle. Completing this step three more times brought her back to home plate which completed the challenge that was asked of her.

The success that the students had with this activity will help them as we embark on the new math unit. The confidence of many students was elevated, even if they were not able to recreate the diamond, but only able to find the dimensions. This activity is not something that I will assign a mark to, because the students opened their minds to view what I asked as a challenge and to solve it how they saw fit. In the spring I will bring this class outside and ask them to create a life-size diamond as a class project. They will all be able to take part, but at that point I will be looking at how students can adapt to other ideas and solutions to the challenge.

I will suggest three key points to promote the most success from this activity. First, be sure to give your students the freedom to explore. Second, have a wide variety of manipulatives and tools available because students will find different solutions to the challenge. Lastly, create a healthy and positive learning environment where students are not afraid to have different solutions to those of their peers. I hope this activity makes your students think as creatively as mine did.

Resources

Diagrams and other explanations and activity ideas can be found at: http://math.about.com/od/pythagorean/ss/pythag.htm

Images on the activity page, found in the back, were retrieved from Google images.

MY TURN: Michelle Naidu

I tried this activity with a mixed ability group in their resource room period. Students were in grades 9 to 11, but the majority belong to alternative math programming and function below grade level.

I was unhappy with how this activity unfolded with these students. I chose to include the use of string as a manipulative from the beginning of the activity, and in hindsight would have only offered it to students after they had struggled with the question for a bit. Given right at the start, the string distracted from the question – the string became the primary focus of the students instead of a visual to assist them with the math.

Secondly, with this particular group of students, doing this as a stand-alone activity was unsuccessful. We have not been doing challenge type problems for very long, and so they are hesitant to try different things and are doubtful about previous mathematical knowledge they can recall. These students were not able to link the idea with making a baseball diamond (a square) with right triangles. For this particular group, I believe the activity would have been more of a success had it been done in the context of geometry, even if not directly related to right angle triangles, to help their memory.

This is certainly an activity I would like to try again, being more mindful of timing next time.

MY TURN: Lindsay Shaw

I did the baseball problem in a grade 10 Foundations of Mathematics and Precalculus class. The students had already taken the Pythagorean Theorem as a part of their Trigonometry unit. For the students, I used it as a review of a concept learned. The students really liked the concreteness that it offered to Trigonometry. I have repeatedly told the students to remember that "the hypotenuse is the longest side" and this problem made that stick. Before when I was saying that "the hypotenuse is the longest side" they heard me, but they did not internalize the idea. However, with this question the students saw a reason for knowing "the hypotenuse is the longest side". In order to throw from home to second to tag someone out, of course, it is a longer throw than from home to first. The problem for the students seemed so obvious when you say it like that, but it was exactly the problem they needed in order to make the link and to remember that "the hypotenuse is the longest side". I liked this problem and so did the students; they took it on and solved it quickly, independently and logically. It is a problem I will use in the future to introduce the Pythagorean Theorem and trigonometry instead of a review.

POINTS ON A CIRCLE

Randi-Lee Loshack

Introduction: I found that when students are encouraged to look for patterns in a problem, it provides a low entry point for all students to become engaged with the problem. From there, one can extend the problem to have students find an equation that represents the pattern.

Materials/Resources: Paper and pencil

Grade level: Middle school aged students all the way up to grade 12. Students will approach the question differently based on their skill set.

Problem: If 8 distinct points are placed on a circle, how many line segments could you create? An example of 4 distinct points is given as clarification.

Extensions:
Students should quickly be able to draw the 8 points on a circle and begin counting the number of segments possible. Some students will see the pattern quickly and extend it to find the answer. Give them more or less points on the circle and ask them to check if their pattern still holds. Once they establish that the pattern holds for all points on a circle, have them try to represent the pattern with an equation. Have students test the equation on the examples they have already answered. If they are still requiring extension, have them count how many triangles are possible when you join points on a circle. It may also be interesting to link the equation created with Math A30 combinatorics and Math B30 sequences.

This particular question came from the 2010 Gauss Math contest in which it stated "Distinct points are placed on a circle. Each pair of points is joined with a line segment. If 8 distinct points are placed on a circle, how many line segments would there be?" A version of this question is also contained in Burt Theissen's *Mathematics A30* textbook within combinations. However, rather than approaching this question from the premise of combinatorics, I approached it as an investigation question, where students had to draw in the lines (chords) that joined the points and find how many they could create. I used this question initially with a grade 9 Mathematics class and extended the question to a Mathematics A30 class.

