vinculum

Journal of the Saskatchewan Mathematics Teachers' Society
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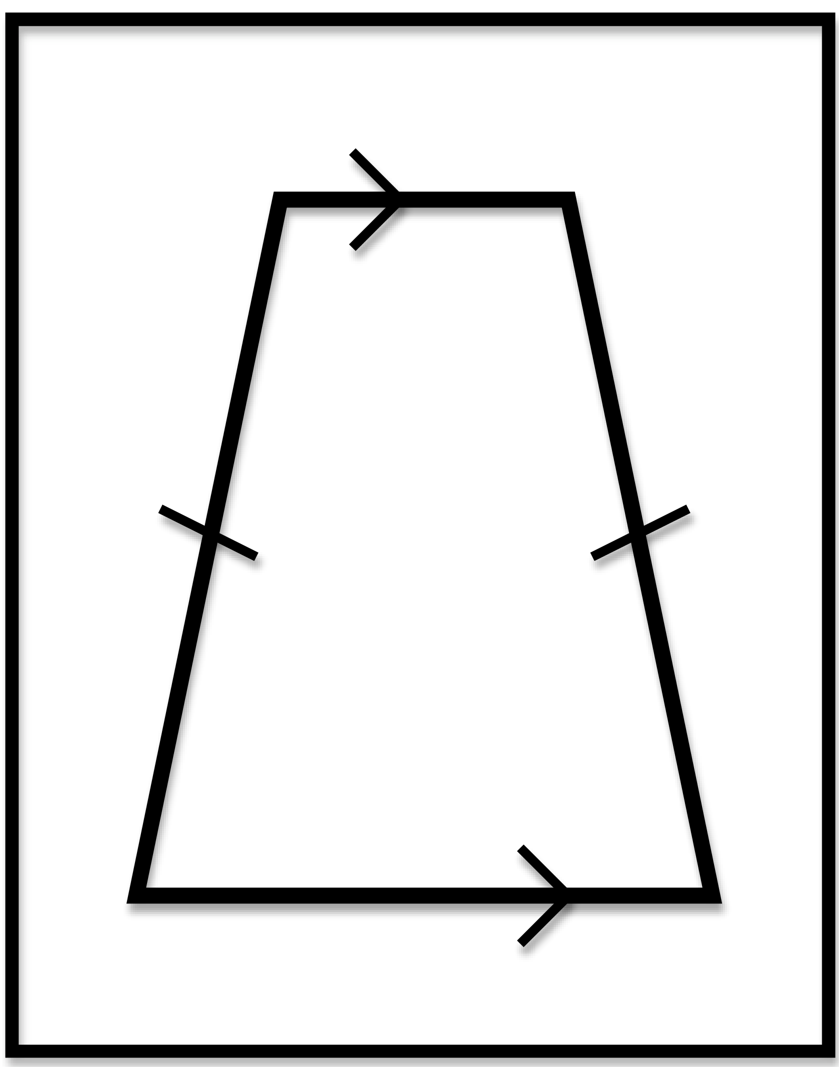
Special Issue of the Journal of the Saskatchewan Mathematics Teachers' Society

The Collected Works of Rick Seaman



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Journal of the Saskatchewan Mathematics Teachers' Society

SMTS objectives – as outlined in the January 1979 SMTS Newsletter – include:

1. To improve practice in mathematics by increasing members' knowledge and understanding.
2. To act as a clearinghouse for ideas and as a source of information of trends and new ideas.
3. To furnish recommendations and advice to the STF executive and to its committees on matters affecting mathematics.

vinculum's main objective is to provide a venue for SMTS objectives, as mentioned above, to be met. Given the wide range of parties interested in the teaching and learning of mathematics, we invite submissions for consideration from *any persons interested in the teaching and learning of mathematics*. However, and as always, we encourage Saskatchewan's teachers of mathematics as our main contributors. *vinculum*, which is published twice a year (in April and October) by the Saskatchewan Teachers' Federation, accepts both full-length **Articles** and (a wide range of) shorter **Conversations**. Contributions must be submitted to egan.chernoff@usask.ca by March 1 and September 1 for inclusion in the April and October issues, respectively.

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EDITORIAL: CHANGE(S), FIVE YEARS LATER

Egan J Chernoff

Over five years ago, in *vinculum 1*(1), my first editorial for the Journal of the Saskatchewan Mathematics Teachers' Society (SMTS) began as follows:

> Stated as a proverb, change is the only constant. However, and without getting into a discussion of the derivative, how much change is occurring at a particular point may vary. In other words, while change is the only constant, rate of change is not, necessarily, constant. For example, points in time may experience more or less change than other points in time. At the present point in time, and with respect to the teaching and learning of mathematics in the province of Saskatchewan, we are in the midst of major change. (p. 2)

For the remainder of that editorial, I detailed the change(s) that was (were) occurring at that particular point in time: Saskatchewan's recent and pending adoption of new mathematics curricula for grades K-12; the potential for a Saskatchewan and/or Western Canadian version of the 'Math Wars'; the newly elected SMTS executive; and, numerous changes to the Journal of the SMTS (e.g., a new editor; the announcement of associate editors, an editorial board, and an editorial advisory board; a change in the SMTS journal's name, from *The Numerator* to *vinculum*; and, my attempt at a logo for the journal).

Looking back over my last five years of editorials, from *1*(1) to *4*(1), I see, now, that a theme has emerged: I have documented change(s) to the teaching and learning of mathematics in the province of Saskatchewan. For example, in *1*(2) I detailed the first semester activities from the members of the newly founded *M(ath)Ed Cohort* (a Master's of Education Program for in-service mathematics teachers in the Department of Curriculum Studies in the College of Education at the University of Saskatchewan, with the support of the SMTS). From a where-are-they-now perspective, I am proud of the change(s) exacted by the (official and unofficial) members of the M(ath)Ed Cohort. For *2*(1), I foreshadowed how a change in the mathematics curricula, which (arguably) introduced a "new" approach to the teaching and learning of mathematics, would lead to change in related domains (e.g., textbooks, professional development, and others); however, I did not anticipate that these change(s) would lead to a multi-year debate (from September 2011 to present day) over the "new" approaches to the teaching and learning of mathematics, which I collectively refer to now as the 'Canadian Math Wars'. As another example, in *2*(2) I looked back over my first two years as editor and detailed some of the change(s) that I, personally, had gone through after embracing my role as "accidental-editor" (p. 2) of *vinculum*. My editorials for *3*(2.1960), *3*(2.1970), *3*(2.1980), *3*(2.1990), and *3*(2.2000), which were designed to "set the stage" for each decade of our celebration of 50 years of the SMTS, documented major moments that changed world and Canadian history, popular culture, and mathematics. Which brings us to *4*(1 & 2).

For the remainder of this editorial, in keeping with the theme of documenting change(s) to the teaching and learning of mathematics in the province of Saskatchewan, I will detail some of changes that are occurring at this particular point in time. First, this is my last issue as editor of *vinculum*. (With apologies to George Costanza: Alright, that's it for me! Be good everybody!) Second, I am pleased to announce that my close colleague Gale Russell (a past editor of the journal of the SMTS) will, once again, take over as editor of (what is now known as) *vinculum* (or whatever she decides to name the journal). Lastly, in perhaps the biggest change to the teaching and learning of mathematics in Saskatchewan over the past two decades, Dr. Rick Seaman is retiring from the Faculty of Education at the University of Regina. I am honoured that my final issue as editor of *vinculum* houses and preserves 'The Collected Works of Rick Seaman'. I would be remiss not to mention that Gale, just recently, has accepted the position of Assistant Professor, Secondary Mathematics Education in the Faculty of Education at the University of Regina – Rick's former position. (Given the former and the latter, talk about going out on a high note.)

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INTRODUCTION

Rick Seaman

Introduction

Why have I spammed the *Journal of the Saskatchewan Mathematics Teachers' Society* (JSMTS) for the past two decades with my submissions? What was my master plan? Is there an underlying subliminal message? What have editors Don Kapoor, Mike Fulton, Gale Russell, Jennifer Von Sprecken, Lindsay Collins and Egan J Chernoff had to put up with, and why? In retrospect I have, for this Introduction, attempted to reorder the submissions in a meaningful order and with anticipated forgiveness it is time to come clean with respect to my intentions! My only qualifier is that each article contains at least one mathematical process. However, I have "cherry-picked" what I have included in this Introduction.

Looking Back [see p. 68]

I begin with the article 'Looking Back' (Seaman, 2009) as a "set" to the introduction of this special issue of *vinculum*. It briefly provides some background about me as a high school senior, high school mathematics teacher, doctoral student and today a mathematics education professor. The references in this article indicate some of the gold I panned from the relevant literature while I was working on my dissertation. Please keep this article in mind as you read my brief review of the other articles I submitted to JSMTS over the intervening years.

I've got a secret: Math anxiety [see p. 14]

The voices of three female elementary teachers provided a glimpse of the impact math anxiety had on their lives and teaching (Seaman, 1998, 1999). This article was a call for all mathematics teachers to reflect on their practice with respect to the impact of the affective domain, and to reaffirm the distinction between teaching people as opposed to just content – which, in my view, is a helpful mindset to begin any teaching year.

Cognitive Strategy [see p. 5]

In the two articles 'Writing in Mathematics' (Seaman & Kapoor, 1992) and 'A Representational Model for Problem Solving in Grade Nine' (Seaman & Kapoor, 1993) I wrote about the importance of communication in mathematics and having a strategy for solving algebraic word problems. I regard these articles as the beginning of my journey. This is particularly clear later (Seaman, 2005) when I presented an example of assessing what is important in the problem solving process and provided a clearer idea of the destination.

Seaman (2004) illustrates how representation can be used to teach students how to think like an expert; Seaman, Seaman and Stange (2002) demonstrated the application of a visualization/representation strategy to attach meaning to the fraction symbol; pattern recognition (Maeers & Seaman, 1998; Seaman, 1999) and reasoning (Seaman 2001, 2003).

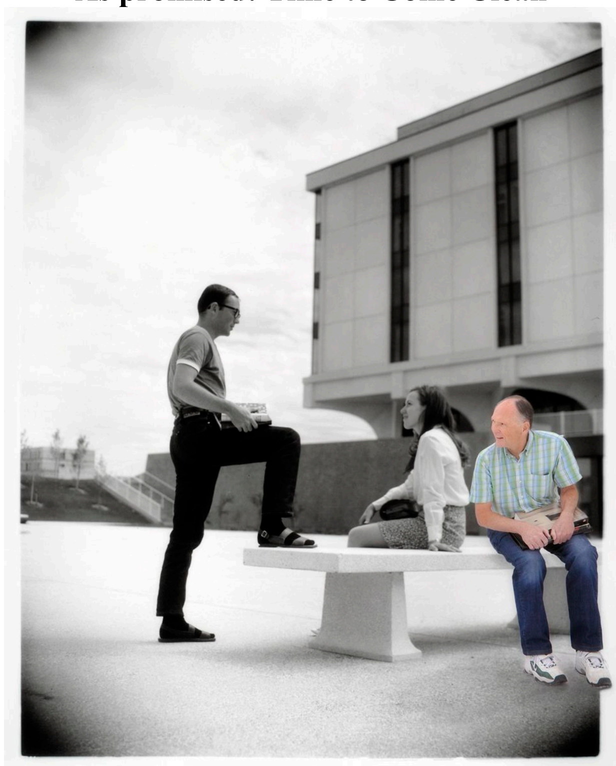
Sharing Real World Applications [p. 22]

In the next stage of my thinking I submitted an example of a real-world application of mathematics in football (Seaman & Forsberg, 2000) which was followed by student examples to be shared with future colleagues: "Value" Marketing food (Philp, Daw & Seaman, 2006), Wascana Lake Urban Revitalization Project (Bell, Biech, Johns & Seaman, 2007) and ethanol as an energy source (Pokoyway, Renneberg & Seaman, 2010). Don't tell Egan but it is time for another submission! It's our secret.

Collaboration [p. 60]

Finally, I suggested that a regular feature in the JSMTS would encourage more mathematics teachers sharing ideas for teaching mathematics in the article 'Let's Talk About Our Ideas' (Seaman, 2008). Today, this goal would probably be best accomplished with the assistance of social media.

As promised: Time to Come Clean



On May 10th, 2011 a picture of me was posted on the University of Regina blog website http://www2.uregina.ca/yourblog/?p=2510. It was a picture of me 'back in the day' in black and white, more recently combined with a picture of me today, in colour, and accompanied by the following narrative:

> At the time the black and white photo was taken I was wondering where my mathematics degree would take me and what would be my contribution to society.
>
> I went on to obtain my Masters of Mathematics and Bachelors of Education and teach mathematics in high school in Regina. In the meantime, I was also a sessional lecturer for the Department of Mathematics and Statistics at the University of Regina. I returned to university to earn a Doctorate in Curriculum and Instruction and I am presently an associate professor of Mathematics Education in the Faculty of Education at the University of Regina.
>
> As I reflect back and then look to the future, my hope is that our mathematics education students will begin their teaching careers with the pedagogical understandings it took me a career to obtain. The continuum from black and white to colour to me signifies the passing of the torch and my small contribution to society.

In appreciation,

Rick

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WRITING IN MATHEMATICS

Rick Seaman and Don Kapoor

Introduction

In 1988, Saskatchewan Education gave each teacher in the province of Saskatchewan a copy of *Understanding the Common Essential Learnings: A Handbook for Teachers*. The handbook was developed to give teachers an overview of each Common Essential Learning as it applies to their teaching. The Common Essential Learnings (C.E.L.s) are Communication, Numeracy, Critical and Creative Thinking, Technological Literacy, Personal and Social Values and Skills, and Independent Learning. These C.E.L.s are "a set of six interrelated areas containing understanding, values, skills and processes which are considered important as foundations for learning in all school subjects" (Saskatchewan Education, 1988, p. 7).

In March 1989, a commission of 23 American professional mathematics educators prepared a set of standards that were published in the *Curriculum and Evaluation Standards for School Mathematics* for the National Council of Teachers of Mathematics (NCTM) (The Council, 1989). The second standard is "Mathematics as Communication," and indicates that students are to develop their use of signs, symbols and terms in mathematics. "This is best accomplished in problem situations in which the students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine and consolidate their thinking" (Curriculum and Evaluation Standards for School Mathematics, 1989).

Writing in mathematics

Mathematics and writing in the classroom? What place has writing in Mathematics? "The traditional view has been that students learn to write in English classes and to compute in mathematics classes and 'never the twain shall meet'" (Davison & Pearce, 1988). This paper will define the types of writing and look at various uses of writing in the mathematics classroom. This paper will also demonstrate the value writing has for the student and for the teacher.

This emphasis on writing in mathematics is gaining ground. Pearce and Davison (1988) state:

> that writing can lead to a deeper understanding and improved mastery of a topic. This is because writing is a mode of language that involves the active manipulation of knowledge. Creating an original piece of writing requires students to analyze and synthesize information, focus their thoughts, and discover new relationships between bits of knowledge. Writing about something involves many of the thought processes teachers would like to foster in their students. Consequently, writing can be an instrumental tool to promote learning in areas not usually associated with writing (p. 6).

In the mathematics classroom different types of writing may occur. Davison and Pearce (1988) have classified the types of writing in mathematics classes into five categories:

1. Direct use of the language (copying and transcribing information).
2. Linguistic translation (translation of mathematical symbols into words).
3. Summarizing/interpreting (summarizing, paraphrasing and making personal notations about material from texts and other sources).
4. Applied use of language (situations where a mathematical idea is applied to a problem context).
5. Creative use of language (using written language to explore and convey mathematically related information).

Seaman, R. & Kapoor, D. (1992). Writing in mathematics.
Saskatchewan Mathematics Teachers' Society Journal, 28(3), 46-49.

Pearce and Davison (1988) studied 31 teachers in Billings, Montana area to determine the amount, kinds and uses of writing in junior high mathematics classrooms. They found that writing rarely happens in mathematics, especially beyond the direct use of the language. It was also found that writing was not a planned activity. Although their sample was small, the authors followed up by investigating mathematics teachers outside the area and found the findings valid. A review of the literature suggests ways that writing may be implemented in the mathematics classroom.

Linguistic translation

Symbolic vocabulary does not have a "one-to-one correspondence between sound and combinations of letters" (Dunlap & McKnight, 1978). For example, 346 is read "three hundred forty-six"; the one-to-one relationship does not exist. As a result, the translation of symbolic vocabulary into language must be memorized. A word normally represents one concept whereas symbolic language may be represented as a series of steps. But the direction for symbolic vocabulary will vary from expression to expression and must also be memorized. For example, in mathematics the directionality for reading symbolic vocabulary is not necessarily from left to right.

$$(1/2 + 3/4; \quad \sum i^2; \quad 23\sqrt{461}; \quad 456)$$

Also, in a symbolic expression, one cannot omit any portion of the expression and still get the message. Consequently, assignments that have students explain the meanings of symbolic language are important for the teacher to know whether the students understands how to read the symbols (Dunlap & McKnight, 1978; Krulik, 1980).

Davison and Peace (1988) go further and suggest that students work on translating their solutions to word problems into a complete sentence. Students could also write out all the steps needed to solve an algorithm. This gives immediate feedback for the teacher to see if the student truly understands an algorithmic process such as division. This could be taken one step further, having the answers given to other students for verification or to translate back into symbolic vocabulary.

Summarizing

The activity of summarizing can serve the purpose of clarifying the mathematical process in the student's mind, and of helping the student communicate how well the content is understood (Davison & Pearce, 1988; Johnson, 1983; Mett, 1987). Summarizing can take several forms, but in written format is often referred to as journals, logs, diaries, expository writing or prompts where the teacher usually provides the context for the writing.

Burton (1985) discusses the use of journals as focusing on "What I thought about what I did – a kind of brainstorming with oneself." Journal writing provides a permanent record of the mathematics course that ordinary class notes cannot provide. Burton says these recorded personal reactions to experience are valuable.

Watson (1980) suggests that students can do their journal writing whenever it is convenient for it to be done with no particular required topic. She feels the dialogue journal, requiring teacher response, allows teachers to understand how students feel. Teacher comments are appreciated by students who, through comments, are made to feel that the teacher cares (Burton, 1985; Mcintosh, 1991; Mett, 1987; Nahargang & Peterson, 1986; Watson, 1980).

Mett (1987), on the other hand, believes the teacher should not comment in the journal, as it is a very personal document. Nahargang and Peterson (1986) use journals to allow students to demonstrate “coverage on an understanding of mathematical concepts at their own rate, while at the same time providing the teacher with a unique diagnostic tool,” revealing misunderstandings and confusion. A student may have misunderstandings that do not show up on homework assignments and tests but are obvious in a journal entry (Davison & Pearce, 1988). Using journal entries as a diagnostic tool for student anxieties, concept misunderstandings, and process errors is valuable for teachers to see the whole student, and enables the teacher in giving appropriate help (Davison & Pearce, 1988; Geeslin, 1977; Miller & England, 1989; Mett, 1987; Nahrgang & Peterson, 1986). Writing prompts serve as an informal method of evaluation and assessment, and can be viewed by students and teachers as an integral part of the instructional process. Examinations often come too late to benefit students in the learning process (Miller, 1991).

Miller and England (1989) define four kinds of prompts: Contextual, Instructional, Reflective, and Miscellaneous.

Contextual Prompts are affective and subject-oriented. Such prompts might be, “Explain how you feel about yourself in this class...Tell me something I need to know about you and algebra...Do you think algebra is important?”

Instructional Prompts influence subsequent teaching. They may reveal a teacher’s poor choice of words in describing a concept, and therefore change instructional practices through teacher self-evaluation. Misconceptions are found before examinations, as can be attitudes, anxieties, and concerns that need to be dealt with. Some examples of teacher-focused prompts are: “Explain what you learned the best today.” A more student-focused instructional prompt would be: “Write a former teacher to indicate what s/he could have taught more or less of”.

Reflective Prompts can be analytical or clarifying. An analytical reflective prompt might be: “Remember when you learned how to _________?’ “Imagine you are writing a note to your best friend to explain how to _________.”A clarifying reflective prompt might be “What would you identify that you have done that helped you the most in this class? Explain.”

Miscellaneous Prompts include prompts for free writing or writings just for fun, i.e., “What is your favorite single digit number?”

Teachers may benefit from student summarizing by listening to how students perceive their classes to determine which methods work better than others and what areas need more concentration (Bell & Bell, 1985; Shaw, 1983). O’Shea (1991) uses diaries to not only understand the problem-solving processes of students, but also to hear their failures and successes. The diary allows expression of student feelings about the process and student assessment of the value of the assignments. Diaries provide the opportunity for the teacher to give feedback by indicating her/his own thoughts as s/he reads the students’ thoughts (Borasi & Rose,1989; O’Shea, 1991).

Letter writing (Schmidt, 1985), rewording problems (Johnson, 1983; Krulik, 1980), having students define terminology (Dunlap & McKnight, 1978), describe algorithms, charts or graphs (Henrichs & Sisson, 1980), processes, properties or concepts in their own words (O’Shea, 1991; Johnson, 1983; Shaw, 1983; Watson, 1980), are all ways of using writing to advantage in the mathematics class.

Applied use of language

Students can make up story problems themselves based on newspaper or magazine articles, stories or pictures, or teacher-provided data sheets (Bell & Bell, 1985; Davison & Pearce, 1988; Fennell & Ammon, 1985; Johnson, 1983; Shaw, 1983). The resulting problems can be used to exchange with other individuals or groups for assignments or tests (Bell & Bell, 1985; Davison & Pearce, 1988; Fennell & Ammon, 1985) or to create publications such as problem-solving booklets (Fennell & Ammon, 1985). This ability to make up problems indicates to students that math problem-solving can be applied to the real world (Davison & Pearce, 1988). Making games, using the "prewriting – writing – rewriting" process involves another application of math and writing (Shaw, 1983).

Creative use of language

Another way to view mathematics is as a subject that has a history and real people associated with it. Students may prepare short papers on events or people in mathematics (Davison & Pearce, 1988), or write a book report on a mathematician or mathematical topic (Burton, 1985; Esbenshade, 1983; Johnson, 1983; Schmidt, 1985; Williams & Mazzagatti, 1986). Preparing project reports or research papers involving weather, sports or in-depth independent research lends a real-world application with a creative angle to math class (Davison & Pearce, 1988; Henrichs & Sisson, 1980; Lipman, 1981). Having students orally report and discuss their work allows students to synthesize their findings and share and communicate this with peers and teachers (Davison & Pearce, 1988; Geeslin, 1977; Schmidt, 1988). Student-developed peer evaluation with teacher guidance helps students be involved in the critical elements of evaluation (Burton, 1985).

A study of the history of mathematical language encourages vocabulary development and spelling as well as contextual meaning (Henrichs & Sisson, 1980). Cloze activity math problems, where the first two pages are given, and the student completes the last page him/herself encourages inductive, divergent thinking (McKim, 1981).

Conclusion

Sterrett (1990) says that for many mathematicians, "the first step" in getting students to write about mathematics is the most difficult. Often, mathematics teachers do not feel "qualified" to work with student writing. Students, in turn, have mixed reactions to being asked to "write" in math class. Sterrett (1990) cites Hersch, another mathematician, "...The day will come, I believe, when the value of writing to learn will be universally acknowledged."

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PATTERN

Mhairi (Vi) Maeers and Rick Seaman

"Pattern is all around us" and "the world is composed of many intricate patterns" are expressions we sometimes hear from students and colleagues. Pattern is certainly very much a part of our everyday lives, but it is the recognition and application of pattern, which enables us to survive more than the fact that pattern 'exists'. One may even wonder if pattern does exist outside our personal interpretation, understanding and application of it. In this introduction to the latest issue of Ideas and Resources for Teachers of Mathematics we would like to introduce the concept of pattern through some ideas from the literature and through a recent personal 'pattern' experience.

Background

Mathematicians have been described as makers of patterns of ideas (Billstein, Libeskind & Lott, 1993, p. 4) and mathematics has been considered the classification and study of all possible patterns (Sawyer, 1963, in Orton, 1993, p. 8). Patterns created by one mathematician may well lay the groundwork for another (Polya has stated that Rene Descartes' ideas helped him with his work on problem solving). Searching for and identifying patterns has long been considered a very important problem solving heuristic. Determining patterns is crucial to police investigation, to household plumbing problems, to young children's daily routines. Anything 'out of the pattern,' for example taking a nap before lunch instead of after lunch can throw a young child 's day askew. We tend to live by personally 'inflicted' patterns in order to survive the stress of daily life. Once something becomes a commonly accepted pattern, then it no longer needs to be calculated; one has established order out of what may appear chaotic—but we know now that chaos has an implied order of its own.

The ability to perceive pattern is very much at the core of mathematical understanding. For the very young child, the ability to sort objects such that all the buttons are in one pile and all the pennies are in another demonstrates an early notion of pattern. Collections of objects, such as stamps, hockey cards, stickers and shells signify to the collector (and the observer) an understanding of what makes a stamp a stamp, or a shell a shell. In other words shells have certain unique properties that make them distinct from stamps, or stones etc. It is the identification of these unique properties or attributes that set objects apart in groups. In pre-kindergarten classrooms, kindergarten and the early grades it is quite common to find children working with sets of objects, sorting, classifying, ordering, lining them up in one-to-one correspondence, making bead strings, acting out patterns, and then using these same objects for early number development, early work with operations and with graphing. Working with sets and the attributes of sets can consume a large percentage of mathematical activity in the early grades. As Cathcart, Pothier, and Vance (1997) indicate "the recognition of patterns is a basic skill that enhances the development of mathematics concepts" (p. 63). They also state that concrete experiences with pattern should precede working with number patterns.

The ability to determine, recognize, and apply pattern, if begun with relevant concrete experiences, will be easily transferred to working with number patterns (e.g., multiplication tables, equivalent fractions, commutative property, operational relationships), problem solving, such as looking for a pattern as a solution strategy, and into algebra, which focuses on the ability to identify, extend, and create patterns with unknowns and variable expressions. Number pattern and mathematical relationships (such as missing addend problems) can be used to help children

Maeers, M., & Seaman, R. (1998). Pattern.
Ideas & Resources for Teachers of Mathematics, 7, 2-5.

understand the concept of variables and the use of numbers instead of (or with) letters (e.g., 5 + x = 7).

In algebra word problem-solving, the use of worked-out examples highlights by means of pattern recognition the underlying structures of problems. For example, if students study the worked-out examples of proportional reasoning questions whose surface features are all different, the time taken to study these worked-out examples would be less than solving the equivalent problems and the pattern with respect to the method of solution would be evident. This notion of surface feature and underlying (pattern) structures can be found in an article by Robins Shani and Richard Mayer (1993), "Schema Training in Analogical Reasoning."

Pattern Resources

The National Council of Teachers of Mathematics [NCTM] (1989) in *The Curriculum and Evaluation Standards for School Mathematics*, outline two standards for working with patterns. Standard 13 for grades K-4 (Patterns and Relationships) states that the "mathematics curriculum should include the study of patterns and relationships so that students can

- recognize, describe, extend, and create a wide variety of patterns;
- represent and describe mathematical relationships;
- explore the use of variables and open sentences to express relationships" (p. 60).

For more information about this standard please see the following website:
http://www.enc.org/reform/journals/ENC2280/nf _28060s13.htm

Standard 8 at the grades 5-8 level (Pattern and Functions) informs us that "the mathematics curriculum should include explorations of patterns and functions so that students can

- describe, extend, analyze, and create a wide variety of patterns;
- describe and represent relationships with tables, graphs, and rules;
- analyze functional relationships to explain how a change in one quantity results in a change in another;
- use patterns and functions to represent and solve problems" (p. 98).

For further information on this standard please see the following website:
http://Vt'Ww.enc.org/reform/journals/ENC2280/nf_28098s8.htm

There is no specific pattern standard at the grades 9-12 level, but Standards 5 and 6 found in the table of contents for the on-line Standards document indicate a need for a fundamental understanding and application of pattern. These standards can be located at: http://www .enc.org/reform/jou rnals/£NC2280/n f 280dtocl.htm.

From 1991-93 the NCTM published two Addenda Series documents, which concentrated on *Patterns (Kindergarten through Grade 6)* and *Patterns and Functions for Grades 5-8*. Both of these documents present many practical activities in which the concept of pattern plays a major role and is used in application to other mathematical concepts.

A delightful book which we recently discovered is entitled *Making Patterns* (1992), a Scholastic publication, written by Helen Pengelly. This book focuses on pattern and presents many practical pattern tasks for young children to engage in. Each task is accompanied by illustrations of children constructing the pattern activity, a running commentary on the process and product of children's work, and ideas for teacher observation and follow-up work. Pengelly writes, "the potential for experiences with pattern to develop mathematical understanding is profound. In fact, pattern can feature as a prominent and unifying characteristic of the curriculum" (p. 4).

Pattern Websites

- Border Pattern Gallery: http://www.math.okstate.edu/-wolfclborderlborder.html

- •Grade 1 Pattern Activity: http://www.dpi.state.nc.us/Curriculum/Mathematics/Mth.LssnPlns/Mth.l.3.4
- •Meteorology – Does Weather Happen Randomly: http://www.cl.ais.net/rlevine/Weathr20.htm
- •The Language of Mathematics: http://www.math.montana.edu/-umsfwest/
- •L-systems-Fractals: http://life.csu.edu.au/ complex/tutorials/tutorial3.html
- •What is a Fractal?: http://www glyphs.corn/art/fractals/what_is.html
- •The Golden mean: http://galaxy.cau.edu/tsmith/KW/golden.html
- •Penrose Tilings and the Golden Mean: http://galaxy.cau.edu/tsmith/KW/goldenpenrose.html
- •Origami Tessellations: http://www.cea.edu/sarah/chris/
- •Pattern Pals: http:/ /cust3.iamerica.net/anyroddy/
- •The Number Bracelet Game: http://www.geom.umn.edu/-addingto/number _bracelets/number bracelets.html
- •3-D Modeling, Fabrication and Illustration: http://www .sover.net/-tlongtin/
- •Tessellations Project (and links to other tessellation sites): http://www.inform.umd.edu/UMS+State/UMDProjects/MCTP/Technology/School_ WWW Pages/Tessellations/TessellationHomePage.html
- •Puzzles Involving the Fibonacci Series: http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibpuzzles.html
- •Teaching Strategic Skills: http://acorn.educ.nottingham.ac.uk/ShelJCent/PubList/tss/
- •The World of M.C. Escher: http://lonestar.texas.net/-escher/
- •Classification of Patterns: http://www.geom.umn.edu/-demo5337/Groupl/

You can view many more pattern sites at the Math Forum → Steve's Dump at http://forum.swarthmore.edu/-steve/. Simply click on "Quick Search" and enter the word pattern. The sites listed above are ones that we visited and found useful as a result of this search. At our own Math Central website [http://mathcentral.uregina.ca/], there are three resources that focus on pattern. They can be found in the Resource Room/Keyword/Pattern.

A Pattern Project

Pattern can be interpreted through all disciplines and can connect all disciplines, an idea similar to the last sentence in the above quote by Pengelly. Just recently a group of five elementary education professors worked with 60 third year students on a semester-long curriculum integration project. The focus of study was PATTERN and our task was to: (1) determine how pattern could be represented and interpreted through each discipline [mathematics (Vi Maeers and Liz Cooper), language arts (Carol Fulton), social studies (Kathryn McNaughton), and the arts (Nancy Browne)], (2) demonstrate to our students our collective perspectives on pattern (through a Hyperstudio presentation), (3) take our students through a variety of pattern-focused activities within each subject area, and (4) work with groups of children in a local elementary school to help them understand different ways of thinking about pattern.

At the beginning of our project (September, 1997), as each member of the curriculum project team tried to define pattern it became clear that each of us had a different definition and indeed quite different ways of thinking about pattern within our separate subject areas. This is in agreement with Tahta, 1992, who stated that the word pattern was used differently in different contexts. Dictionary definitions did not help us reach a consensus. We decided to develop a 'composite' of our definitions and present them to students within the framework of a multimedia (Hyperstudio) project. As each of us worked on this project, collaborating constantly with at least one other member of the team, and talking through our examples to understand how our examples illustrated our sense of pattern within our subject area, we actually began to agree on some big ideas surrounding pattern and extensions from those big ideas.

We began with a statement like "pattern extends from a 'design' such that its composite parts can be distinguished as separate from its surroundings; a unity in time, shape, place." This statement was then illustrated through a mathematical example, a social studies one, a language arts one, and an arts one. Our next big idea, "pattern is a design altered by transformation (translation, rotation, reflection)," was also illustrated by examples from each subject area. Another big idea was that "pattern can be an extension of an original design, a replication of it, or a recursion of it." Our last big idea was that "pattern can be a disruption of an original design" but explicit in this idea is the need to be able to distinguish the original design to be able to perceive the disruption. This last big idea leads us into Chaos Theory and Fractals – an interesting and challenging notion to illustrate in each subject area.

The big ideas outlined above were identified and examined within each subject area first and then, by extrapolation, across the subjects. Our intent was for our students to understand these big ideas and how they could be represented in each of the above subject areas. The mathematics part of the project highlighted tessellations and Escher patterns, social studies highlighted where and how people live in different parts of the world, language arts focused on story and word patterns, and arts education focused on patterns in song and action. Each part of the Hyperstudio presentation highlighted a slightly different focus, but the big ideas were the same. Our Hyperstudio project will soon be available on Math Central – so watch for it. Our project and this introduction close with the following words:

> "An ability to see pattern in our world provides us a base to predict when things do and could change, provides us with a foundation of (and for) order, and provides us with an ability to see boundaries in space, place and time. We use our ability to perceive pattern to enable us to make sense of things in our world, to understand society better."

We hope that as you read this latest issue of *Ideas and Resources for Teachers of Mathematics* you will find suggestions for activities at different grade levels that enable both you and your students to explore both the process and product of pattern.

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I'VE GOT A SECRET: MATH ANXIETY

Rick Seaman

I have taught high school mathematics for 23 years and have also instructed undergraduate mathematics at the university level as a sessional lecturer for 18 years. One of the courses I have taught is Math 101, a degree requirement class for general arts and elementary education. In my first time teaching this class as a six-week summer course, I asked my wife (an elementary school teacher) who I might find in this group. She told me to expect a significant number of "middle-aged female elementary school teachers" taking this as their final requirement for their degrees. (Over the years, teachers here have been required to have a degree to teach, with many taking evening and summer courses.) When I came home from the first class session, my wife asked me who was in the class. Amazed, I said she had been correct. There was an extraordinarily high number of "middle-aged female elementary school teachers" – how did she know? She paused thoughtfully and said, "Be kind. They are terrified."

My curiosity piqued, I asked the students their feelings. My wife was right. They were terrified. Repeated teaching of this course, with the same experience each time, led me to interview three female elementary school teachers in depth, and resulted in 'I've Got a Secret: Math Anxiety'.

Alice, Barb and Chris (not their real names) are elementary teachers each with over 20 years of teaching experience. All three women are highly intelligent, articulate, and painfully aware of their limitations in mathematics; however, they have become equally aware of their strengths in teaching mathematics within the constraints of grade level or concept level. Chris remembers doing "really practical problems" in arithmetic with her grandmother that were "fun." Chris adds that after arithmetic math "was never practical. I had this idea, you kind of just memorized it, answers came from the air, or God knows what. I never figured it out." Alice found mathematics after arithmetic limited in applicability and the subject seemed impersonal. She felt no "bond" with mathematics, saying, "it was not something I was particularly interested in." Barb also saw no meaningful application; math to her was merely being given an assignment, and then "they (the math teachers) sat down." There was little connection between the math's symbolic language and the real world. "You get to a test and it all looks like Greek to me," said Alice, "They're just things that float in the air. You get through it, and repress it for life." It was apparent that once past the arithmetic stage, and into the more abstract aspects of the mathematics I stage, these women no longer saw the applicability of what was expected. Alice states: that "if you don't pass this, you don't get your year- well, do you know what that would mean? I mean, that has got to be the ultimate, the ultimate embarrassment personal failure."

That they had received poor teaching was a strongly held conviction by all three women. Alice believes her difficulties with math began when, as a "gifted student" she was involved in a pilot project where, "we literally had to teach ourselves." Barb describes her teachers as "very, very poor math teachers ... I don't remember any teaching." Chris wonders if she had good teachers, would it answer for her the question, "Whether I am really learning disabled in math, or stupid, or what? Looking back I don't think my teachers knew how to teach math."

Seaman, R. (1998). I've got a secret: math anxiety.
Ideas and Resources for Teachers of Mathematics, 10, 2-4.

Teachers alone are not responsible for any emotion underpinned by qualities of fear and dread in mathematics, an emotion that has been referred to as mathematics anxiety. Perhaps changing classroom practices to textbooks and curricula that encompass more issues that are relevant to adult daily working, living, or story plots that capture human interest would reduce math anxiety for students.

Over the course of one week, Alice, Barb and Chris, three "math anxious" primary level female teachers, were interviewed concerning their perceived math anxiety. The first two interviews took place at the home of the interviewer and the other in the individual's classroom after school. These places were determined by the interviewees. Each interviewee was told that the interviewer had a genuine interest in knowing more about math anxiety. They were also told that a transcript would be made and that they were guaranteed anonymity. The interviews developed from general conversations led by the interviewer. Each interview lasted approximately 20 minutes.

The three interviews turned out to be one with a person I knew well, one with a person I knew not as well, and one with a person I did not know. All three women had a genuine concern to analyze their math anxiety, to understand it better, which ultimately made the interviews flow- because they actively participated. The introspection involved appeared to be a useful exercise for all three women and all expressed their appreciation of the invitation to express their thoughts and views on math anxiety.