Observations

Grade 9 mathematics. The great part of this question is that all students were able to enter into the investigation and connect the dots on the circle. Some students drew in all of the lines and added them up, while others recognized that there was a pattern and extended it to find the answer. Students appeared to be more engaged in this question than the lesson that had preceded it, and there was a buzz, during the last 15 minutes of class while students had their heads down calculating with the occasional pop-up of their head to discuss their findings with their desk-mates. For those students who found the pattern early and came up with a number before other classmates, I extended the question to involve 16 points on a circle. From there, I then asked them what about 1000. The students gave me a defeated look because they did not want to add the numbers 1 to 999. I then asked them to represent the pattern using algebra. This stumped many of them since the question was asked during the second month of grade 9, and most of them struggled to remember how to represent an equation from grade 8. We had not covered equations or algebraic representation yet, so many of them were stopped in their tracks. I had two students from a combined number of 57 students who were able to represent it algebraically. Students enjoyed the problem solving activity even though it was not tied directly to the rational numbers unit they were working on.

Math A30 observations. I approached the question in much of the same manner as the math 9 class. I introduced the problem with about 5 minutes left in class and asked them to use the rest of class to come up with a solution. All students were able to answer the question and most were able to see a pattern develop as they created the lines. I was surprised at how involved the students were in solving the question. The next day we talked about the question and many students stated that there was an obvious pattern. I then asked them to represent the pattern algebraically so they could solve 1000 points on a circle. Many became frustrated after 10 minutes and we discussed it as a class. After a discussion, I then asked them to solve the problem using combinations and many of them were able to apply the steps necessary to get the answer. From there I asked them why the combination formula and the sum formula yielded the same answer. I was disappointed that only a few students actually tackled the problem. Many students gave up if the answer did not come to them after 2 minutes.

Conclusion

All students who were introduced to the initial question attempted to answer the problem. I feel because it has such a low entry point, it gave all students a chance for success. At times, trickier "brain teaser" type math problems can leave some students behind. It also has curricular ties to grade 9 math with respect to solving equations, as well as to Math A30 combinatorics and the Math B30 arithmetic sum formula. Having the students generate their own sum formula from an investigative question is a powerful tool. In this way it is not just a memorized formula, but their own constructed formula from which they can answer related problems in mathematics. I am hoping that by introducing such types of questions into the mathematics classrooms it will enhance the problem solving ability of my students. As well, for myself, it is helping me see the extension in some of the mathematics that I am introducing to students.

MY TURN: Gale Russell

I gave this problem to two friends, one of whom, Susan, considers herself not mathematical, while the other, Lynn, enjoys playing with mathematical puzzles. I started by sketching a series of circles with different numbers of points on them and asked Lynn and Susan how many chords could be drawn using the points on each circle. They determined the number of chords and I posed the problem of how to determine the number of chords without drawing a picture.

Lynn quickly began making sketches, often incomplete, and adding quantities together. When she summarized her strategy, she said "it's like factorial, except you add rather than multiply the numbers". She then provided an example where if you wanted to know how many chords there are when there are 17 points on the circle you would need to add 1 through to 16. She explained that she saw the pattern because each time she added one more point to the circle there was one less new chord formed than the number of points.

Susan wanted to use the numbers she knew, so she started by finding the difference between the number of chords and points on each circle. Susan then looked for what she needed to add to these differences in order to get the right number of chords. Ultimately, she dropped off the difference she had calculated as she started to see the same property as Lynn.

Watching Susan's developing strategy, I too learned something. In solving this problem, I always just kept drawing circles with more points on each new circle, but I learned from Susan's work that if I just added points to an existing circle I would see the *(n-1)* part of the summation formula for the sums that both Susan and Lynn came to recognize as being the solution. Just watching that one strategy gave me a new way of visualizing and remembering the formula for this problem, and its more famous counterpart – the handshake problem.

MY TURN: Julie Helps

I introduced this problem to a grade 10 Foundations of Mathematics and Pre-Calculus class first thing in the morning. They have had multiple experiences this semester with identifying and continuing patterns, so this problem was a fun challenge for them.

The students were again able to use the problem solving strategy of solving a simpler problem. Many started by drawing a circle that contained two points, and found the one obvious chord. They increased to a circle containing three points, then four, and most wrote out the pattern they were seeing. Most of the students recognized that the pattern was not linear, but continued to extend the pattern until they got to a circle containing 10 points, which was their goal. Some students had adding issues, but otherwise the majority of students were successful in finding the correct number of chords.

Students who struggled with finding the pattern were still able to determine the correct number of chords, since the problem could also be solved by simply creating the sketch with 10 points. About 10 of the students were able to take the question further, and determine a general formula to represent the situation. Even those students who were not able to put it all together were able to explain why the equation was divided by 2, and why they had to subtract 1 from the initial number. The students (and I) really enjoyed having multiple ways to solve the problem.