The reason for selecting female primary teachers was through a desire to narrow the research, to hear what they would say about their anxiety 'then and now'; to hear what impact math anxiety had on their career choices and how it affected their careers, and to see if any of these women believed that their math anxieties were directly related to gender. I was interested in what insights I could glean concerning math anxiety as a high school math teacher. The interviewees understood the objectives and what would be done with the data collected and the resulting transcripts. Each participant received a copy of her interview plus a summary of the themes found in the interviews.

Math has often been treated as a solitary subject – one relegated to working in relative personal isolation. Alice described that as an adolescent, you feel alone in your shortcomings and "you run around trying to hide it." Barb said she would "never think to ask (the teacher) anything." While students working in groups could help address these concerns, the role of the teaching in reducing math anxiety would not diminish.

A supportive teacher who provides a healthy classroom climate is crucial to developing positive attitudes toward mathematics. Ensuring acquisition of prerequisite skills, teaching for understanding and using math when teaching other subjects are among suggestions for preventing math anxiety. Also, the creation of a climate where it is acceptable to both question and make mistakes might help take the fear out of mathematics. A healthy climate that allows for errors and questions might have negated the kind of embarrassments and traumas described by these women.

Events and feelings from elementary school days are subject to vivid recall. These memories are continually linked to attitudes to mathematics and teachers. Alice, Barb and Chris expressed clear memories of specific happenings not usually successful with learning, or lack of it, and teachers. Alice remembers not knowing what to do in the pilot program; Barb remembers a World War II veteran, shell-shocked, shuffling, non-verbal teacher, with mouth agape; and, Chris still remembers the embarrassment of being told that a younger boy in another class could do something in math that she and the rest of the girls in her class were unable to do.

Seaman, R. (1998). I've got a secret: math anxiety.
Ideas and Resources for Teachers of Mathematics, 10, 2-4.

Conversely, good experiences are vivid memories, as are good teachers. Alice recalls the "best teacher in the whole world" as an applied statistics teacher in university, who was positive and supportive and made math make sense. Barb remembers more of the contrast between her math teachers and her other teachers. Chris remembers a university professor who tutored her free all summer so she could pass her test. When she did, they both cried. Chris still keeps in touch with her. Mathematical success appeared to be influencing the career choices for these women.

Math avoidance may be seen as a "cause as well as a symptom" of math anxiety. Math avoidance leads to limiting careers to those that neither require mathematics training nor its use. All three women acknowledged that careers were, to some degree, determined by their avoidance of mathematics courses out of necessity or fear. Alice avoided grade twelve physics, which did not allow her entrance into any careers that require physics. Barb found that Teachers' College was the only admission she could get without grade 12 math. While she feels she would still have selected education as a career, she would likely have attended a different college if attended a different college she had math. Chris would have chosen architecture had she not avoided math in her British education.

Fear of the tests in mathematics seemed to be part of the avoidance aspect of math anxiety. When Alice had to do a math test, she took her fifty percent with relief. Chris panicked when she had to do a mastery test and achieve ninety percent to pass. Barb couldn't figure out how or why "other kids managed to pass." There appeared to be a general fear of contact with mathematics, including classes, homework, and tests.

An important concern regarding math anxiety is whether female elementary teachers pass their anxiety on to their students, particularly female students. Is gender bias the reason for math anxiety and would remedial math programs alleviate anxiety and break the cycle? Or does math anxiety beg attention across gender?

Having acknowledged math anxiety, the three female elementary school teachers indicated they are not only comfortable teaching mathematics but often empathetic with students and indicate they are better than average at teaching math. They seemed eager to break the cycle of poor attitudes generating poor attitudes and provide their pupils with positive experiences in learning mathematics. They all stated, however, that they have a limit to what they are able to teach in mathematics, and generally, the delineation is high school. Alice will not work in high school because she feels she does not have the background in mathematics; Barb says she would be "basket case" if she had to teach high school math; and Chris says she is happy teaching elementary math because she knows her skills are above those of her students. Alice, Barb and Chris had found their 'comfort zone' for teaching mathematics. In response to the constraints math anxiety places on teachers' comfort zones, all three women suggest that for those who must teach mathematics, methods or support groups would be of minimal help.

Barb believes she has become a particularly good math teacher and suggests that part of the reason is that her initial insecurity about teaching math caused her to work and prepare extra carefully for it. She believes she is creative in her math teaching and not bound by the textbook sequence. She also finds that she never gets frustrated when students "don't get it." She just approaches their difficulty from another angle. Alice believes she is a more empathetic teacher, and she is careful to teach children the "system" of mathematics – the prerequisite knowledge base needed for future mathematics. Chris is careful to attend workshops to make her a "better" math teacher. She it is important to make children think and consider all possibilities. These teachers, while displaying residual effects of math anxiety, are quite confident within their "comfort zone.

If one was searching for a connection between gender and math anxiety, it was not evident in this study. Much has been made of the connection between women and math anxiety, but the only difference between male and female math anxiety may be that females are more likely to report it, perhaps, because it is "safer" for women to admit to math anxiety than for men.

Not once did any of these women suggest that they did poorly or avoided mathematics because of the fact that they were female. It does seem, though, that math anxiety takes its place permanently if nothing is done to modify it. Alice says, "Math anxiety stays with you forever: It's that little bit of you that thinks that you've fooled the world. But if anybody dug deep enough, they'd find you are really stupid in math. It is sort of always playing 'I've got a secret'."

"Today I know it exists," states Alice, "it's an avoidance thing is what it is. But at least I recognize it for what it is. Unfortunately, I think when you are an adolescent it's a very personal thing, but I believe now if I worked on it now I would get it...." Barb sees her math anxiety as a habit, an attitude. "Like smoking it was really cool – now I'm stuck with it. It's no longer useful. Same with math, it's no longer useful to me at all to be a dunce at math. But I'm stuck with it to a certain extent." Barb has stayed clear of situations where her math anxiety would show but today she states "I think it is unthinkable to kids now to let opportunities pass by them like I did." Chris concludes. "I think I'm still math anxious, but it's not quite as bad." She has joined the new mathematics committee and "it hasn't been too bad, but once in a while we have these activities we have to do and I've felt like a real nincompoop."

There appears to be no definite origin for math anxiety. Some argue biochemical brain differences; some behavioral responses; and, good teaching seems to be able to eliminate it. Perhaps the first step for educators is to become more aware of math anxiety and its relevance to good teaching. Eliminating math anxiety may reduce the secrets students keep about their lack of mathematical prowess and may open more doors in the future.

Seaman, R. (1998). I've got a secret: math anxiety.
Ideas and Resources for Teachers of Mathematics, 10, 2-4.

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THE FIBONACCI SEQUENCE

Rick Seaman

History

Leonardo of Pisa (1175-1250) better known as Fibonacci is one of the few mathematicians in the Middle Ages to be mentioned today in mathematics courses. This is, of course, not a mere coincidence. The science of mathematics developed extremely slowly during that period, and there were very few noted mathematicians.

In 1202 (his second version written in 1228 exists today) Fibonacci wrote Liber Abaci ("Book of the Abacus"). The book is a comprehensive work containing almost all the arithmetic and algebraic knowledge of that day. Since Fibonacci received his early mathematical training from Moslem tutors, he quickly recognized the superiority of the Hindu-Arabic decimal system, with its positional notation and zero symbol, over Roman numerals. So through this book he defended the merits of Hindu-Arabic notation. Although this book had little influence on merchants in his native Italy, it played an important role in the development of mathematics in Western Europe in the course of several centuries that followed.

In the l9th century a French number theorist, Edourd Lucas, is credited with naming the number sequence that appears in the following problem in Liber Abaci, the Fibonacci sequence.

> The number sequence 1,1,2,3,5,8,13,21… is called the Fibonacci Sequence. Each number is generated by adding the two previous numbers. For example, 8 + 13 = 21 and the next number, 34, would be the result of adding 13 + 21.

PROBLEM – Gardner (3): "Suppose," Leonardo wrote, "a male-female pair of adult rabbits is placed inside an enclosure to breed. Assume that the rabbits start to bear young two months after their own birth, producing only a single male-female pair and that they have one such pair at the end of each subsequent month. If none of the rabbits die, how many pairs of rabbits will there be inside the enclosure at the end of one year?" (p. l16).

Complete the following table:

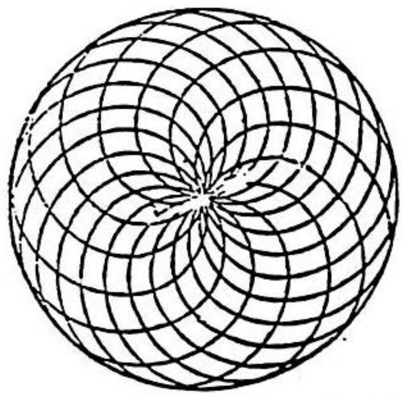
MONTH	RABBIT PAIRS BRED DURING MONTH	PAIRS OF NONBREEDING RABBITS	PAIRS OF BREEDING RABBITS	TOTAL # of PAIRS AT END OF MONTH
1	F1 = 1	1	1	F3 = 2
2	F2 = 1	2	1	F4 = 3
3	F3 = 2	3	2	F5 = 5
4	F4 = 3			F6 =
5	F5 = 5			F7 =
6	F6 =			F8 =
7	F7 =			F9 =
8	F8 =			F10 =
9	F9 =			F11 =
10	F10 =			F12 =
11	F11 =			F13 =
12	F1 2=			F1 4=

What is so remarkable about this table?

Fibonacci is Everywhere!!

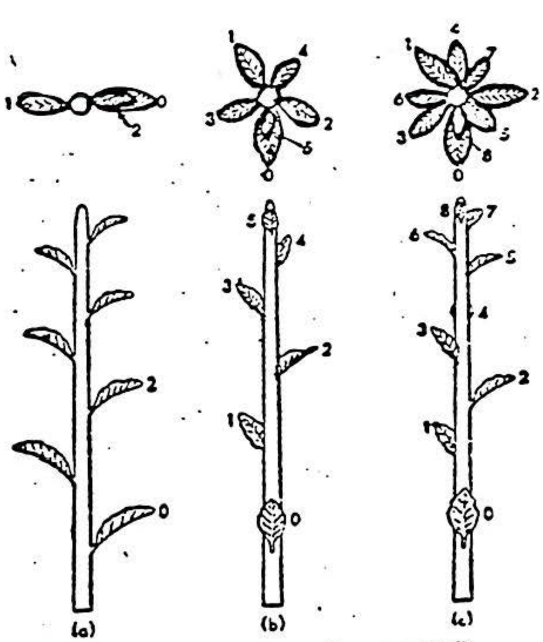
The number of spiral floret formations visible in many sunflowers, spiraled scales on a pinecone and segments on the surface of a pineapple have been found to match Fibonacci numbers.

Seaman, R. (1999). The Fibonacci sequence.
Ideas & Resources For Teachers of Mathematics, 10, 12-16.



Distribution of seeds in a sunflower head

Northrop (8) examined the arrangement of leaves- buds, branches- on the stalk of a plant. “Fix your attention on some leaf near the bottom of a stalk on which there is a single leaf at any one point. If we number that leaf 0 and count the leaves up the stalk until we come to one which is directly over the original one, the number we get is generally some term or other of the Fibonacci series”(p.55). Again, as we work up the stalk, let us count the number of times we revolve about it. This number, too, is generally a term of the series.



The arrangement of leaves on a stalk

(a)

(b)

(c)

(a) Northrop (8) shows a ½ spiral where 1 represents the number of revolutions and 2 the number of leaves.

(b) can be called either a 2/5 or 3/5 spiral, depending upon whether, looking down from the top, we wind about the stem in clockwise or counter-clockwise fashion. To fix our ideas, we shall agree to take the longer path and call this a 3/5 spiral. Then the arrangement shown in

(c) is a spiral – not a spiral.

Such arrangements are expressed by the following ratios: 1/2, 1/3, 2/5, 3/8, 5/13, 8/21, and so on. The meaning of this notation is two-fold. As before, 3/8 means that it takes three revolutions and eight steps to get to the leaf vertically about the zero leaf. Or another way of thinking about this would be: Since it takes eight leaves to cover three revolutions, the average angle of rotation in passing from one leaf to the next is 3/8 of 360°. Broussear (1).

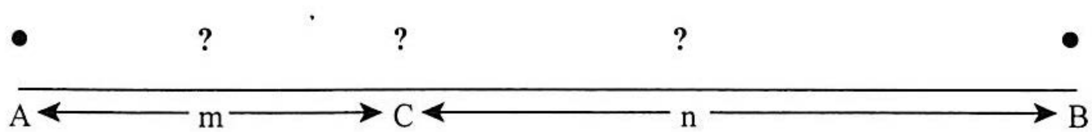
If we make a table of angles corresponding to the various cases we have:

PHYLLOTAXIS TYPE (PT)	ANGLE° = (PT) 360°
1/2 (0.5)	180°
1/3 (0.33…)	120°
2/5 (0.4)	
3/8 (0.375)	
5/13 (0.38461)	
8/21 (0.38095)	
13/34 (0.38235)	

We seem to be approaching a PT of _______ and an angle of ______.

The Divine Proportion

In geometry, the golden section was the name given in the 19th century to the proportion derived from the division of a line into what Euclid called "extreme and mean ratio". For example, consider line segment AB and the point C.



The division required is affected by choosing C so that the ratio of AC to CB is the same as that of CB to AB.

$$mn=nm+n$$

Let $x= nm$, then $x=1x+1$ and $x2-x-1=0$ and $x=1\pm 52$ therefore $nm=1+52$ = τ and n = τm, a direct proportion.

Lucan Picioli (1445-15 14) wrote in 1509 that the golden section (divine proportion) has various mystical properties and exceptional beauties both in science and in art. People consider the rectangle whose sides are in this approximate ratio to be most pleasing to the eye. 3 by 5 and 5 by 8 cards and postcards are examples. The golden rectangle has been found in the architecture of ancient Greece and in other works of art. The Parthenon at Athens and Da Vinci's "St. Jerome" are two such examples.

A fascinating aspect of the Fibonacci sequence is the fact that if we take the ratio of consecutive terms, the value gets closer and closer to τ.

n	F_{n+1}/F_n	n	F_{n+1}/F_n
1	1/1=1	2	2/1=2
3	3/2=1.5	4	5/3=1.66…
5		6	
7		8	
9		10	
		20	1.6180339985

↓

τ ≅ 1.6180339887

QUESTION: Now that you know the value of τ, what is the value of its reciprocal?

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$1/\tau =$ _________

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2014, Volume 4, No. 1 & 2, 22-26.

IN PREPARING FOR FOOTBALL GAMES, NOTHING IS LEFT TO CHANCE

Rick Seaman

The incentive to write 'In Preparing for Football Games, Nothing is Left to Chance' came from an experience I had teaching mathematics to middle years students. It was fall and I also was in the middle of coaching high school football. As a defensive coach, I was looking at the data that I had summarized from the videotapes of my next opponent's offensive plays. It dawned on me that what I was doing would be a practical application of the middle years mathematics data management strand.

So in class, I indicated to my students the different ways I had represented the data so I could get a better understanding of my opponents' offensive tendencies. Then, I went on to show them how I had applied pattern recognition, inductive reasoning and probability to predict what the opponent might do in certain situations. The class was fascinated with the conjectures I had made and could see how videotapes from their football team's other opponents could also be used for further exploration and subsequent conjectures.

Since then I have updated the representations that I use and placed them here in the context of preparing for a championship game. Enjoy the actual preparation as you see how you might apply this example to the data management strand. As an educator you will be able to identify with the multiple representations that are used by coaches (teachers) to help players (students) better understand their opponents' tendencies (concepts). This open-ended activity provides students the opportunity to explore, make conjectures and then verify them when their team plays the opponent they have analyzed. Although, I have used football as an example this activity) could also be applied to other sports. Let your imagination be your guide.

This article deals with a football team's weekly defensive preparation for a game on Saturday. The week that will be used as a template is one that led up to the Canadian Junior National Championship in 1998. Junior football players are high school graduates who are at most 22 years of age that my or may not attend university. The game is played on a 65 X 110-yard field with 10-yard deep end zones. The offensive and defensive teams each have 12 players on the field at one time, and the offensive team has only three attempts to gain ten yards in order to make a first down. As the defensive team's game preparation is discussed, the reader will recognize the different forms of representation used for constructing understanding and for communicating information and understanding between players and/or coaches (Greeno & Hall, 1997). It is helpful for the reader to imagine representation on a concrete-abstract continuum adapted from Hyde and Hyde (1991):

Physical Objects
    Bodily Actions & Movements
        Spoken Language
            Pictures, Diagrams, Charts
                Symbols
                    Written Language

Seaman, R. (2000). In preparing for football games, nothing is left to chance.
Ideas and Resources for Teachers of Mathematics, 12, 6-11.

The defensive game preparation for the week is divided into two parts: Pre-Practice and Practice. The representations, although not complete, illustrate how coaches use them to prepare the team defensively for their game on Saturday.

The Week

Pre-Practice

After each game, the defensive coaches will look at a video, filmed from both the sideline and end zone, that pictorially represents their game. This video helps the coaches assess how well the team has played and anticipate what their next opponent might believe is a defensive weakness.

Subsequently, a defensive meeting is called and the game video is projected onto a big screen. The Defensive Coordinator's concerns are then discussed with the defensive team. Following the meeting, some players will take home their own copy of the game video to further assess their play.

The next task for the defensive coaches is to look at the game video of the next opponent. In preparation for the National Final, the coaches had two game films for this purpose.

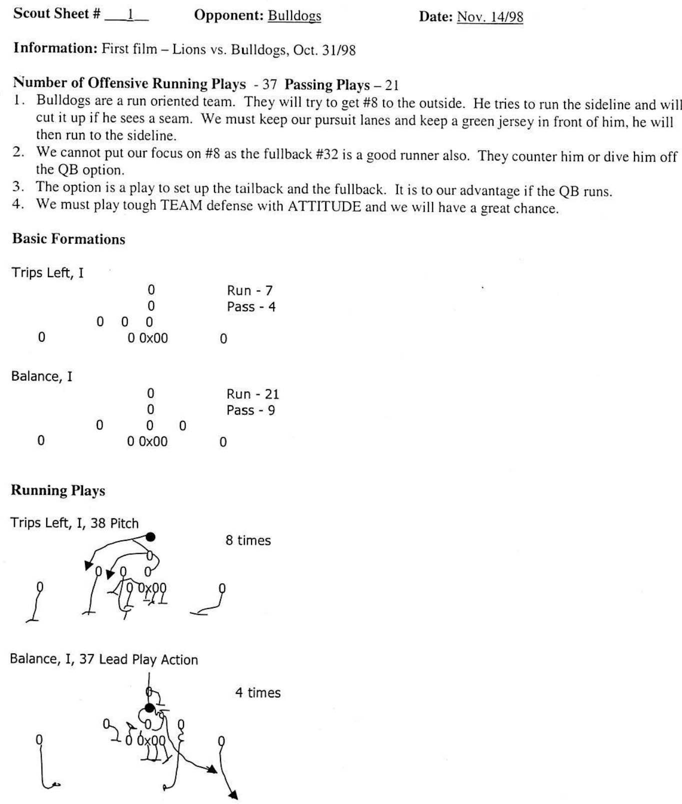
While looking at the game video, information from the game is simultaneously put into the computer by the Defensive Coordinator. One possible way of sorting this information is summarized on the chart entitled: Offensive Tendency Breakdown Sheet (Figure 1).

Down Distance	Hash	Field Position	Run Gap	Pass	Direction	Formation	Play	Gain
1/10	Right	Their 35	C		Strong Side	Trips Left, 1	38 Pitch	23 yards
1/10	Left	Our 52		Flats	Strong Side	Balance, 1	37 Lead Play Ac.	Incomplete
2/10	Left	Our 52		Deep Seam	Weak	Ace Right	Back Up Seam	Incomplete

Figure 1. Offensive Tendency Breakdown Sheet

The chart above is but one possible sort for the defensive coaches to get some idea of what the opponent is attempting to do on certain downs and distances, position on the field, and formations. One's imagination is the only constraint for what information might be analyzed. The Defensive Coordinator then summarizes the information that he believes is important to prepare for the game on Saturday on a Scout Sheet. (A partial example of the Scout Sheet is illustrated in Figure 2.)

This information is Xeroxed and passed out to all the defensive coaches and players and then discussed when they meet to watch the film of their upcoming opponent. Pass/run tendencies and the number of times they run certain plays versus the opponent's formations are discussed. Some players at this time will also obtain a copy of the video and scout the player that they will play against.



Scout Sheet # 1 **Opponent:** Bulldogs **Date:** Nov. 14/98

Information: First film – Lions vs. Bulldogs, Oct. 31/98

Number of Offensive Running Plays - 37 **Passing Plays** – 21

1. Bulldogs are a run oriented team. They will try to get #8 to the outside. He tries to run the sideline and will cut it up if he sees a seam. We must keep our pursuit lanes and keep a green jersey in front of him, he will then run to the sideline.
2. We cannot put our focus on #8 as the fullback #32 is a good runner also. They counter him or dive him off the QB option.
3. The option is a play to set up the tailback and the fullback. It is to our advantage if the QB runs.
4. We must play tough TEAM defense with ATTITUDE and we will have a great chance.

Basic Formations

Trips Left, I

Run - 7
Pass - 4

Balance, I

Run - 21
Pass - 9

Running Plays

Trips Left, I, 38 Pitch

8 times

Balance, I, 37 Lead Play Action

4 times

Figure 2. Scout Sheet

As a secondary coach, one will look at the video and represent where the ball was thrown from its position on the field on a diagram. The coach is attempting to gain a greater understanding as to what the opponent's passing tendencies are regarding their position on the field (see Figure 3)

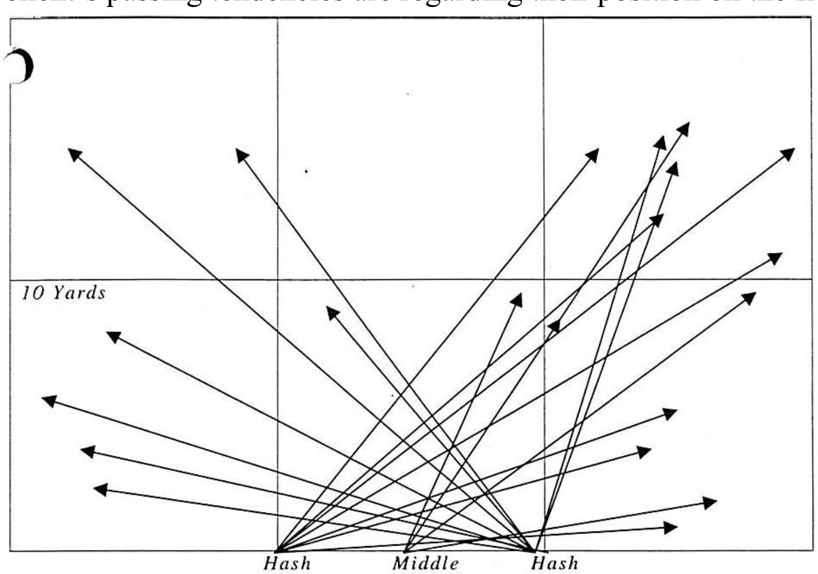


Figure 3. Opponent's Passing Tendencies Regarding Their Positions

From this, the coach can ascertain that the Bulldogs have a tendency to throw to the strong or wide side of the field. Also, when they do go the other way, the ball was thrown to the closest receiver to the quarterback out of the Ace Right formation (see Figure 4).

Seaman, R. (2000). In preparing for football games, nothing is left to chance. *Ideas and Resources for Teachers of Mathematics, 12*, 6-11.

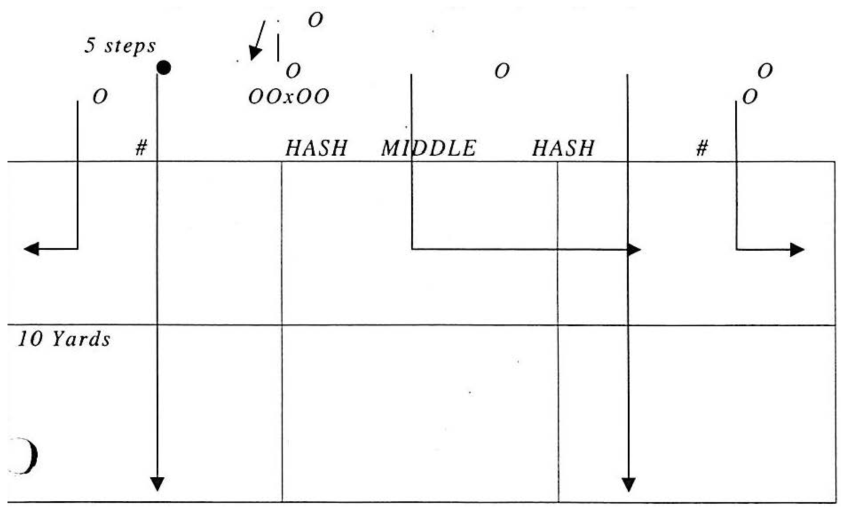


Figure 4. Back Up Seam Pass from the Ace Right Formation

Also the secondary coach can represent the opponent's pass plays with a diagram to check where the wide receivers line up widthwise (split) versus the pass play being run. For example, with a narrow split are the wide receivers running outside routes? With a wide split are they running inside routes? In preparation for this particular game, the secondary coach found a tendency from these diagrams that indicated that with a certain split one could expect an outside route. The first time the Bulldogs gave the secondary this read the player adjusted and the Bulldogs were unsuccessful. However, when another player was playing the same position later on in the game, he did not notice, and the Bulldogs had a long pass completion!

At practice, the Bulldog's pass plays are represented on diagrams and shown to the scout team who then represent the opposition 's passing offense by acting out the plays against the defense. At any time the coaches can stop or repeat a play to help the defensive backs and/or linebackers understand what the opponent is trying to do and communicate how the team is going to defend a certain route. Tendencies from the Scout Sheet and the secondary coach's diagrams are reviewed with the defensive backs at this time. After practice, these players are asked to use the Scout Sheet running and passing plays to mentally review their defensive assignments and reactions.

From the computer analysis the Defensive Coordinator represents running and passing plays that are of concern to him and on what down and distance the defensive team might expect them on diagrams. The Defensive Coordinator also makes up his preliminary Defensive Ready Sheet of defenses he will use in certain Situations the game on Saturday (see Figure 5).

DEFENSIVE READY SHEET vs. Bulldogs Nov. 14/98

Single Blitzes	Stunts	1/10	2/7+	2/4 –6	2/-3	3/-3	3/1	Inside Our 20
Mac	Ox	Flip Red	Load Green	Alberta Red	Alberta Silver	Load Purple	Monster Silver	Sting Purple
Twist	Loop	Base Bleu	Under Gold	Under Silver	Base Brown	Jumbo Red	Lightning Purple	Crash Brown

Figure 5. Defensive Ready Sheet

Mac, Ox, Flip, Load, Alberta, Monster, Sting, Twist, Loop, Base, Under, Jumbo, Lightning and Crash tell the seven people closest to the ball (linemen and linebackers) where to line up and how to play their position. The colors blue, brown, gold, green, purple, and red tell linebackers and defensive backs what their pass/run responsibilities are for that particular play.

The Defensive Ready sheet is used next to make a Script to be put to use daily at practice during different defensive periods (see Figure 6).

SCRIPT vs. Bulldogs Date: Nov. 12/98

Down/Distance	Formation	Play	Defense/Blitz	Play No.
1/10	Trips left I	38 Pitch	Flip, Red	1
2/7+	Ace Right	Back Up Seam Pass	Load, Silver	2
1/10	Balance I	37 Lead Play Action	Load, Green/Slam	3

Figure 6. Script

The combination of representations, Scripts, and using diagrams of plays maximizes the use of practice time. A scout team can use the play diagrams to act them out against the linebackers and defensive linemen during the scripted inside run period, against the linebackers and defensive halfbacks in the scripted pass skeleton, and also the defense during the scripted team defense period. The Script facilitates the players' and coaches' communication and understanding of their opposition.

Summary

Each week, the defense has at most four practices of one hour and 45 minutes in duration to get ready for their next opponent. Multiple representations from concrete-abstract like acting out opponent's plays, meetings, videos, diagrams, charts, symbols and written language are all used to facilitate the coaches and players communication and understanding. Thus, when it comes to representation and preparing for football games, nothing is left to chance.

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REPRESENTATION: TWO VIEWPOINTS FOR CLASSROOM COMMUNICATION

Rick Seaman

It is helpful for teachers and students to imagine representations on a concrete-abstract continuum (fig. 1.1); however, the different forms of representations should not be looked upon as the primary focus of instruction (National Council of Teachers of Mathematics, [NCTM] 1998). Such an approach to instruction limits their value in the teaching and learning of mathematics. This article discusses two viewpoints of how representation might be seen as a tool for communication: first, as one that supports students' mathematical thinking and second, one that guides teachers' teaching of mathematics.

Manipulatives
↘
Real World Scripts
↘
Spoken Language
↘
Pictures/Diagrams/Charts
↘
Written Language
↘
Mathematical Symbols
↘
Mathematical Formulas

Figure 1.1. Concrete-Abstract Continuum

Students' Mathematical Thinking

Representations can be used to facilitate students' mathematical thinking. This section discusses some implications representations have on problem solving, and inductive and deductive reasoning.

Problem Solving

Representing the problem is embedded in most problem-solving strategies when students are reading the problem for analysis and are searching for a solution. Consequently problem solving is an opportunity for each student to represent problems somewhere on the concrete-abstract continuum. For example, consider the following problems:

Problem One: Ten trees are to be planted in exactly five rows with four trees in each row. How can this be done?

Problem Two: Lee is able to bench press at most 260 kg not including the bar. How many different ways can the weights – 5 kg, 10 kg, 25 kg, and 40 kg – be placed on the bar if it will hold up to seven weights on each side while keeping the bar balanced?

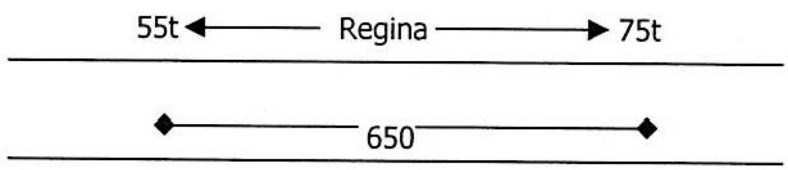
Problem Three: Take a piece of paper and tear it in half. Place the two pieces of paper on top of one another and tear them in half. This process yields four pieces of paper. Place them on top

Seaman, R. (2001). Representation: two viewpoints for classroom communication.
Sum Thing to Think About, May 2001, 13-21.

of one another and tear them in half again. Imagine continuing this process through 25 such tears. How many layers will there be in the pile of paper?

Problem Four: Two cars leave Regina traveling in opposite directions, one going 75 mph and the other 55 mph. When will they be 650 miles apart?

Most students would consider using a diagram to represent the first problem and a chart for the second problem. It should be noted that students' representations are not unique and that students should be encouraged to discuss their representations with others. However, these two examples were deliberately chosen to hopefully provide a consensus with students regarding their choice of representation. If students classify their knowledge according to what they believe is most useful in helping them solve a problem then they might classify the latter two problems as diagram and chart problems respectively. Some students might consider the third problem to be a diagram or chart problem. However, most students would actually take paper and perform the tears implying the importance of the manipulative and refer to it as a manipulative problem. The fourth problem highlights the importance of diagrams in making understanding more profound and deeper. Let t be the time in hours that each car is travelling, then the diagram for the problem would look something like:



The diagram can be represented in written language that can be translated into mathematical symbols. Consequently this problem can be classified according to being a diagram problem or a 'whole equals the sum of its parts' problem since the distance one car travels plus the distance the other car travels equals the total distance traveled by the two cars.

These classifications maybe referred to as the underlying or deeper structure of the problem with respect to that student. Since the classification depends on how students use their knowledge, deeper structures are not unique. Students using relevant representations to classify their problems become more expert-like in their thinking and minimize the amount of material that has to be recalled.

Inductive Reasoning

Inductive reasoning might be seen as part of the mathematical thinking process in which information about some members of a set is used to make a conjecture about other or all members of the set (O'Daffer 1990). For example a teacher might ask his/her students, "1, 3, 5, 7, what is next?" The students should reply, "9". The teacher replies, "Correct, that is an example of inductive reasoning. You made a conjecture that was based on a limited amount of information." The teacher then asks "2, 4, 6, 8, what is next?" The students answer immediately "10". To which the teacher replies "Wrong! It is: 'Who do you appreciate? – Me'". The attempt at humor is employed by the teacher to demonstrate that there is no certainty when using inductive reasoning.

Representations can help the students to form conjectures. For example, students are asked to evaluate this expression:

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$$

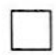
and generalize the result.

The student might try to apply a 'simpler but related problem' strategy and represent the 0 problem with mathematical symbols.

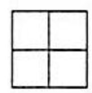
$$1 = 1$$
$$1 + 2 + 1=4$$
$$1 + 2 + 3 + 2 + 1=9$$
$$1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$$

This student inductively might conclude the answer is 121 since the answer to the question appears to be the square of the middle term. Such students might even go further and generalize: $1 + 2 + 3 + 4 + ... + n + ... + 4 + 3 + 2 + 1 = n^2$.

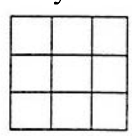
For others, they might draw a diagram or use squares for each situation. 1 = 1 x 1 would be represented by a one by one square of area one.



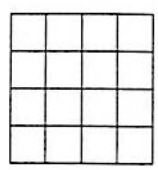
1 + 2 + 1 = 4 = 4(1 x 1) would be represented by four one by one squares of area one.



1 + 2 + 1 = 4 = 4(1 x 1) would be represented by four one by one squares of area one.

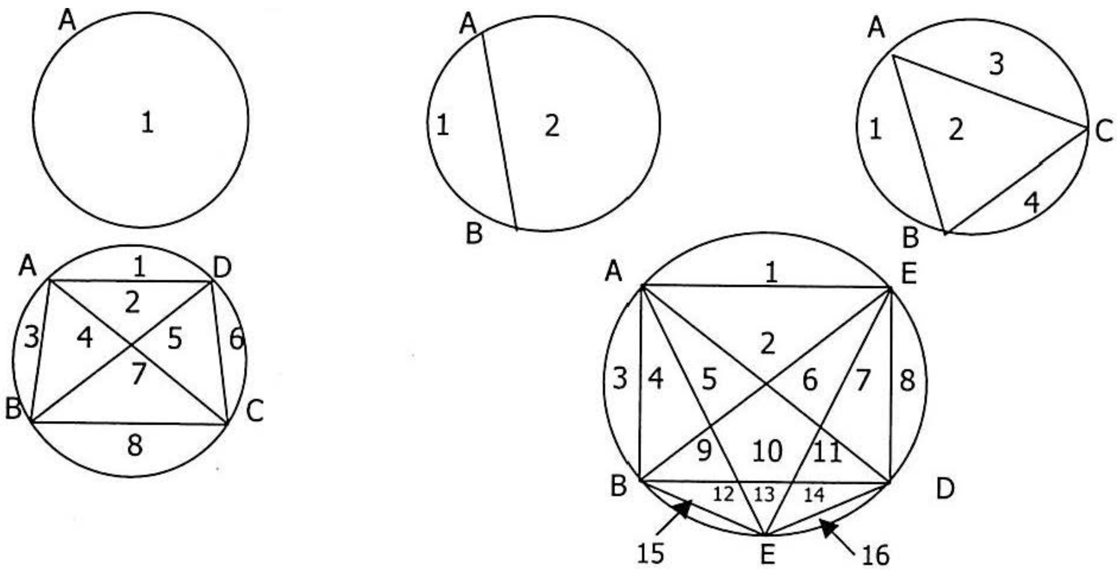


1 + 2 + 3 + 4 + 3 + 2 + 1 = 16 = 16(1 x 1) would be represented by 16 one by one squares of area one.



This student might inductively conclude the answer is 121 since if the middle term is 11 it gives you the length of the square whose area is the answer to the sum.

An example that demonstrates the risk involved when you reason from a few examples to a conjecture is the following problem. As one can conclude from counting the regions formed by chords connecting different points on the circumference, the maximum number of regions for one to five points is respectively, one, two, four, eight, and sixteen.



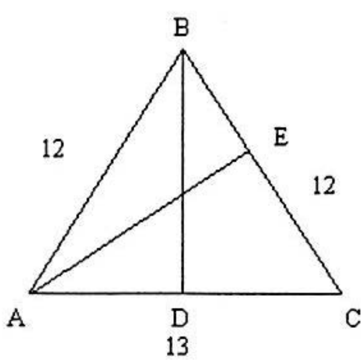
A pattern seems to be developing that for each additional point on the circumference of the circle the maximum number of regions doubles! One conjectures the relationship to be $R = 2^{n-1}$, where R is the maximum number of regions into which a circle can be dissected and n is the number of points used on the circumference of the circle. Still, if one places six points on the circumference of the circle the maximum number of regions is 31 and not 32. The representations initially supported then refuted the conjecture. Not until deductive reasoning confirms a relationship can one be sure about a conjecture arrived at by inductive reasoning. Maier (1988) deduced that $R = 1 + C(n, 2) + C(n, 4)$ where R is the maximum number of regions into which a circle can be dissected and n is the number of points used on the circumference of the circle. Inductive reasoning, even with its inherent uncertainty, when used with different representations may provide the bridge to the certainty afforded by deductive reasoning.