IS IT FAIR?
Tamara Schwab

Two players, G and H, decide to play a 10 round game of dice where each simultaneously rolls a regular six-sided die. After each roll, the smaller number is subtracted from the larger number. If the difference is 0, 1, or 2, Player G gets 1 point and if the difference is 3, 4, or 5, Player H gets 1 point. The player with the most points wins the game. Is this game fair?

Materials: six-sided dice, paper, and pencils.

Aims, goals and outcomes: This lesson is based on a problem with the potential to connect to a large range of outcomes at multiple grade levels. The aims, goals, and outcomes are expanded upon in the review section due to their broad nature for this problem. Some possible extensions are listed below. I recommend that extensions be allowed to develop naturally from the students whenever possible.

Working through the problem: The "Is it Fair?" problem is appealing because it easily lends itself to allowing students to take responsibility for their own learning, so I have structured the pull-out sheet for the problem to facilitate this goal. First, decide how you want your class to work – I recommend pairs or groups. Then, give the question to your class without showing them the dice (they should think of the need for dice for experimentation when they are working on the problem). If choosing mixed-ability groupwork, remind groups to talk to each other as they work and explain what they are doing because it may inspire different/new ideas from others in the group and helps prevent frustration in struggling students.

Allow students to discuss whether or not they think the problem is fair and try to explain why they have come to their given conclusions. Once they have talked through the problem, ask them to find a way to demonstrate that their conclusion (for or against fairness) is correct. Encourage students to take control of their learning and develop their own approaches. Go between groups asking questions when you see that they are stuck (e.g., Can you find another way of showing each other what you are trying to say? Does that way of showing the problem give every possibility?), without guiding them toward a specific method or the 'correct' answer. You can expand the problem as appropriate for each group.

Possible extensions: I chose this question because it is so easy to adapt for different classrooms and grade levels. The purpose is to take it and make it your own, or better yet, let your students own it! Here are a few possible extensions: Is it possible to make the game fair? Are differences of 1, 2, or 3 and 4, 5, or 6 more or less fair? Is the game fair if you use a different die (e.g., 12 sided)? Find the probability of various outcomes. How can the data be represented (e.g., circle graphs)? These do not have to be used, but hopefully they give some ideas for adapting and expanding the original problem.

I used small groups when introducing the problem to a grade 7 class and, focusing on the students taking responsibility for their own learning of mathematics, allowed groups to take different approaches to solving the problem. For the most part, they began by agreeing that the problem was unfair. Some hesitated, saying it may be 50/50, but most returned to their belief that the game was unfair because there were more smaller (0-2) answers than larger (3-5) answers. This stage is important for them to do on their own because it allows the students to start figuring out what they already know rather than having them try to understand how you would work through the problem. It also gives the students a starting place for how they are going to attempt to solve the problem.

When asked to demonstrate their belief that the game was unfair (or fair in a few cases), quite a few automatically asked for dice which lead to the whole class conducting an experiment first. Many groups stopped after 10 rolls (or 1 'game'). I want students to start critiquing their own methods of problem solving as they use them because it is an important part of having students taking responsibility for their own learning. So I spoke to the groups that stopped after 10 rolls, asking if 10 rolls are enough to show that the game is actually unfair. As a result, some groups played more 'games' while others decided that 10 rolls is enough to demonstrate their belief. Once groups finished their experimentation with the dice, I asked them to find a different way of demonstrating the fairness of the game. The approaches used varied, including: subtraction tables, t-charts, listing the sample space, using fractions for rolling individual dice, and so on. Some groups needed more probing questions than others to work through the approach(es) they selected to demonstrate their belief regarding the fairness of the problem, but all groups eventually came to a justifiable conclusion that the game was unfair.

Review

The "Is it Fair?" lesson worked well and all students were able to participate at their own level of mathematical ability and complete the activity. The use of groups allowed students to speak about the mathematics they were using and learn together from each other instead of just me. I really enjoyed the fact that there were so many possible extensions for groups who were able to complete the base problem quite quickly. No one was sitting in their desk bored or off-task; it was a fun and engaging lesson. I would definitely use it again.

Specific aims, goals and outcomes

As further described on the page 52, this lesson has the potential to connect to a large range of outcomes at multiple grade levels. Some possible extensions are listed above, but I recommend that these and other extensions be allowed to develop naturally from the students whenever possible. This lesson incorporates all of the aims and goals in the mathematics curriculum: logical thinking, number sense, spatial sense, and mathematics as a human endeavour. Deeper understanding and inquiry, which are strongly recommended by the curriculum, are also integral parts of this lesson.