Deductive Reasoning

O'Daffer (1990) stated that "deductive reasoning is a mathematical reasoning process in which valid inference patterns are used to draw conclusions from premises" (p. 378). Essentially, one is concerned with whether something follows necessarily from something else. Think about the question asked by Gail Burrill (1998).

Question: Consider the isosceles MBC in which AB and BC are 12, AC is 13, and AE and BD are altitudes. How long is DE? ·

After some investigation of the question, the following theorem could be formulated:



Theorem: *Consider the isosceles ΔABC above, in which AE and BD are altitudes. Show that AD = DE?*

Seaman, R. (2001). Representation: two viewpoints for classroom communication.
Sum Thing to Think About, May 2001, 13-21.

This theorem illustrates the importance of representations that are the crucial part of the statement of a theorem and are included to take the place of verbal descriptions. If the representation is connected only to the theorem being proven it might distract and be counter-productive to students' efforts to focus on proving the theorem. In this case, what does the student have to add or eliminate when analyzing the diagram? Is this a two or three-dimensional diagram?

There is a shift in the role of the representation as the student attempts to prove the theorem. The two right angles in the triangle might suggest that a construction is possible. That is, that a circle of diameter AB with points B, E, D, and A on its circumference can be constructed (Fig. 1.2). This is a critical move in that the student might notice that as a result of the construction that $<ABD = <AED$ since the two inscribed angles intercept the same arc. Next from the diagram and the givens the student also realizes that $<CAE = <ABD$ since ΔMBC is isosceles and ΔMBD is similar to ΔCAE. Therefore, $<CAE = <AED$, ΔMED is isosceles and AD= DE. This example reveals that diagrammatically supported arguments are actually easier to formalize before the actual proof is formally written up. Or for a more elegant method, you can just note that D is the mid-point of BC, while the right angle at E means that DE is the radius of the circle with center D an radii BD, DC and DE. Thus DE = 6.5.

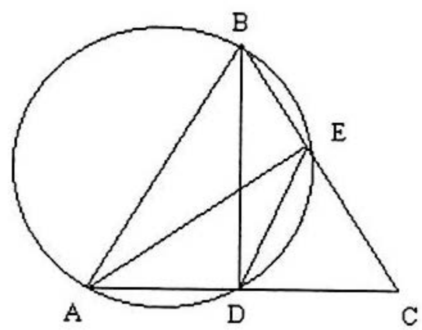


Figure 1.2. Geometrical Construction

Students should be cautioned that poorly drawn diagrams can also be misleading in that deductive reasoning can be swayed by what the eye sees in the diagram. Northrop (1964) underscores this pitfall by the following argument that there are two perpendiculars from a point to a line.

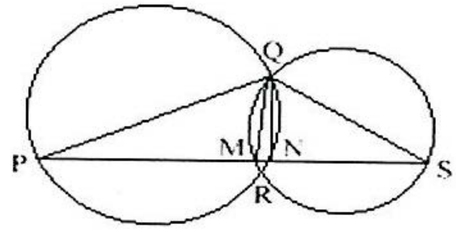


Figure 1.3. Geometrical Fallacy

Let any two circles intersect in Q and R. Draw diameters QP and QS and let PS cut the circles at M and N respectively, as in Fig. 1.3] Then $<PNQ$ and $<SMQ$ are right angles. (An angle inscribed in a semicircle is a right angle.) Hence QM and QN are both perpendicular to PS (p. 103).

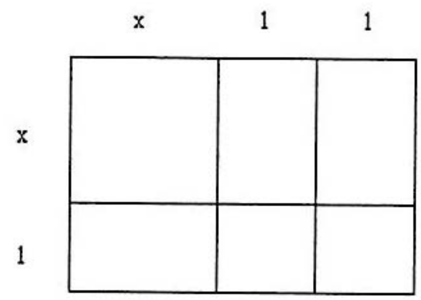
The fallacy becomes apparent when it is recognized that the diagram has been poorly drawn with QP and QS approximating where the diameter should be. Another concern is that some diagrams are misleading when they undergo changes of scale, a common ploy for statisticians. Difficulties also arise when students use diagrams to engage in infinite or limiting processes. Sometimes diagrams seem to even go against our intuition. For example, when a solid of revolution is produced by rotating $y = 1/x$ about the x-axis on the interval $[1, \infty)$ has an infinite surface area but a finite volume!

As this section highlights, the main strength of the representation lies in the opportunities for illumination and insight. Representations put to use intelligently can make students more expert-like in their problem solving, and support the inductive and deductive reasoning processes. The other viewpoint of how representation might be seen as a tool is the effect of representation on the teachers' teaching of mathematics.

Teachers' Teaching of Mathematics

The concrete-abstract continuum of representation has implications for the teaching of mathematics. Forms of representation need not be taught as though they are ends in themselves. Instead, they can be considered as useful tools for constructing understanding. For example, suppose the foundational objective of a lesson is to have grade nine students multiply a binomial by a binomial. For instance: $(x + l)(x + 2)$. With the concrete-abstract continuum of representation in mind the teacher might begin the lesson by applying the distributive law and symbolic language producing: $(x + l)(x + 2) = x(x + 2) + l(x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$.

If some students are having difficulty the teacher may draw a rectangle of area



$(x + l)(x + 2)$ where the sum of the area of the individual parts is $x^2 + x + x + x + 1 + 1 = x2 + 2x + x + 2 = x^2 + 3x + 2$. Consequently, with the whole being equal to the sum of its parts $(x + 1)(x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$. Writing the areas in this manner demonstrates that the product of two binomials can be obtained by distributing the first term in the binomial on the left through the second binomial and then repeating the process with the second term. If a student is still having difficulty the teacher may use the same ideas as contained in the diagram above but with a more concrete representation called Alge-Tiles™.

For younger students the teacher might begin with more concrete representations (pattern blocks, fraction blocks, number lines, geoboards, Cuisenaire™ Rods) and work towards understanding and more abstract representations. Suppose you have drawn two parallel lines on the board with two fixed points on the bottom line. Two lines are then drawn from each of the base points to meet on the top line, forming a triangle. The point on the top line slides back and forth, forming several triangles. If the students are having difficulty understanding that all the triangles have the same area the teacher might use such software as the Geometer's Sketchpad® to represent the situation and automatically make the area calculations. Negative numbers that defy intuition can become more understood to students if teachers use representations that furnish insightful understanding. Teachers might use verbal analogies that deal with temperature, borrowing money from the bank, and/or the plus/minus rating given hockey players. As a result, the number line can be extended to include negative numbers and then students learn to symbolically represent negative numbers.

Teachers might think that concrete manipulatives are used primarily with the K-9 student. Number lines can be used to represent real numbers and ideas that go against the students' intuition. For example, to demonstrate the density of rational numbers and the existence of irrational numbers. Also, a two-dimensional plane can represent the 'imaginary numbers' where the real numbers are positioned on the horizontal axis. This representation gives the students a picture of the complex number system that is not intuitively obvious to students. If the emphasis is to highlight the closure of sets with respect to an operation, the teacher might represent the complex number system with a Venn diagram beginning with the natural numbers. With either representation the student gains a more overall understanding of the complex number system.

In analytic geometry, when teaching the ellipse, a piece of string is an ideal representation for the sum of the two focal radii. The string can be used to draw the ellipse, lead to a definition of the ellipse, and see the effect of moving the foci on the shape of the ellipse. The meaningful equality between the length of the major axis and the length of the line segment from one focus to the end of the minor axis on the ellipse is also apparent when using the string.

In algebra, one example is to prepare the desks in the classroom to form a rectangular grid before the students enter the classroom. When each student sits down they are given an ordered pair depending on where they sit. By having the students represent the Cartesian Plane by 'real world scripts' the teacher can assess their understanding of ordered pair, relation, domain, range, function, slope, whether ordered pairs satisfy straight lines, points of intersection etc. When considering relations some representations (tables, various forms of equations, mapping diagrams, graph paper, technology) are better than others depending on the lesson objectives of the teacher.

In geometry, when students first learn to prove theorems, some students may need to initially use two columns of statements/reasons to represent their proofs. Some students might use one side of the page that contains rough work of representations that are consulted when attempting to develop a proof and the other side to contain the formal proof. Others might represent their argument verbally in paragraph form.

Teachers should encourage students to represent problems in multiple formats. For instance in data management, bar graphs, pie graphs, stem-and-leaf graphs, box-and-whisker graphs, and line graphs should be seen as different ways that students might represent the data in order to understand and/or communicate it to someone else.

Students might then recognize that some representations are better than others are when constructing understanding or communicating information. Consider the following problem that can be represented in symbolic or written language:

8, 5, 4, 9, 1, 7, ____

Eight, five, four, nine, one, seven, _____

What number comes next?

The answer becomes more obvious with the latter representation. Students should have the freedom to represent the problem as they see fit which gives the teacher an opportunity to complement the students for their different representations and approaches to solving a problem. For example, a teacher might be teaching a unit on solving linear equations. The students are given a problem: The sum of two numbers is 1000 and the difference of the two numbers is 100. The students might represent the problem as a chart, one linear equation and/or as two equations in two unknowns. The teacher then discusses the many different ways the problem can be solved (including the method suggested in the curriculum guide) with the students. The teacher acknowledges the 'theories' that the students are constructing and attempts to encourage or revise those theories based on the students' representations. The teacher in this type of classroom goes from being a 'sage on the stage' to a 'guide at the side'. When students ask for help the initial step is not to show the students how to do the question but rather to listen to the students' theories about how they would represent the problem and respond accordingly. By acknowledging the different ways that students represent problems the learning environment has been altered so that students with varying strengths and needs can achieve the lesson objectives.

Whereas teachers can pick and chose their representations for instruction on the concrete-abstract continuum, textbooks are locked in time. Future textbooks might tend towards at least two worked examples to explain mathematical procedures and concepts with multiple representations of written language, symbolic language, pictures, diagrams, and charts. Such an example might explain how to add two integers with different signs by describing an analogy in words. Then the addition of the two integers could be represented in symbolic language followed by a pictorial representation. As a result, the example is organized inductively starting with a familiar analogy, followed by multiple representations and finishing with a conjecture.

Summary

The mathematics teacher should initially see representation as a concrete-abstract continuum (fig. 1.1) that has implications for the students' thinking and their teaching. Students' thinking becomes more expert-like when they classify problems according to the most relevant representation that helped them solve the problem. Representations have also been seen to support the inductive and deductive reasoning processes. Perhaps, the mental hierarchy teachers might have consists of three levels: the concrete-abstract continuum for representation level, induction level and deduction level. Pedagogically representation, though not a primary focus of instruction, can help students construct understanding and communicate information and understanding without teachers changing their foundational objectives.

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WHAT DOES 1/6 MEAN?

Rick Seaman, June Seaman, and Ev Stange

To view this entire article please visit our website at: http://mathcentral.uregina.ca/SMTS/news-june-2002.htm.

Secondary mathematics teachers have been known to state publicly that if students could only add, subtract, multiply and divide fractions they would have a chance at being successful at learning high school mathematics. They might also wonder what representation(s} elementary teachers utilize to help students understand how to carry out these operations on fractions. One prerequisite for success with fractions is for students to be able to attach meaning to the fraction symbol. A way this understanding might be facilitated is by working with a concrete representation of the fraction symbol. English (1999) states that before we use a concrete representation as an analogue "with our students, it is wise to consider whether it clearly portrays the intended mathematical idea" (p. 30). English goes on to caution that poor analogues require the student to understand the concept being taught just so they can understand the analogue. This paper will suggest instructional methods (Mayer, Sims, & Tajika, 1995} to improve understanding of the fraction symbol emphasizing:
worked-out examples using words, symbols, and pictures,

a) inductive organization of material beginning with a familiar situation to the students (analogue) and
b) ending with formal statements of the meaning of the fraction symbol reinforced by a teacher/student dialogue.

At each step opportunities for practice are given to allow students to gain complete understanding. The paper concludes with examples of how these instructional methods can be applied to facilitate student's understanding of adding fractions.

Attaching Meaning to the Fraction Symbol

Sobel and Maletsky (1999) assert "the initial concept of a fraction is that of a fractional part of a whole, a geometric concept" (p. 91). The following will connect geometrical representations familiar to the student to the meaning of fractions in that each student will be given 12 equal-sized tiles that represent the pieces of a chocolate bar.

Worked-Out Example A

You have just bought a chocolate bar, which contains six equal-sized pieces in total [The following are verbal, symbolic, and visual representations of the analogue].

Six pieces of chocolate represents six parts out of six equal-sized pieces is written 6/6.

The teacher would reinforce the verbal, visual, and symbolic representations of the analogue by reading the verbal representation out loud and pointing to the corresponding parts in the visual and symbolic representations. The teacher/student dialogue format would follow.

Sample

Teacher: Make a chocolate bar with six equal pieces using your tiles.

Student: *[student performs the action]*

Teacher: I'll show you how we write about the pieces (of chocolate). The bottom number tells us how many equal parts here are altogether [On the blackboard the teacher would write /6 and point to the six and ask, "How many equal parts is this chocolate bar cut into?"]

Student: Six.

Seaman, R., Seaman, J., & Stange, E. (2002). What does 1/6 mean?
SMTS: The Numerator, 2(1), 3-4.

Teacher: That's right. What does the bottom number tell us again?
Student: The bottom number tells you how many pieces the whole chocolate bar is cut into.
Teacher: Right [and again writes /6 on the board and points to the six].
The teacher would give extensive opportunities for the student to create chocolate bars from a written cue. For example:
Teacher: [The teacher writes the symbolic representation of a chocolate bar on the board e.g.. 6/6. There would be many such examples.] Show this chocolate bar with your tiles [points at 6/6].
Student: *[student shows this with the tiles]*
Teacher: Good.
The teacher would provide the visual cue. on a chalkboard and ask the students how to describe it symbolically. For example:
Teacher: [The teacher draws a visual representation of a chocolate bar on the board.] The student is then asked. "How would this be written?"
Student: Six over six.
Teacher: Yes. Write that on your paper (individual chalkboard or whiteboard).
Student: Writes 6/6.
Teacher: Good.

Worked-Out Example B

You have just eaten one piece from the six-piece chocolate bar. That means that you have eaten one piece out of six equal-sized pieces of the chocolate bar [The following are verbal, symbolic, and visual representations of the analogue].

Each piece of chocolate represents one part out of the six equal-sized pieces written 1/6

Three pieces of chocolate out of the six equal-sized pieces of chocolate would be written 3/6

The teacher would reinforce the verbal, visual, and symbolic representations of the analogue by reading the verbal representation out loud and pointing to the corresponding parts in the visual and symbolic representations. The teacher/student dialogue format would follow.

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THREE REAL-WORLD EXAMPLES OF PROPORTIONAL REASONING

Rick Seaman

In the *Principles and Standards for School Mathematics* (2000) the Connections Standard states: "Instructional programs from prekindergarten through grade 12 should enable all students tore recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics" (p.64). As an example of the Connection Standard, the Principles and Standards for School Mathematics (2000) employed a proportional reasoning example where the teacher asks students, "How is our work today with similar triangles related to the discussion we had last week about scale drawings?" Also, the *Curriculum and Evaluation Standards* (1989) states that in grades 5-8 "students need to see many problem situations that can be modeled and then solved through proportional reasoning" (p. 82).

'The File'

As a mathematics teacher I keep a file on my desk in which I put clippings, articles and the like that I think have some promise as real- world applications to supplement my teaching of mathematics. After about four months I look through 'the file' to see what has been collected and to my amazement I have found threads that I did not realize were there. The following three real-world examples of proportional reasoning were generated from such a file. Some examples might be considered too dated for current classroom use, but if they were considered good applications at one time they are not any more dated than a 'traditional' word problem so I continue to use them. I have incorporated sites from the World Wide Web to provide the background to the examples wherever possible.

#1. Spinks vs. Tyson.

The first example is the Michael Spinks and Mike Tyson showdown for the heavyweight boxing crown that occurred on June 27, 1988. Both fighters went into the boxing match with undefeated records, Spinks at 31-0 and Tyson at 34-0. Michael Spinks retired after the match. The following doze exercise from an Associated Press article provides the backdrop for also discussing the ethics behind the amount of money that athletes are paid.

Not Quite Minimum Wage

ATLANTIC CITY, N.J. (AP) - Michael Spinks was paid at the hourly wage of $ _____ million US for lasting 91 seconds against Mike Tyson in Monday night's heavyweight championship fight.

Spinks was guaranteed $13.5 million for the fight, which comes to $ ____for each of the eight Tyson punches he absorbed, or $ ____for each second of the fight.

Tyson will receive around $20 million US, give or take a million or two, depending on the gross take from ticket sales, pay-per-view and closed-circuit receipts.

That comes to $_____ a second or $____ million an hour – or $____ million for each of the two punches landed by Spinks.

People who paid $1500 for a ringside seat ended up paying $____ a second for the privilege. Those who paid $35 to watch it on closed-circuit TV coughed up ____ cents a second.

Donald Trump, the casino owner who put the fight together; came out a double winner: He was expected to reap a take of $60-$80 million form the fight, as well as millions more from customers dropping money in his casinos along the boardwalk.

Seaman, R. (2003). Three real-world examples of proportional reasoning.
The Numerator, 2(2), 6-8.

Thanks to Tyson's punching power, the high-rollers had an extra hour on their hands Monday night to drop more money at Trump's tables.

The signing of shortstop Alex Rodriguez on December 11, 2000 to a 10-year 252 million-dollar contract by the Texas Rangers brings to mind another possible example by applying information found at rangers.mlb.com.

 # 2.Gerard Bull and the Iraqi Babylon gun.

According to Mark Wade at www.astronautix.comjlvs/babongun.htm (5 September 2002) "From March of 1988 until the invasion of Kuwait in 1990, Iraq contracted with Gerard Bull to build three superguns: two full sized 'Project Babylon' 1000 mm guns and one 'Baby Babylon' 350 mm prototype.... The recoil force of the gun would be 27,000 tonnes – equivalent to a nuclear bomb and sufficient to register as a major seismic event all around the world. Nine tonnes of special supergun propellant would fire a 600 kg projectile over a range of 1,000 kilometres, or a 2,000 kg rocket-assisted projectile." Bull from his research was convinced that such a gun could be used to launch artillery shells into enemy territory. In particular Iraq wanted the gun to fire chemical, biological and/ or nuclear warheads at either Iran or Israel. One weakness of the gun was that once built it could not be moved and could only be fired in one direction.

On March 22, 1990 Bull, a Canadian-born U.S. citizen, was assassinated in Brussels, Belgium. This came as no surprise especially when one considers the arena in which he chose to do business. For those interested in learning more about Gerald Bull the film 'Doomsday Gun' was for HBO Showcase, and aired for the first time on July 23, 1994. For more information go to www.google.com, key words: Doomsday Gun. The following question provides a springboard to discuss Middle East issues with students.

- Where could Iraq position two super guns so that one could reach Tel Aviv in Israel and the other reach Tehran in Iran? For a map of the area go to www.mapquest.com.