More specifically, for the grade seven class I worked with, this lesson applies directly to the statistics and probability outcome SP7.3: Demonstrate an understanding of theoretical and experimental probabilities for two independent events where the combined sample space has 36 of fewer elements [C, ME, PS, R, T]. Although there are problems in many textbooks that enable you to accomplish this outcome, the "Is it Fair?" problem allows you to specifically focus on allowing the students to take responsibility for their own learning while covering the curriculum.

Components of the Numbers outcome N7.5: Develop and demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences) [C, CN, ME, PS, R, V], are inherent in the work used to solve this problem. All of the students used fractions within their individual approaches to the solving of the problem. The "Is it Fair?" problem incorporates fraction skills without making fractions boring or superficial, both complaints that I have heard frequently in the past.

Statistics and probability outcome SP7.2: Demonstrate an understanding of circle graphs [C, CN, PS, R, T, V], can also easily be incorporated. Although I chose not to go in this direction when I used the lesson this time, the lesson was so successful that I would definitely incorporate it next time.

MY TURN: Gale Russell

I introduced this problem to my friends Susan (a self-described math-phobic) and Lynn (someone who enjoys looking for math jokes) by saying that they were to play a game in which they each rolled a die, the resulting difference was to be determined, and one or the other would get a point depending on the difference. I then told them that I was thinking that if the difference was 0, 1, or 2, then Lynn would get a point, but if it was 3, 4, or 5 Susan would get the point, but I was not sure if it would be fair. At first, both Susan and Lynn felt it would be fair, but when I asked them to explain why, their thinking took on different approaches.

Lynn abruptly decided that the game would not be fair, and when Susan asked why, Lynn started to talk about what would happen if she rolled a six. Lynn noticed that Susan was not following her reasoning, so she suggested that Susan write out all possible combinations. Interestingly, the word combination was used consistently throughout the course of their examination of the problem, and their representations remained consistent with the use of that term. For example, when Susan wrote out the possibilities, she started with one roll being 6 and listing all of the possible rolls for the other die, but when she then moved on to the one roll being 5 she stopped the second roll at 5 rather than at 6. I inquired why she did this and she said because 5,6 was already taken care of with 6,5 previously. Similarly, Lynn constructed a grid of the sample space, but excluded any repetitions.

At this point, Lynn, counting up the number of differences less than 3 was convinced that the game was not fair. Susan, on the other hand, argued that it could not be decided one way or the other because there was probability involved and that we cannot know which roll will happen because each roll is random. Susan also spoke of the different combinations being equi-probable, but she was unable to reconcile her conception of probability and randomness with the solving of the problem.

MY TURN: Jacquie Johnson

I gave this problem to a sixth grade class. I gave the directions to the game orally, which seemed a bit problematic for the group even with a demonstration on a SmartBoard. However, after a few attempts and students self-monitoring, we had it sorted out. Perhaps written directions would have helped. It does not take long to play the game using ten rolls so it was OK that we had a bit of a slow start. The students predicted that the game was fair before we started. They were a bit surprised by the outcome, and were also very engaged in the problem. They were curious as to what had happened. According to the results it did not seem to turn out fair. A couple students began to organize the data for what outcomes could occur and soon the rest of the class started to organize in the same way. Once we counted the chances for player one and player two all the students seemed to be comfortable with the results. It gave us a chance to discuss theoretical and experimental probability using a very accessible situation. We even had time to do the extension of trying to make it fair, this also created some valuable discussion.

AN ARC MIDPOINT COMPUTATION LESSON

Gregory V. Akulov and Oleksandr (Alex) G. Akulov

This lesson first appeared at (and is reprinted with permission from): http://mathcentral.uregina.ca/RR/database/RR.09.10/akulov2.html.

The well-known midpoint formula states that, if the straight line's segment *AB* (see Figure 1) has the ends at $x = a$, $x = b$ and midpoint M at $x = \mu$, then $2\mu = a + b$, or $\mu = (a+b)/2$. It states also that the same relationship holds for *y*-values.

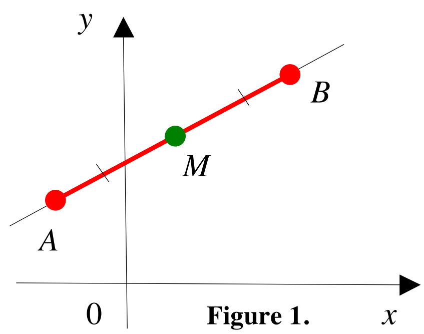


Figure 1.

Circular analogue of midpoint formula is a new topic.