#3. Buying textbooks in university can be expensive.

Every semester university students must empty their wallets as they purchase textbooks. Second-hand textbooks are scarce and it seems that textbooks are regularly changed so this option is not always available. Students in second year have learned to wait until classes begin to find out which texts are really necessary but they acknowledge that if they are taking one of the sciences they have no choice but to pay top dollar for the textbook. One reason given students for the high cost of texts is that for a small market there are higher publishing costs. The National Association of College Stores (Sept. 5, 2002) at www.nacs.org/common/research/textbook$.pdf broke down the price of a book [The prices "are averages and do not represent a particular publisher or store."] as follows:

A) Publisher's Paper, Printing. Editorial Costs: 32.1 cents. All manufacturing costs from editing to paper costs to distribution, as well as storage, record keeping, billing, publishers' offices, employee's salaries and benefits.

B) Publisher Marketing Costs: 15.3 cents. Marketing, advertising, promotion, publisher's field staff, professors' free copies.

C) College Store Personnel: 11.4 cents. Store salaries and benefits to handle refund desk extra textbooks back to the publisher.

D) Author Income: 11.5 cents. Author's royalty payment from which author pays research and writing expenses.

E) Publisher's General and Administrative: 9.9 cents. Including federal, state and local taxes, excluding sales tax, paid by publishers.

F) Publisher Income: 7.0 cents after taxes. After-tax income from which the publisher pays for new product development, author advances, market research and dividends to stockholders.

G) College Store Income: 4.7 cents Pre-Tax. Note: The amount of federal, state and/or local tax, and therefore the amount and use of any after-tax profit, is determined by the store's ownership, and usually depends on whether the college store is owned by an institution of higher education, a contract management company, a cooperative, a foundation, or by private individuals.

H) College Store Operations: 6.8 cents. Insurance, utilities, building and equipment rent and maintenance, accounting and data processing charges, and other overhead paid by college stores.

I) Freight Expense: 1.3 cents. The cost of getting books from the publisher's warehouse or bindery to the college store. Part of cost of goods sold paid to freight company.

Represent this data as a pie graph and then discuss how the price of textbooks might be lowered.

The figures you have been working with are in US dollars. Some Canadian bookstores also use these figures what would they be in Canadian dollars and how would this change your pie graph?

Final Thoughts

There is no pretence that these three real-world examples are a complete selection of examples are a complete selection of examples and that all possible questions have been posed. They are indicative of the type of examples that can form a foundation to demonstrate the usefulness of proportional reasoning. The teacher’s awareness of such examples and the creativity brought to these examples will further reinforce the usefulness of proportional reasoning.

Seaman, R. (2003). Three real-world examples of proportional reasoning.
The Numerator, 2(2), 6-8.

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BUILDING ON STUDENTS' INTUITIONS TO REPRESENT PROBLEMS ALGEBRAICALLY BUT LIKE AN EXPERT

Rick Seaman

Cynthia Barb and Anne Larson Quinn (1997) wrote an article in *The Mathematics Teacher* entitled "Problem Solving Does Not Have to Be a Problem." The authors described how they developed "students' problem-solving skills by building on their intuitions and helping them to refine various strategies rather than by just making sure we got through a long list of problems" (p.538). The authors further state, "if problem solving is initially intuitively based, the algebraic strategies become a welcome shorthand method" (p.541). Below are the seven problems from the article that the authors categorized according to their surface features as either salary, work, time-distance or mixture problems.

Problem 1: Jessica's day job pays \$8.00 an hour and her night job pays \$3.50 an hour. She worked a total of 49 hours in one week and earned a total of \$293.00. How many hours did she work at each job? (p. 538)

Problem 2: Your neighbor has ordered 300 ft. of dirt to plant a garden and do some landscaping. His wheelbarrow can hold 5 ft.. It takes him fifteen minutes to fill and transfer the wheelbarrow to the garden. You decide to help out and know that you can fill and transfer your own 10 ft. wheelbarrow in the same amount of time. How long would it take to move all the dirt if you and your neighbor work together? (p.538)

Problem 3: An executive leaves home on a business trip traveling 50 miles per hour. One hour later, her husband finds an important briefcase that she left and starts after her at 70 miles per hour. Assuming that no one is stopped for speeding, how long will it take to catch her? (p. 538)

Problem 4: A 40 percent disinfectant solution is to be mixed With a 20 percent disinfectant solution to obtain 10 liters of a 30 percent solution. How many liters of the 40 percent solution and many liters of the 20 percent solution should be used? (p. 540)

Problem 5: How much of a 40 percent disinfectant solution should be mixed With how much pure water, 0 percent disinfectant, to obtain 20 liters of a 20 percent solution? (p. 540)

Problem 6: How much of a 40 percent disinfectant solution should be mixed with how much pure water to obtain 20 liters of a 10 percent solution? (p. 540)

Problem 7: How much of a 50 percent acid solution should be mixed with how much pure acid, or 100 percent acid, to obtain 10 liters of an 80 percent acid solution? (p.540)

I would like to comment on how the approach taken by these authors might be enhanced to help students when problem solving to not only build on their intuitions to represent problems but to do so algebraically and like an expert.

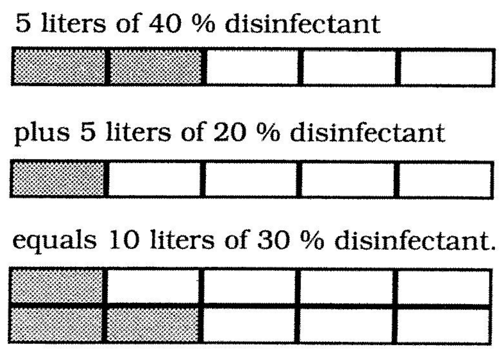
Beginning with problem one several students in the article used guess-and-check to find the number of hours for each pay rate that would yield \$293. "For instance, testing twenty hours at \$8.00 and twenty-nine hours at \$3.50 yields an answer of \$160.00 + \$101.50 \$261.50, which is too low" (p. 538). One student chose random values while another student made estimations after calculating the range of Jessica's total earnings. These students tacitly worked their strategies around a part-whole principle. That is, the amount of money Jessica earns at her day job per week plus the amount of money she earns at her night job per week equals the total amount of money she earns per week. Making explicit an expert-like connection between arithmetic and algebraic word problems at the principle level could enhance the authors' approach to problem

Seaman, R. (2004). Building on students' intuitions to represent problems algebraically but like an expert. *The Numerator, 3*(1), 6-8.

solving. The strategy would become one of translating the part-whole principle into an equation. For example, let x be the number of hours that Jessica worked at her day job. Then 49x is the number of hours Jessica worked at her night job. The translation of the part-whole principle above then becomes 8x + 3.5(49x) = 293.

Similarly guess-and-check could be used in problem two to make explicit the part-whole principle. That is, when moving the dirt the part of the task completed by one neighbor plus the part of the task completed by the other neighbor equals the part of the task completed. This translates algebraically into the equation 1/15 t + 1/7.5t = 1 where t is the time, in hours, it takes moving the dirt while working together.

In problems four and five, Barb and Quinn again used an intuitive approach to solve these problems. For example, in problem four, the first guess might be that of equal portions of solution.

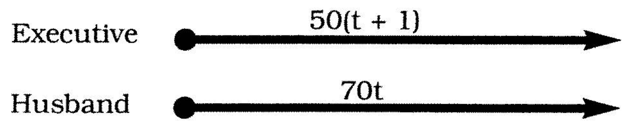


The above diagrams represent the students' guesses with the shaded part indicating the corresponding amount of disinfectant. Diagrams like this help students to 'discover' the part-whole principle of the problem. That is, that the amount of disinfectant in the 40% disinfectant solution plus the amount of disinfectant in the 20% disinfectant solution equals the amount of disinfectant in the 30% disinfectant solution. This part-whole principle then translates algebraically into the equation .40x + .20(10-x) = .30(10) where x is the number of liters of 40% disinfectant solution and 10 - x is the number of liters of 20% disinfectant solution. The part-whole principle also applies to problems six and seven but not the equal portions of solution.

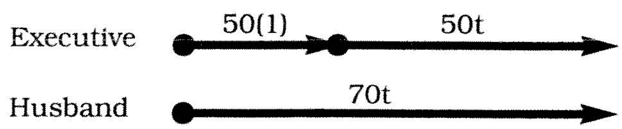
Now consider problem three where the diagram might indicate a potentially new underlying principle. Using successive approximations of one, two, and three hours a student of Barb and Quinn worked with a time line diagram like the following and arrived at the correct time it takes the husband to catch up with his wife.

Such a diagram might lead students to conjecture that the underlying principle for this problem is equal quantities since the distance the executive travels is the same as the distance her husband travels to catch up with her. A diagram that is used to represent the problem algebraically might look something like the following:

Seaman, R. (2004). Building on students' intuitions to represent problems algebraically but like an expert.
The Numerator, 3(1), 6-8.



where t is the length of time, in hours that it takes for the husband to catch up with his wife [executive] who has been traveling for t+1 hours. The equality principle translates algebraically into the equation $50(t+1) = 70t$. However, it is possible that some students might modify this diagram and make explicit the part-whole principle [not to scale]



Their explanation being that the distance the executive travels for the first hour plus the distance the executive travels while her husband tries to catch her equals the total distance her husband travels. For these students the part-whole principle translates algebraically into the equation $50(1) + 50t = 70t$. This points out that problems may have more than one underlying principle, and their application depends on how expert-like students mentally categorize information. As a result, it is important that students justify completely why they believe a problem has a particular underlying principle.

The authors used time lines and diagrams to make the part-whole principle implicit for the students but such classifications of problems must become explicit for students to appreciate that one principle applies to all seven problems. By becoming expert-like in their classifications of problems students would recognize that to solve these seven problems they would need to learn only one principle that links the four surface features together.

Summary

Problem solving, initially intuitively based, helps students to successfully represent problems at the concrete level while acknowledging the 'natural ways' of solving problems. Students should then explore, make conjectures regarding a problem's underlying principle and justify their reasoning. It then becomes a matter for students to show how a written principle translates into an algebraic equation. Maybe when attempting to solve a future problem these expert-like students would consciously retrieve problems that they have seen before with the same principle. These students may even transfer this way of thinking to other disciplines.

Reference

Barb, C., & Quinn, A.L. (1997). Problem solving does not have to be a problem. *The Mathematics Teacher, 90* (7), 536-542.

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PROBLEM REPRESENTATION: ASSESSING WHAT IS VALUED

Rick Seaman

The *Curriculum and Evaluation Standards* (NCTM, 1989) state that, "If problem solving is to be the focus of school mathematics, it must also be the focus of assessment ... [and assessment] should provide evidence that they [students] can apply a variety of strategies to solve problems" (p.209). The *Assessment Standards* (NCTM, 1995) add, "The purpose of evaluating a student's achievement in mathematics is to open a public window on that student's mathematical thinking processes" (p.58). The Principles and Standards (NCTM, 2000) go on to assert that assessment include "what kinds of mathematical knowledge and performance are valued" (p.22). The question is, how can teachers assess that which they value in the students' problem-solving process, in particular, problem representation? In this article, which is based on an integration of my research and practice, are two examples of a student's work in which an analytic scoring scale was used to assess her problem representation before she actually began to solve the problem.

Assessment as a Cyclic Process

Planning assessment

Assessment has been described as a cyclic process of planning, gathering information, interpreting results and making decisions (NCTM, 1998). In planning for assessment the teacher considers phases of the problem-solving process that are of particular interest and then devises an instrument that yields the greatest amount of feedback. With respect to problem representation some questions are:

- Is the student initially reading for understanding? That is, is the student able to slow down their thinking and indicate what has just been read including the goal of the problem? Most students try to analyze and solve the problem first.
- Is the student reading for analysis? Is the student analyzing the problem and making a note of what is given in the problem? Is the student aware of any givens that are not numerical?
- Does the student realize the difference between reading a problem for understanding and analysis and appreciate their respective roles in the problem-solving process?
- How does the student represent the problem? Does the student realize that there are different possible abstract-to-concrete representations of the problem? Does the student recognize that some representations are better than others in helping to solve the problem? Does the student know that different forms of representations are not necessarily taught as the objectives of the teacher's lessons and they will be recognized for 'their' representations? Does the student see different representations as an aid to their understanding and communication?
- Does the student use representation(s) as an aid to decide on a deeper structure for the problem? Can the student justify their choice of deeper structure? Is the student becoming more expert-like in the classification of problems according to what they believe is most relevant for solution (deeper structure) rather than the context, the same quantity such as age, weight, or time used in the problem, and/ or the question asked in the problem (surface structure)?

Detailed answers to these questions provide insights into each student's thinking while solving problems and emphasize the phases of the problem-solving process stressed during instruction.

Seaman, R. (2005). Problem representation: assessing what is valued. *The Numerator 4*(3), 20-29.

Gathering information
One way to collect this information is to employ structured worksheets (SW) when assessing (see fig. 1). But how do you make up a SW? One way is to construct the SW by using questions that guide and assess phases of the problem-solving process that you value. Although the SW will also contain room for problem solution this article will be restricted to assessing students' problem representation by means of an analytic scoring scale and conclude with suggestions for further assessment.

Analytic scoring is a method of assessment that assigns point values to phases of the problem-solving process that are valued by the teacher. Each phase of problem representation on the SW was scored as follows:

2 - reflects that information given is complete.

1 - some appropriate information is given.

0 - no response or no idea what to do.

The analytic scoring scale (see fig. 2) employed is an application of the rubric developed in Hutchinson (1993) and Seaman (1995) to assess particular phases of the problem-solving process. Following the suggestion given in the *Principles and Standards* (NCTM, 2000) that assessment and instruction must be integrated so that assessment becomes a routine part of the ongoing classroom activity rather than an interruption" (p.23) one-question quizzes were given with room for students' problem representation on one side with scoring and comments on the back. One will find that the analytic scoring of problems quickly becomes less time consuming as you gain experience doing it and it helps facilitate students' understanding of problem representation through the feedback given to students. See figures 3 and 5 for actual student work. Example two (the second quiz) was handed out to the students in class ten days after example one (the first quiz).

Example 1: Consider Lori's work on the quiz in figure 3. For all four questions Lori receives a score of 1 out of 2 for the respective four phases:

1. Lori could have been clearer in describing what she had just read: For example, some cars are traveling and we want to know when they are certain distance apart. Lori indicated some analysis when she discussed the fact that there were two cars.
2. She made no mention of the fact that the two cars left Calgary the slower one going east and the faster one going west.
3. Lori did not label east and west on the diagram or indicate the distances that each car travels in the diagram. It was assumed that the C indicates where Calgary was situated in the diagram.
4. She classified the problem as a 'Whole is equal to the sum' problem rather than completely as 'Whole is equals to the sum of its parts' and did not give a reason why.

On the back of the one-question quiz (see fig. 4) Lori receives feedback on her problem representation.

Example 2: Consider Lori's work on the quiz in figure 5. Lori receives scores of 2, 2, 1, and 1 for the respective four phases with the following comments:

1. Lori's description of the surface structure and of the goal of the problem is clear. Lori has indicated some analysis when she mentioned there were 200 people surveyed.
2. Lori has identified all relevant relationships in the problem.
3. Although Lori has indicated most of the relationships in the diagram she forgot to define the universal set (U) for this problem.

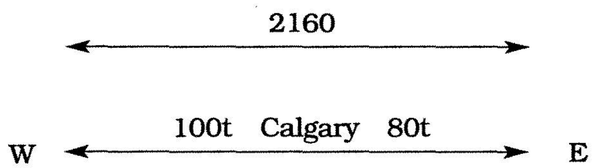
4. She classified the problem as a 'Venn Diagram' problem but did not give the reason why.

On the back of the one-question quiz (see fig. 6) Lori receives feedback on her problem representation.

Interpreting results

Example 1: The feedback on Lori's problem representation summary sheet indicates a need for improvement in all four phases of problem solving especially in the justification of her choice of deeper structure. Ideally, a better labeled diagram would make explicit the justification for the deeper structure, which could then be described verbally and translated into mathematical symbols. That is, three different representations of the same deeper structure:

1. Let t be the time in hours it takes for the two cars to be 2160 km apart.



2. The distance one car traveled west plus the distance the other car traveled east equals the distance the two cars are apart after a certain time period.
3. 100t + 80t= 2160

The SW reinforced an instructional objective of having the student apply multiple representations for constructing understanding and for communicating information while problem solving.

Example 2: Lori on this problem has shown improvement in identifying the surface structure, the goal of the problem, and in identifying the givens in the problem. She needs more work on labeling diagrams properly and justifying her choice of deeper structure of the problem.

Students are given structured worksheet summary sheets (SWS) that they will keep in their portfolios to summarize their results over a number of quizzes (see fig. 7). After every one question quiz the students will record their results on the SWS and after four or five quizzes students should be able to identify which phase of the problem-solving process is giving them the most difficulty and 'negotiate' with their teacher some help for that phase. In particular, I had a repeating grade nine student use the SWS to identify two phases of problem representation that he wanted to work on with me to improve his problem solving. That memory of that student negotiating his specific needs in problem solving will forever remain with me.

Making decisions

In this article word problems, an analytic scoring scale, and structured worksheets were used as a vehicle to assess students' problem representation while problem solving. The objective is that students gain control over the problem-solving process, comprehend the difference between reading a problem for understanding and for ana1ysis, value different forms of representation, and exhibit expert-like behavior in the classification of problems, and all this before they actually solve the problem!

Some further extensions for assessment include asking students to indicate when they made the decision as to what the deeper structure of the problem is by placing a "/" in the text of the problem. This gives the teacher further feedback as to when the student is making a decision regarding the deeper structure of the problem and whether it is legitimate or the student is cueing in on the surface structure and/or goal of the problem. Also students could be asked where they have seen a problem like this before with respect to deeper structure. This question asks the student to explicitly retrieve a problem that will hopefully help solve the problem that they are presently working on. When students begin to make such comments as, “It's just a whole equals the sum of its parts problem!" it indicates that they have begun to retrieve like an expert other problems that will help them solve the present problem.

There is more to teaching problem solving than paying lip service to problem representation. If it is important then it should become part of the assessment. One way is to assess using an analytic scoring scale and use structured worksheets that feature problem representation. For the student, it is hoped that this focus on problem representation and subsequent assessment leads to success not only in problem solving but establishes a way of thinking that will transfer to other subjects and everyday life.