Arc midpoint computation

Let origin-centered arc *AB* of radius r (see Figure 2) have the ends at $x = a$, $x = b$, and midpoint *M* at $x = \mu$. Then

$$2\mu = \pm\sqrt{(r+a)(r+b)} \pm \sqrt{(r-a)(r-b)} \;,$$

where the first radical has "$-$" iff (*Note: the word iff originated in the 1950's as shortening of if and only if*) the arc makes a negative *x*-intercept, and the second radical has "$+$" iff the arc makes a positive *x*-intercept. *Note, that the arc midpoint computation works identically for both coordinates.*

Examples

A. Origin-centered arc of radius 50, located as shown at Figure 2, has the ends at $x = 14$ and $x = 25$. Find *x*-coordinate of its midpoint.

B. An arc of radius 40 is centered at origin. It starts and ends at $x = -24$ and $x = 9$. What is *x*-coordinate of arc's midpoint, if it is located in quadrants I and II?

C. An arc has center at $(0,0)$ and radius 82. It starts at $y = 18$ in quadrant II, passes through quadrant III and ends in quadrant IV at $y = -1$. What is *y*-value of arc's midpoint?

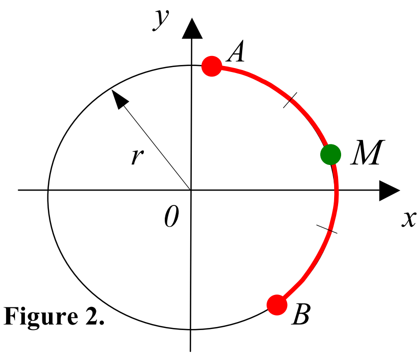


Figure 2.

Solutions

A. $r = 50$, $a = 14$, $b = 25$, both radicals go with "$+$", and $20\sqrt{3} + 15$ is the answer; ***B.*** $r = 40$, $a = -24$, $b = 9$, first radical goes with "$+$", second radical goes with "$-$", hence the answer is $14 - 4\sqrt{31}$; ***C.*** $r = 82$, $a = 18$, $b = -1$, both radicals go with "$-$", making answer $-45 - 4\sqrt{83}$.

Arc midpoint computation was suggested by first attempt shown in [1]. Details of proof involved identities received from [2].

References

1. Oleksandr (Alex) G. Akulov (2009). Dot product finds arc midpoint. Math News, U of W. Volume 111, Issue 6, p. 5.
2. Gregory V. Akulov (2010). The slope of the angle bisector relationship in applied and theoretical problems. Vinculum, SMTS. Volume 2, Number 1, p. 49.

PRODUCTS WITHIN THREE CONSECUTIVE NUMBERS

Problem Statement

Choose three consecutive numbers. Investigate the square of the middle number and multiply the outer two numbers. Is there a relationship between the two products and why? Report on your findings. (Boaler, 2002, p. 57).

Problem Description

Depending upon the participants' past experiences and knowledge, some time might need to be spent on understanding consecutive numbers. Explanations for the findings can be made through different types of representations.

Possible Extensions Questions

What types of numbers do the findings extend to?

How would consecutive rational numbers, consecutive rational numbers, and so on be defined?

So what?

References

Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning. Revised and expanded edition.* New York, NY: Routledge.

THE LEAP FROG ACTIVITY

Problem Statement
In this activity, use a line of coloured counters (or act it out, getting some friends to play the roles of frogs and toads). Place 3 frogs (let's say red counters) in a line on one side and 3 toads (let's say green counters) on the other side with one space between the two sets. How many moves must the frogs make to completely change position with the toads? They can hop one space forward or over (around) one opposing player at a time.

(Source www.Google.ca /images)

Basic Rules of leap Frog
The frogs and toads have to change places making as few moves as possible. They can slide to an adjacent empty space or hop over one other frog or toad to an empty space on the other side.
Frogs and toads cannot move backwards and can only hop over one frog or toad at a time.

Extensions and Questions
Have students complete the task within a specific time.

Change the conditions of the movements using (one or two movements or a combination).

Have students set out a row of chairs and sit on either side. Boys can be placed on one side and girls on the other.

Does it always take the same number of moves every time Leap Frog is played?

Can you find a rule for the number of moves, no matter how many frogs or toads?

What happens when you have one less or one more coloured counter to either side?

SQUARES GAME

Question

You have a square game board with dimensions *n x n*. The game board has a penny in every square except for two. One corner of the board is left empty, and the corner furthest from this empty square has a die in it. What is the smallest number of moves required to get the die into the initially empty cell given that the only valid move is to move a marker or the die into an adjacent empty cell (not diagonal)?

1. Try the game with a 3 x 3 game board.

2. Try the game with a 4 x 4 game board.

3. Repeat the game with a 5 x 5 game board, then a 6 x 6 game board.

4. Fill in the following table using your values:

Side Length	Number of moves

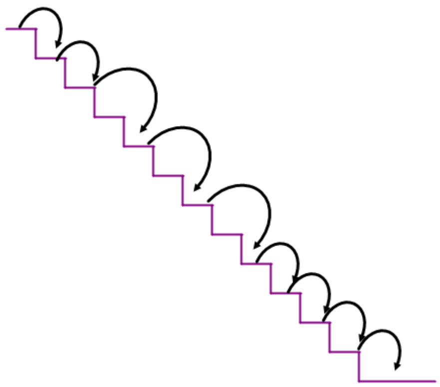
5. What pattern do you notice? Predict the result of a 7 x 7 game board.

EULER'S FORMULA

Is there a relationship (pattern) between the number faces, vertices and edges in polyhedrons? What is it? Does the relationship hold true for composite objects (Euler's formula)? With very little investment in time and energy you can have students engaging in this classic problem. All you need are some regular polyhedrons and perhaps some composite objects in order to let the students do some counting and recording and soon they will be on their way to discovering some mathematical beauty that exists in our world.

FACES	VERTICES	EDGES

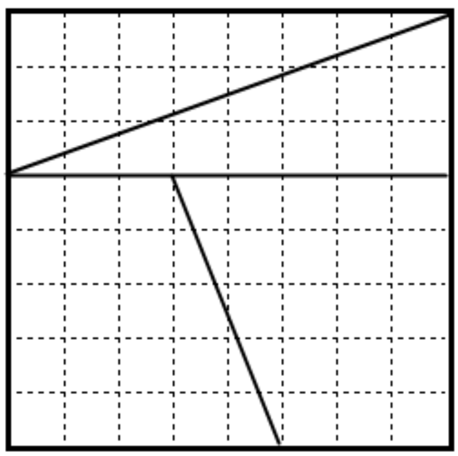
THE STAIRCASE PROBLEM

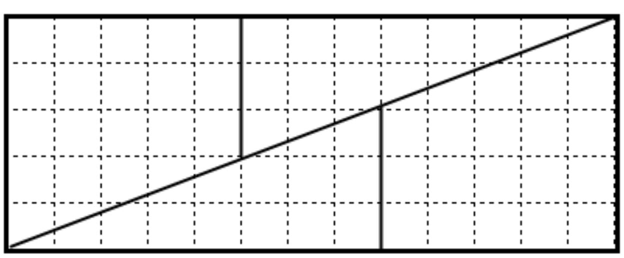


Liam's house has a staircase with 12 steps. He can go down the steps one at a time or two at a time. For Example: He could go down 1 step, then 2step, then 2 steps, then 2,2,1,1,1,1.

In how many different ways can Liam go down the 12 steps, taking one or two steps at a time?

Extension: Is it possible that the area in an 8×8 square can be changed into the 5×13 rectangle below?





HOW MANY BOOKS ARE THERE IN OUR SCHOOL DIVISION?

Problem Description

Book your school's library and bring students to the library. You may wish to discuss the importance of estimation in math and in "real life" before you begin the problem. Ask you students the question, "how many books are there in our school division's elementary or high schools". Direct your students to record the steps / thinking that they go through in order to solve the problem. Their goal is to provide you, the teacher, with an estimated number of books and the process that they used to come up with their estimation. Keep in mind, your students will need to determine how many schools there are in the school division. I found it helpful to provide them with internet access. In true Fermi style, you may ask them to estimate how many schools there are in the division. Keep in mind that, many school librarians will be able to tell you exactly how many books are in your library, so you can see how close your student's estimate was to the actual answer.

Student Direction

Students will begin in a number of different places. Some may begin by figuring out how many elementary schools there were in your division via access to the internet. Others will immediately go to the shelves and begin counting books. Still others may sit down and begin discussing a plan to solve the problem. The order that the students follow is not important.

ALARM CLOCK COUNTING

The students are presented with a narrative situation where they glance at their alarm clock to see if they are running behind schedule. Unfortunately, they had left a book across the bedside table, and it has completely blocked the lower half of the familiar neon patterns that make up the clock's digits. After sufficiently explaining the graphic, the teacher tells the class that they needed to be up by a certain time in order to make an appointment. The teacher can modify the pattern on the clock face and the time the students must awake. Using this information, the class is asked to answer one remarkably complex question, "Are you late?"

HOW MANY CARAMELS FIT INTO THE HIGH SCHOOL GYMNASIUM?

What is a Fermi Question

A Fermi question requires estimation of physical quantities to arrive at an answer. A Fermi question is posed with limited information given and requires students to ask many questions. This type of question demands communication and emphasizes process rather than "the" answer.