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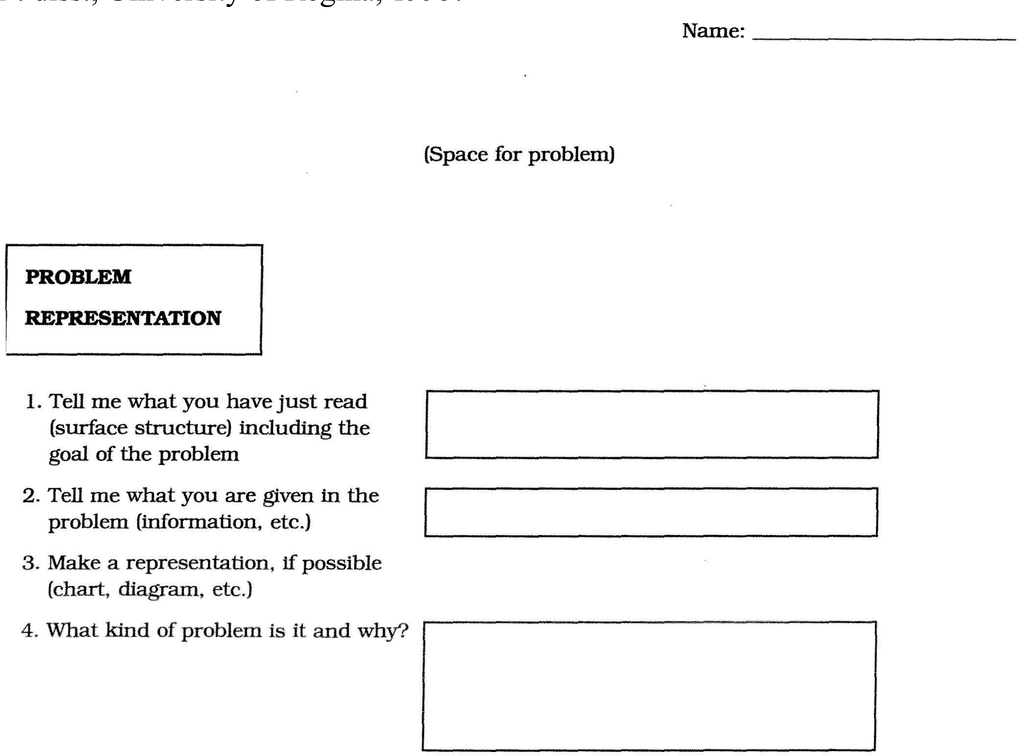


Name: ______________________

(Space for problem)

PROBLEM REPRESENTATION

1. Tell me what you have just read (surface structure) including the goal of the problem
2. Tell me what you are given in the problem (information, etc.)
3. Make a representation, if possible (chart, diagram, etc.)
4. What kind of problem is it and why?

Fig. 1. Structured Worksheet

Seaman, R. (2005). Problem representation: assessing what is valued.
The Numerator 4(3), 20-29.

1. Tell me what you have just read (surface structure) including the goal of the problem. This refers to the extent the solver can describe the surface structure and goal of the problem.

2 = The solver is able to describe the surface structure of the problem with minimal analysis and the purpose for which the problem is undertaken (goal).
1 = The solver has identified at least one of the above.
0 = The solver is unable to describe the surface structure or goal of the problem.

2. Tell me what you are given in the problem (information etc.). This refers to the extent to which the necessary relationships in the problem information have been stated.

2 = The solver has identified all relevant relationships given in the problem.
1 = The solver has identified at least one but not all, relevant relationships given in the problem.
0 = The solver has identified none of the relevant relationships given in the problem.

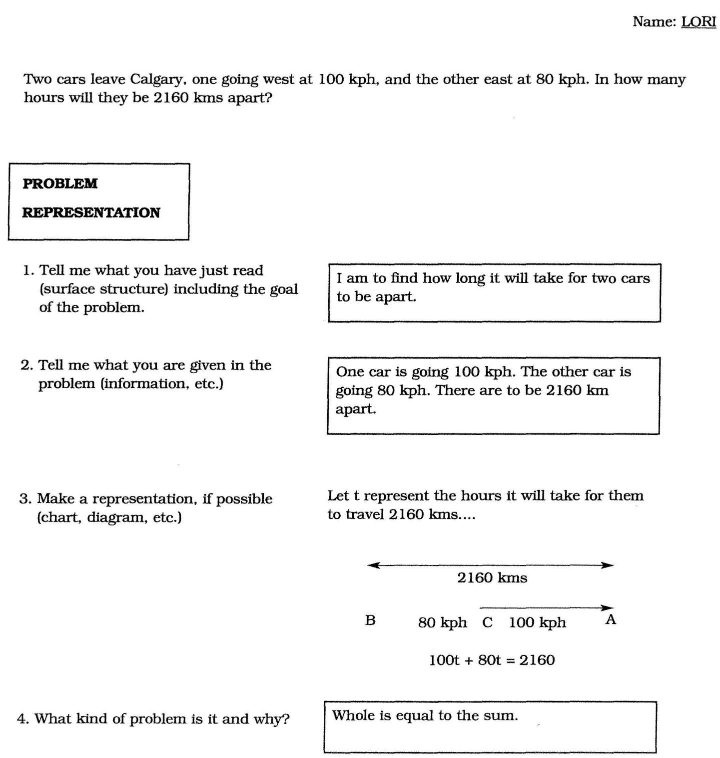
3. Make a representation, if possible (chart, diagram, etc.).

2 = Correctly drawn and labelled chart, diagram, etc.
1 = Correctly drawn and incorrectly labelled chart, diagram, etc.
0 = No charts, diagrams are presented.

4. What kind of problem is it and why? This refers to the extent to which the solver identifies the problem as one with a deeper structure.

2 = Name or description of type of problem, based on deeper structure, is accompanied by a reason based on deeper structure.
1 = Name or description of type of problem based on deeper structure.
0 = No naming or description of type of problem based on deeper structure.

Fig. 2. Analytics Scoring Scale.



Name: LORI

Two cars leave Calgary, one going west at 100 kph, and the other east at 80 kph. In how many hours will they be 2160 kms apart?

PROBLEM REPRESENTATION

1. Tell me what you have just read (surface structure) including the goal of the problem.
   I am to find how long it will take for two cars to be apart.

2. Tell me what you are given in the problem (information, etc.)
   One car is going 100 kph. The other car is going 80 kph. There are to be 2160 km apart.

3. Make a representation, if possible (chart, diagram, etc.)
   Let t represent the hours it will take for them to travel 2160 kms....

$100t + 80t = 2160$

4. What kind of problem is it and why?
   Whole is equal to the sum.

Fig. 3. Structured Worksheet.

PROBLEM REPRESENTATION SUMMARY

Name: Lori **Date:**

2
1 #1 #1 #1 #1
0
1 **2** **3** **4**

PROBLEM REPRESENTATION RESULTS FOR EACH OF THE FOUR PHASES IN EXAMPLE #1

---

1. Describes surface structure, goal.
2. Describes information, explicit relationships.
3. Makes a representation.
4. Identifies a deeper structure for problem and justifies choice.

Comments: Lori scored one for each of the four phases in example #1 as indicated above.

1. Mentioned goal but I believe you could have written this up a little better. For example, some cars are traveling and we want to know when they are certain distance apart. Also the mention of two cars indicates that you are mixing reading for analysis with reading for understanding.
2. Mention two cars left Calgary the slower one traveling east and the other west.
3. Indicate on the diagram, which car is traveling east, which car is traveling west, and an expression for the distance each car is traveling in the diagram (e.g., 100t and 80t).
4. Give a reason for your choice of deeper structure, which I think you meant to be 'Whole is equals to the sum of its parts' ["The distance the faster car traveled west from Calgary plus the distance the slower car traveled east from Calgary equals the distance the two cars are apart after a certain time period"].

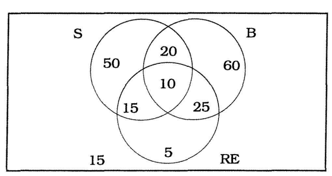
Fig. 4. Problem representation summary.

Name: LORI

In a recent survey of 200 people, the following information was obtained about their investments: 95 invest in stocks, 115 invest in bonds, 55 invest in real estate, 30 invest in stocks and bonds, 25 invest in stocks and real estate, 35 invest in bonds and real estate, and 10 invest in all three. How many invested in only real estate but not stocks or bonds?

PROBLEM

REPRESENTATION

1. Tell me what you have just read (surface structure) including the goal of the problem.	A survey of 200 people was taken about their investments and we are to find out how many invested only in real estate.
2. Tell me what you are given in the problem (information, etc.)	200 people -numbers given in question-95 stocks, 115 bonds, 55 real estate, 30 stocks and bonds, 25 stocks and real estate, 35 bonds and real estate, 10 all three.
3. Make a representation, if possible (chart, diagram, etc.)	

Let S represents the amount of Stocks.
Let B represent the amount of Bonds.
Let R represent the amount of Real Estate invested.

4. What kind of problem is it and why?	Venn Diagram (Survey)

Fig. 5. Structured Worksheet.

Seaman, R. (2005). Problem representation: assessing what is valued.
The Numerator 4(3), 20-29.

PROBLEM REPRESENTATION SUMMARY

Name: Lori **Date:**

2	#2	#2	_	_
1	_	_	#2	#2
0	_	_	_	_
	1	**2**	**3**	**4**

PROBLEM REPRESENTATION RESULTS FOR EACH OF THE FOUR PHASES IN EXAMPLE #2

---

1. Describes surface structure, goal.
2. Describes information, explicit relationships.
3. Makes a representation.
4. Identifies a deeper structure for problem and justifies choice.

Comments: Lori scored out of two a two on the first two phases, one on the next two phases for example #2 as indicated above.

1. How about: After a survey is conducted concerning certain peoples' investments, they want to find out how many people have invested in a certain category but not in the rest. I am assuming that by your statement "only in real estate" you meant "invested only in real estate but not stocks or bonds".
2. When you mention "numbers given in question" I want to remind you that the givens in problems don't have to be only numbers.
3. You forgot to define and label the diagram with the universal set (U). For example, let U represent the 200 people that were surveyed about their investments in stocks, bonds, and real estate. Also I believe in your let statements you meant the number of people investing in .... And R should be in the diagram rather than RE?
4. What was your reason for the choice of the deeper structure? Was the Venn diagram critical in helping you solve the problem?

Fig. 6. Problem representation summary.

PROBLEM REPRESENTATION SUMMARY

Name: **Date:**

2	#2	#2	_	_
1	_	_	#1, #2	#1, #2
0	_	_	_	_
	1	**2**	**3**	**4**

PROBLEM REPRESENTATION RESULTS FOR THE FOUR PHASES IN EXAMPLES #1 AND #2

Fig. 7. Problem representation summary.

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2014, Volume 4, No. 1 & 2, 50-54.

THE TRUE COST OF "VALUE" MARKETING

Nicole Philp, Bonnie Daw and Rick Seaman

"Would you like to super size that?" This question, so common in today's fast-food outlets, epitomizes the "more for your buck" marketing strategy that gives consumers the impression that they are receiving more value for their money. Fast-food outlets are using this notion of "value" marketing to up-sell their portion sizes of single-serve foods such as French fries and soda drinks. In the "Value" Marketing activity that follows, grade ten algebra students will look at the implications 'super sizing' is having on our health by studying the relationship between the number of calories and the price of portion sizes of various food items served in fast-food outlets.

The NCTM's *Principles and Standards for School Mathematics* (2000) encourages teachers to include activities that help students "develop and evaluate inferences and predictions that are based on data" (p. 324). By graphically representing the number of calories and the cost of fast food, students gain a "deeper understanding of the ways in which changes in quantities can be represented mathematically and of the concept of rate of change" (p. 305). The following activity will also allow students to see mathematics in a real situation that personally affects their lifestyle choices.

Activity – "Value" Marketing

This was introduced by having students place their "order" at a fictional fast-food outlet. Rather than being served the actual food, students were given the number of teaspoons of lard that represented the fat content in their order (Kuhl, 2003). This lard activity led into a class discussion about the recent concern regarding the fast-food industry's contribution to the increased percent of overweight adults and children. The students were then presented with the news articles, "Children fat, and getting fatter" (Leblanc, 2003) and "Obese Kids Eat More Fast Food" (Associated Press, 2003). One student, after reading that one in three children are over-weight, made a connection to his own classroom with the assumption: "that means at least ten of us in this room are overweight?!"

The connections students were able to make between the news articles and their own lives motivated them to "actually use math," as one student put it, to discover more about the effects of "value" marketing. The objectives of the "Value" Marketing activity were to present students with a real-world situation that could be represented without difficulty by a best-fit line. Students were to meet these objectives by:

1. Organizing and analyzing data and communicating results in oral and written forms
2. Applying the concepts of slope and linear relationships to a real-life situation
3. Making connections between a graphical representation and a corresponding algebraic equation

In groups of two or three, students were given an activity sheet containing data regarding the number of calories and the price of various food items from a particular fast-food outlet. For in assessment students were told to place the price of a food item on the y-axis and the number of calories of the item on the x-axis. The students discussed finding an appropriate scale for their x and y-axes, choosing points for the line of best fit, and determining the corresponding equation.

Following these group discussions regarding the activity, each student in the group was then responsible for completing a part of the project. One student plotted the points and drew the line of best fit while the other student(s) determined the slope and equation of the line. Each group

Philp, N., Daw, B., & Seaman, R. (2006). The true cost of "value" marketing.
The Numerator 5(2), 15-19.

shared their findings with the class by discussing their fast-food outlet's method of "value" marketing. One student explained, "the slope shows that the price rises somewhat, but not nearly as much as the calories". Another student expanded on this observation saying, "71 cents buys an extra 500 calories or 23% more money buys 125% more calories!" Based on this class discussion, students were individually asked to write a letter explaining whether they agreed or disagreed with the fast-food outlet's method of "value" marketing, and to use their graphical and algebraic results to support their arguments. The students' letters indicated a general understanding that although fast-food outlets make it "cheaper to get the combo than the single items," consumers are partially to blame for supporting this type of marketing.

Student Assessment

The Principles and Standards (2000) states that the "assessment of students' understanding can be enhanced by the use of multiple forms of assessment such as ... group projects and writing questions" (p. 372). Through this "Value" Marketing activity, students represented the data graphically, algebraically, orally, and in written form. A rubric was designed to assess students' graphs, mathematical calculations, and letters (see Figure 1).

The sample letter in Figure 2 is a student's interpretation of the graph they drew (see Figure 3) using the data provided in the activity sheet. This student initially interpreted the graph of the cost of a food item versus the number of calories it contained as a direct relationship where the cost increased with the number of calories. After determining a linear equation relating the cost and number of calories, the student then used the slope of the equation to further interpret the graph by stating, "for 41 cents a customer is taking in 200 calories." As was the goal of the activity, this student was able to read the graph and communicate her understanding of the implications "value" marketing is having on our society. Her statements and her graph, however, indicate there are some elements she has failed to consider. The arrows extending the line of best fit indicate students did not consider the domain and range of this relation, as negative values for price and calories are not feasible. Students also failed to consider that rather than being a linear relationship; the graph is really a set of unconnected ordered pairs.

Teacher Reflection

The impetus for this activity arose when our math education professor approached us, his interns, with the resource package "From Wallet to Waistline" (National Alliance for Nutrition and Activity, 2002). He offered it to us as an opportunity to use a real-life resource in our classrooms. The project was also designed to fit into the aims and goals of the curriculum by instilling in students an appreciation for the role of mathematics in society, as well helping students gain an awareness of health issues. Not only did the students use mathematics to find lines of best fit and discuss their implications, but the project also encouraged students to debate the issue of responsibility. One student's argument that "people should restrain themselves," was quickly countered by another that, "if they [fast-food outlets] didn't exist, the temptation wouldn't be there." Some students took the debate further and questioned who was going to pay the "tax hike" when the hospitals begin to "overflow with obese patients."

After studying the relationship between the number of calories and the cost of a fast-food item, students gained a deeper understanding of this issue both on a personal and societal level. Although students were initially hesitant and unsure about how to apply textbook concepts to real-world situations, the discussions that evolved throughout the project indicated that not only were students able to apply these textbook skills, they would also be capable of making an informed decision the next time they ordered at a fast-food outlet.

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Activity Sheet

The True Cost of "Value" Marketing

Directions:

1) Plot the given data.
2) Locate the line of best fit.
3) Find the slope of the line of best fit.
4) Find the equation of the line of best fit.
5) Based on your results, write a letter to your teacher discussing whether you agree or disagree with the method of "Super Sizing".
6) Hand in: Per pair: This sheet and your graph

Per person: Letter to your outlet

Outlet	Item	Size	Calories	Price ($)
"	Value" Meal	Medium	1270	3.93
		Large	1510	4.39
		"Value"	1710	4.80

Slope Calculations:

Linear Equations:

Philp, N., Daw, B., & Seaman, R. (2006). The true cost of "value" marketing.
The Numerator 5(2), 15-19.

Figure 1

Rubric for "Value" Marketing Activity

Name: ______________________________ Date: __________

Mark out of 16: ______

Criteria	4	3	2	1	0
Group Work	The group is on task. Each member contributes to the project and encourages each other.	The group is usually on task and everyone contributes to the project.	The group is somewhat off-task and not everyone contributes to the project.	The group is rarely focused.	The group can't stay focused and no work is getting done on the project.
Graph	The graph is neat and accurate. It has a title and the axes are labeled with titles that represent the units being used. Scales for axes are well-chosen. The two points used for the slope calculations are well-chosen and clearly marked.	The graph is missing one of the required elements.	The graph is missing 2-3 of the required elements.	The graph is missing 4-5 of the required elements.	The graph is missing all 6 of the required elements.
Slope & Equation	The slope is accurately calculated with well-chosen points. All calculations are included and are neatly done. The equation of the line is correct and written legibly on the graph.	Missing 1 of the required elements.	Missing 2 of the required elements.	Missing 3 of the required elements.	Missing 4 of the required elements.
Letter	Explanation of what the graph and slope represent is insightful. Letter is knowledgeable and persuasive. Letter is in proper form and free from grammatical errors.	Explanation of what the graph and slope represent is clear, although not insightful. Letter is knowledgeable but lacks persuasiveness. Contains a few grammatical errors that do not deter from explanation.	Letter displays evidence of understanding of what the graph and slope represent but the written explanation is not clear. Contains grammatical errors .	Explanation of what the graph and slope represent lacks understanding. Letter contains several grammatical errors.	Graph and slope are poorly explained and show a lack of understanding of linear functions. Letter is illegible.

Philp, N., Daw, B., & Seaman, R. (2006). The true cost of "value" marketing. *The Numerator 5*(2), 15-19.