Your Fermi question
How many caramels will fit into the gymnasium?

Outcomes and Indicators (Foundations of Mathematics and Pre-Calculus 10)
1. Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

2. Demonstrate understanding of SI and imperial units of measurement including
    - relationships between and within measurement systems.
    - volume of spheres, and right cones, cylinders, prisms, and pyramids
    - linear measurement

You may work in groups or by yourself, but each person must hand in their answer. Your answer and write up must include:

- Your conclusion
- All of the work that lead you through to your solution, should be clear
- All justifications for your answers

You will be marked on:

- Your mathematical thought
- Your write up
- And how logical your answer is /30 marks

Feel free to use the materials provided (clinometers, tape measure, ruler, caramel)
(adapted from http://www.elemath.ca/Challenges/fermi.htm)

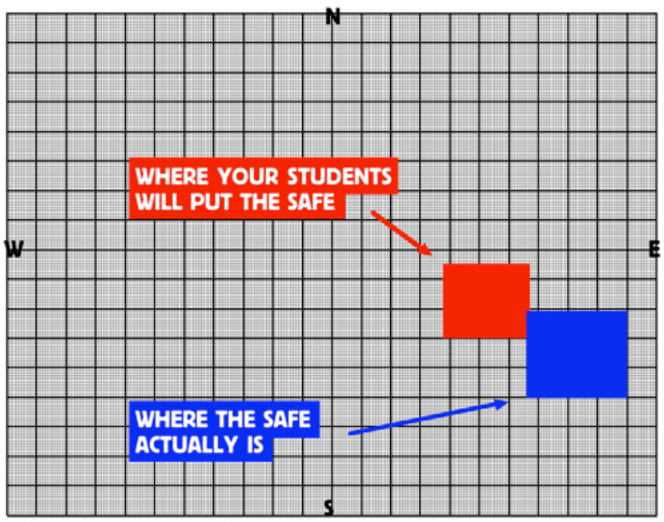
<u>**THE ITALIAN JOB**</u>

Problem

- View clip here: http://www.mrmeyer.com/wcydwt/italianjob.mp4
- Using the grid paper provided, locate where Edward Norton should paint his square of explosive paint in order to blow up the floor and secure the safe.

Problem description

Play the clip for students as many times as required for them to have the information they need. By providing the students with grid paper where every minor line represents one inch and major grid marks occur every 10 lines, you are inviting the majority of your students to make a common error. This problem appears simple enough for students to confidently do alone (and neither painter had access to any help). Once completed, invite a student to draw their safe on the board, and let conversation go from there.



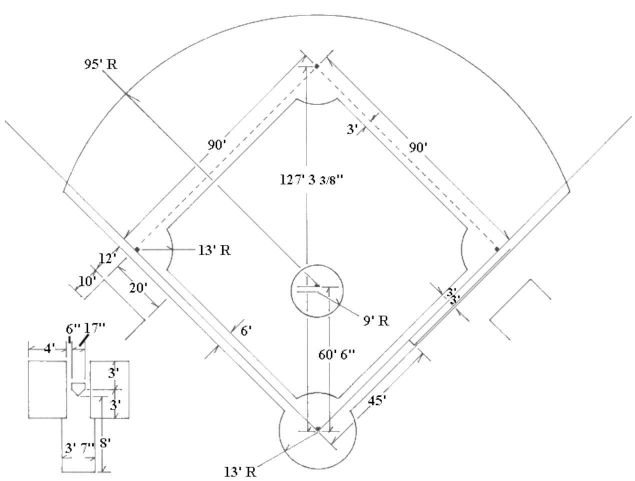
DID PYTHAGORAS PLAY BASEBALL?

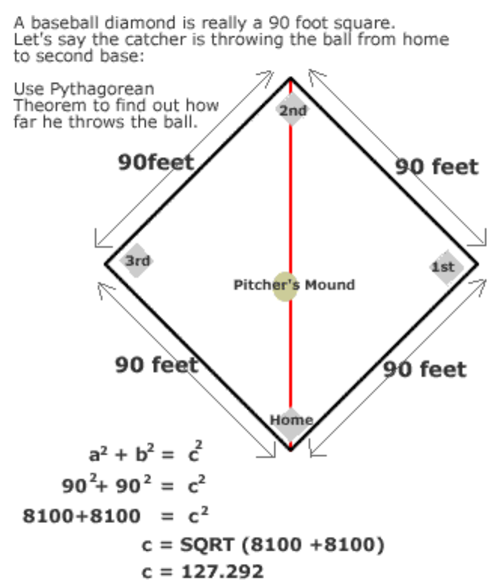
Introduction
I wanted an activity that would introduce students to the Pythagorean Theorem. I also wanted the activity to show students a real life application of the theorem. This activity allows the students to be creative in exploring manipulatives and tools to find a solution to the problem.

Problem
With the manipulatives and tools available to students, recreate a scaled model of a baseball diamond. A desired scale can be assigned in order to assess other outcomes as well.

Grade level
This activity can be used to introduce/assess outcomes involving squares, square roots, Pythagorean Theorem, ratios and scale, right angle triangles, and many more.

Things to know
- The Pythagorean Theorem states that the sum of the squares of the length of the legs of a right angle triangle is equal to the square of the length of the hypotenuse.
- Formula: $c^2 = a^2 + b^2$
- The following are two images that show the necessary dimensions of a baseball diamond:





POINTS ON A CIRCLE

Introduction
I found that when students are encouraged to look for patterns in a problem, it provides a low entry point for all students to become engaged with the problem. From there, one can extend it to students to find an equation that represents the pattern.

Materials/Resources
Paper and pencil

Grade Level
Middle school aged students all the way up to grade 12. Students will approach the question differently based on their skill set.

Problem
If 8 distinct points are placed on a circle, how many line segments could you create? An example of 4 distinct points is given as clarification.

Extensions
Students should quickly be able to draw the 8 points on a circle and begin counting the number of segments possible. Some students will see the pattern quickly and extend it to find the answer. Give them more or less points on the circle and ask them to check if their pattern still holds. Once they establish that the pattern holds for all points on a circle, have them try to represent the pattern with an equation. Have students test the equation on the examples they have already answered. If they are still requiring extension, have them count how many triangles are possible when you join three points on a circle. It may also be interesting to link the equation created with Math A30 combinatorics and Math B30 sequences.

IS IT FAIR?

Two players, G and H, decide to play a 10 round game of dice where each simultaneously rolls a regular six-sided die. After each roll, the smaller number is subtracted from the larger number. If the difference is 0, 1, or 2, Player G gets 1 point and if the difference is 3, 4, or 5, Player H gets 1 point. The player with the most points wins the game. Is this game fair?

Materials: six-sided dice, paper, and pencils.

Aims, Goals and Outcomes

This lesson is based on a problem with the potential to connect to a large range of outcomes at multiple grade levels. The aims, goals, and outcomes are expanded upon in the review section due to their broad nature for this problem. Some possible extensions are listed below. I recommend that extensions be allowed to develop naturally from the students whenever possible.

Working Through the Problem

The "Is it Fair?" problem is appealing because it easily lends itself to allowing students to take responsibility for their own learning. First, decide whether you want your class to work, I recommend pairs or groups. Then, give the question to your class without showing them the dice (they should think of the need for dice for experimentation when they are working on the problem). If choosing mixed-ability groupwork, remind groups to talk to each other as they work and explain what they are doing because it may inspire different/new ideas from others in the group and helps prevent frustration in struggling students.

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Possible Extensions

I chose this question because it is so easy to adapt for different classrooms and grade levels. The purpose is to take it and make it your own, or better yet, let your students own it! Here are a few possible extensions: Is it possible to make the game fair? Are differences of 1, 2, or 3 and 4, 5, or 6 more or less fair? Is the game fair if you use a different die (e.g., D12)? Find the probability of various outcomes. How can the data be represented (e.g., circle graphs)? These do not have to be used, but hopefully they give some ideas for adapting and expanding the original problem.

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Journal of the Saskatchewan Mathematics Teachers' Society

VOLUME 3, NUMBER 2 (October 2011)
VOLUME 4, NUMBER 1 (April 2012)

Selected writings from the Journal of the Saskatchewan Mathematics Teachers' Society: Celebrating 50 years (1961-2011) of vinculum

The teaching and learning of mathematics in Saskatchewan has a long and storied history. An integral part of the past 50 years (1961-2011) of history has been *vinculum: Journal of the Saskatchewan Mathematics Teachers' Society* (in its many different renditions).

The next two issues of *vinculum*, which will present ten memorable articles from each of the past five decades (i.e., 50 articles from the past 50 years of the journal), provide an opportunity to share this rich history with a wide range of individuals interested in the teaching and learning of mathematics and mathematics education. These special issues will provide a historical account of many of the trends and issues (e.g., curriculum, technology) in the teaching and learning of mathematics.

The special issues are meant to serve as a resource for a variety of individuals, including teachers of mathematics, mathematics teacher educators, mathematics education researchers, historians, and undergraduate and graduate students and, further, and most importantly, as a celebratory retrospective on the work of the Saskatchewan Mathematics Teachers' Society.

ISSN: 1920–0765 (print)
ISSN: 1920–0773 (online)