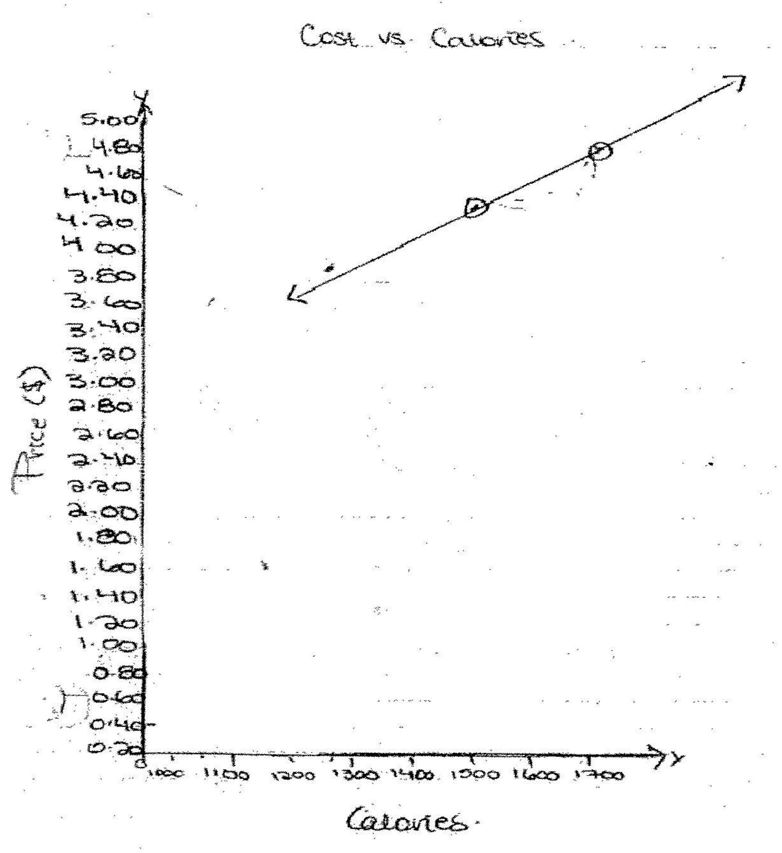
Figure 2

I disagree with the method of 'Super Sizing' because you are making the world gain extreme amounts of unnecessary weight. Children love the food you sell and because you allow them to eat more for less. At the same time you are also increasing their daily amount of calories to a maximum.

The graph below shows that the more you raise the price, the bigger the meal and the more amount of calories your customers take in. The graph also shows that for 41¢ a customer is taking in 200 calories. The future of your business lies in the hands of your customers, and without healthy, living customers you won't have a business.

Sincerely Yours,

Figure 3



vinculum: Journal of the Saskatchewan Mathematics Teachers' Society
2014, Volume 4, No. 1 & 2, 55-59.

THE 'BIG DIG' ACTIVITY

Jennifer Bell, Katy Biech, Dylan Johns and Rick Seaman

What follows is a realistic problem-based learning (PBL) activity (Wikipedia, 2006) that has application to a variety of subject areas. The 'Big Dig' activity was based on the *Wascana Lake Urban Revitalization Project* (Saskatchewan Property Management Corporation, 2006) a project that had gained a lot of media attention in Regina, Saskatchewan, Canada [http://www.regina.ca/] where Wascana Lake (Wascana Centre, 2006) is situated.

During the semester before internship as a class assignment the mathematics majors in the Faculty of Education at the University of Regina [http://education.uregina.ca/] were asked to develop a lesson plan based on the 'Big Dig' that would make connections with subjects such as mathematics, English, social studies, and science. After the lesson plans were returned the professor suggested to the class that they synthesize their ideas into one lesson plan that one of them could teach from during internship. When this particular intern finished teaching the lesson they would then pedagogically discuss the lesson with the next intern who would then teach the lesson to their students and so on. The intention was to not only create a fun and motivating lesson but to also improve the lesson each time it was taught.

Three secondary mathematics majors followed up on this suggestion and used the 'Big Dig' activity during their fall internship. Two interned at one high school in Regina and the third in rural Saskatchewan. The following description is about the lesson taught by the first intern during internship with further comments included in the Intern Reflections section.

The 'Big Dig' Activity

Wascana Centre (Wascana Centre 2006) is a beautifully landscaped park surrounding a 120-hectare man-made lake located in the heart of Regina. A variety of provincial government buildings are found within this 930-hectare parkland development, which attracts around five million visits annually ranging from tourists to local citizens out for a walk or jog. The Wascana Lake Urban Revitalization Project became necessary because the lake had been gradually losing depth and becoming a slough. On average, the lake was only 1.5 metres deep and the growth of weeds was affecting the water quality and capability of the lake to support recreational activities. On October 3, 2003 civic, provincial, and federal governments announced that the Wascana Lake Urban Revitalization Project was to begin with money coming from these three levels of government. The excavation began on January 6, 2004 and was completed on March 21, 2004 just before spring thaw.

As a set to the activity we began by having students share any information they might have about Wascana Lake and the Wascana Lake Urban Revitalization Project. Following the discussion we introduced the 'Big Dig' activity and the rubrics developed to assess the activity. For example a rubric was used to assess the written component of students' work (see fig. 1).

The students were then placed into six predetermined homogenous groups A to F. Each group received a PBL worksheet (Delisle, 1997) to be completed before they began any work on the activity (see fig. 2). It was emphasized that there were no wrong responses and the worksheet was designed to get them started. Each group also received a topic questions sheet (see fig. 3) with subject matter specific to the 'Big Dig'. Groups A and F worked with Cons and Money, Groups B and E worked with Pros and Volume, and Groups C and D worked with History and Time. In order for the groups to be easily identified we gave each group a different colour.

Bell, J., Biech, K., Johns, D., & Seaman , R. (2007). The 'big dig' activity.
The Numerator 6(1), 25-29.

The first day students were given the PBL worksheet (see fig. 2) to complete and hand in. Students quickly discovered that they actually knew a lot about the 'Big Dig'; one student said with pleasure: “Wow! We actually know a lot about this stuff.” At the end of class students were also given a set of newspaper clippings from the *Regina Leader Post* [http://www.canada.com/reginaleaderpost/index.html] on which to make notes [see fig. 4 for an example (Luterbach, 2003).

The second day was dedicated to student research. Students used the newspaper clippings from the previous day plus the Internet to do research on the 'Big Dig' project. They then wrote a first draft of the results of their research.

The third day was used by students to get ready for their presentations. Most chose to make a poster to display information and their findings in the classroom or on school hallways.

The beginning of the fourth day was again spent by students on presentation preparation and watching a video of the 'Big Dig' made by Global Television Regina [http://www.canada.com/globaltv/regina.html]. This video reinforced what the 'Big Dig' project entailed and provided, especially to students, which had not seen the deepening of the lake, more information about the project.

Students spent the fifth and last day watching each group's presentation. After the presentations were completed the students each wrote a paragraph reflecting on the activity. An overwhelming number found the activity engaging and useful: “I didn't know math was involved in activities like this”, “It was fun!”, “We should do more things like this.”, “It didn't even feel like math class!” and “It was nice to spend time outside the classroom and working with new people that I wouldn't work with otherwise.” The students also documented that different subject areas could be connected through activities that related to everyday life.

Intern Reflections

Although no lesson is ever perfect all three of us would use this activity again if given the chance. We discussed, at length, the changes that we would make if we did indeed teach this lesson again. We all agreed that the rubrics used in this activity were effective assessment tools that were both coherent and meaningful for the students; however, the levels within the beginning through exemplary levels in the written component rubric (see fig. 1) could be further refined. For instance, instead of a group receiving a grade of 3, 'developing', for grammar and spelling they might receive a 3.1 while another group may have done better but still deserved the 'developing' level and therefore would receive a 3.8.

We would also put even more emphasis on open-ended questions and investigations to further students' learning. Especially more questions on the worksheets concerning the impact projects such as the “Big Dig” have on the environment.

The timing of the activity could also be altered to best suit the needs of a particular classroom. Instead of five straight days from beginning to end, the activity could be started at the beginning of the semester with the students given class time when possible to work on it. It might also be possible for students to work on the activity in other subject areas.

To enhance the activity further students could take a field trip to Wascana Lake. The Global Television video does a great job of explaining the dig and showing the lake, but a trip to the actual site at the time would be more meaningful and engaging for the students. Also because of its accessibility on the Internet we would now use the 'Big Dig' video clip (Saskatchewan Property Management Corporation, 2006) with our classes instead of the Global Television video.

As interns the 'Big Dig' activity gave us an interdisciplinary approach to teaching mathematics while working with each other to create and then reflect on the success of a lesson just taught. An experience that was both memorable and informative.

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Written Component Rubric	Beginning 2	Developing 3	Accomplished 4	Exemplary 5	Score
Data Collected	Hardly any data collected	Some data collected. Few connections made.	Relevant data collected. Connections between data and PBL question made clear.	Many types of data collected. Numerous connections between PBL question and data.	
Quality of Information (Content)	No specific details. PBL question not addressed. PBL chart not done.	Few details. Details provided could be more relevant. Beginning to address PBL question. PBL chart included.	Some details are provided. Data collection methods made clear and relevant to the topic. PBL included and complete.	Content detailed and directly related to the PBL problem. PBL chart included and complete. Data collection clearly laid out and explained.	
Grammar & Spelling	Very frequent grammar and/or spelling errors,	More than two errors.	Only one or two errors.	All grammar and spelling are correct.	
Neatness	Illegible writing, loose pages. No cover page.	Hardly legible writing or messy type written page. Pages not put together. No cover page.	Legible writing or done on a computer. Clean and neatly bound in some way with cover page.	Neatly handwritten or typed, clean and neatly bound with a cover page.	
Organization / Reflection	Not organized, events make no sense. No justification or reflection included.	Some organization, flow of information jumps around, start and end are unclear. Possible justification within – not clear. Very little reflection. More focus needed.	Fairly well organized. Nice flow of information. Justification for findings are well-written and reflection included.	Good organization, events are logically ordered, clear sense of beginning and end. Justification for findings and well-written reflection included.	
				Total	

Fig. 1. Written Component Rubric.

As a group you need to fill out the following chart in preparation for your research. The chart must be included in the final project (as a pre-research stage). This will be a significant portion of your mark as the planning stage so be sure to be thorough.

Ideas (What do you already know about the problem? How do you plan to represent your information? What approach would you like to take?)	**Facts** (What facts do you know about The Big Dig (these need to be verified/backed-up)	**Learning Issues** (Is there something you do not understand? What do you need to find out prior to your research?)	**Action Plan** (How are you going to find out the missing information? What types of resources do you plan to use?)

Fig. 2. Problem-Based Learning (PBL) Worksheet.

Group A and F - Cons and Money

Cons

1. Before looking at your resources, what are some of the arguments you can think of against deepening Wascana Lake?
2. What are some of the arguments presented in your resources, elaborate upon these, are they accurate.
3. Should they deepen the lake? Discuss.

Money

1. How much will it cost to deepen Wascana Lake? How much does that equate to a m3 of removed soil / sediment?
2. Where is the money coming from? (How could you represent these figures visually or otherwise?)
3. Where did all the money go?
4. How many cars, skateboards or pairs of jeans could you by with all that money?

Group B and E - Pros and Volume

Pros

1. Before looking at your resources, what benefits can you as a group come up with about the deepening of Wascana Lake
2. What benefits for the Dig do your resources offer? Elaborate upon these, are they accurate?
3. Should they deepen the lake? Was it worth all the trouble and effort? Discuss.

Volume

1. How much 'stuff are they taking out of the lake? How much is that? What is the stuff, are there different materials?
2. How much material does each truck hold?
3. Where did all the material go?
4. How many cars, basketballs, or dice would it take to fill the volume of material they are removing?

Group C and D - History and Time

History

1. What is some of the history of Wascana Lake?
2. Has it ever been dug before, how, why, when by who?
3. Who is blessing the lake? Why, How?

Time

4. How long did it take to Wascana Lake?
5. Why is there a deadline, discuss? Did anything slow the digging down?
6. How much fill must be moved an hour I minute I day to meet the deadline?
7. How many episodes of Simpsons, or the OC could be watched over the period of digging?

Fig. 3. Topic Questions Sheet.

Bell, J., Biech, K., Johns, D., & Seaman , R. (2007). The 'big dig' activity.
The Numerator 6(1), 25-29.

Tuesday, August 12, 2003

Hidden costs of deepening lake

Perhaps most people would be in favour of deepening Wascana Lake. However, this is not a simplistic issue.

Regina is privileged to have a most excellent natural treasure in the Wascana Marsh. Many people value it as an opportunity to simply appreciate nature, to have a quiet or reflective walk, jog, hike or take a bicycle trip on the various trails and paths around the edge of the marsh.

Schools and other groups use the marsh to study the natural world.

For the wildlife, this is its home.

Yet instead of protecting, valuing, appreciating and using this as a tourism draw and natural refuge from the city, the trend seems to be to use it for "development" or a dumping ground.

The plan for deepening the lake calls for the dumping of the dredged material to fill the Goose Island area. The idea is to develop this area, providing full public access.

This is the sole remaining off-limits area for wildlife within the entire marsh other than the water itself. The surrounding lawns and managed areas are largely devoid of cover and privacy for nesting waterfowl.

It is perhaps a symbol of the inability of many to value and appreciate the vanishing prairie and potholes that we build a mountain to replace our natural heritage.

This is an ultimate irony.

Yet more of this is apparently planned. The dredged fill will no doubt expand the mountain site to overwhelm the remaining remnant prairie area of the marsh.

Draining the marsh and the lake does have severe ecological impacts. The biota of the lake and marsh was dramatically affected by the previous drainage of the marsh and lake with perhaps the loss of the majority of the turtle populations, amphibians and fish.

Apparently, a recent brief survey for amphibians and reptiles indicated very scarce current populations within the marsh. Part of this may be attributed to the season and low water levels.

If the lake and marsh are emptied again, plus continue to remain dry because of dredging issues until 2004 or possibly 2005, the consequences for the wildlife are obvious.

The marsh needs to be protected and allowed to contain runoff to sustain the remnant populations of amphibians and fish of the area. These can return to the lake if and when the project is finished.

At present, the marsh and lake does provides irrigation water for the Wascana Park system. Plans to expand and develop this system further through using dredged materials and landscaping are perhaps flawed because, as we see this year, water shortages are the predicted trend of the future.

Instead of moving radically towards water conservation through xeriscaping, the Wascana Authority continues with its policy of creating more lawns and landscaping.

Deepening the basin to create more storage of water within the lake does not address long-term predictions for less runoff.

This is short-sighted. Will the marsh be regularly drained in order to maintain the decreasing water levels within the greater basin of the lake?

Can the water quality be improved without addressing issues upstream?

How can we project the amount of silt coming into the system without understanding impacts upstream?

Perhaps the Wascana Centre Authority needs to be subjected to more public scrutiny to evaluate its planning approaches and judgments.

Citizens need to grasp the total financial and ecological impacts of this plan, plus other expenditures, flowing from its approaches.

For example, a greater "naturalization" of the park system would save huge dollars.

We cannot simply pretend that the water flowing into the Wascana Creek system will be adequate to service the needs of expanding lawns, wildlife and boating activities.

Lawns require mowing and maintenance, trees apparently need to be planted and replanted, sprayed, and pruned and fertilized. This all cost dollars.

More lawns attract and sustain more Canada geese. This species will actually increase with "development"; other species will not.

The park could be maintained using other methods to reduce costs. The authority is out of step with modern views, in my opinion.

Can the lake be deepened or should it be? These are important question. How this may be achieved without the destruction of the marsh is another question.

Long-term planning for the entire Wascana Creek system including the portions upstream, the portion within Regina and downstream, is vital. Comprehensive plans are required — not *ad hoc* development.

We need to have a greater vision and understanding of implications and consequences of issues of development.

The current fast-track approach for approval of the Wascana Lake project does not adequately address many of these issues, in my opinion.

BOB LUTERBACH

Regina

Fig. 4. Sample Resource [Regina Leader Post, used with permission]

vinculum: Journal of the Saskatchewan Mathematics Teachers' Society
2014, Volume 4, No. 1 & 2, 60-67.

LET'S TALK ABOUT OUR IDEAS

Rick Seaman

Sharing ideas of how one should begin a year teaching ninth-grade mathematics hopefully is of interest to readers of *The Numerator*. Although not everybody may agree with everything written here [I would hope not], some ideas might be gleaned and hopefully some responses generated in future issues of *The Numerator*. What follows is a synthesis of my own and shared ideas. Wherever possible, I will give credit to those who shared their ideas with me. To these colleagues I extend a sincere 'thank-you'.

In the Beginning…

Initially I would see the ninth-graders to whom I would be teaching mathematics for a 30-minute homeroom meeting. In most cases, the students would come reluctantly from the nearby swimming pool. The last thing on their minds was to have a pencil, paper or pen and be at school. It was in this environment that I stood with objectives to:

•provide information for the year:
•set the tone for the year: and
•put a seating plan in place.

I wanted to minimize any potential problems with classroom management. I am sure it had everything to do with survival, yet I counted the days until I could open up and have fun with them!

Day One +

If I was still not satisfied with the previous day's seating plan because of perceived pockets of potential problems. I had all the students stand up and move to the back of the room. I then proceeded to direct the students to where I wanted them to sit using the following principles:

•split up what I perceived to be potential problem areas;
•fill up the desks starting at the front; and
•move what I thought to be the greatest "problem" student(s) to a desk situated in front of mine.

Quite often these students would ask why they had to sit in front of me. My response was: "I don't have anyone to talk to in this class and I thought you would talk to me." Over the years, I had some pretty good conversations with these students.

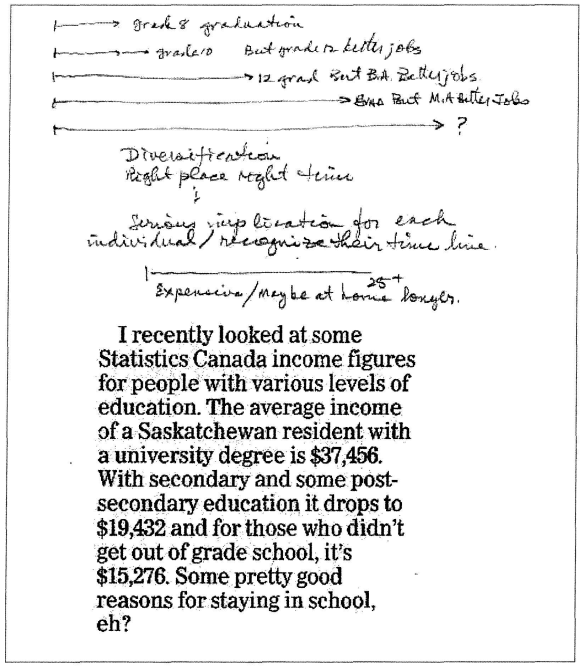
Attitudes and Beliefs

Another principle that I espouse is that if you cannot change a student's negative attitude and/or beliefs about mathematics, you will not be very successful teaching them mathematics. I will share with you some things I tried to help improve students' attitudes and beliefs towards mathematics:

I handed out an executive version of the course outline from the curriculum guide. I then went over it with the students so they could see what was going to be taught and that it was a manageable amount of material.

I would show students a cartoon on an overhead about why a cartoonist needs to know mathematics in order to justify why learning mathematics is important for future occupational choices. I also used some Statistics Canada average income figures [now dated] to indicate the need to stay in school and get an education.

Seaman, R. (2008). Let's talk about our ideas.
The Numerator 6(3), 12-23.


grade 8 graduation
grade 10 But grade 12 better jobs
12 grad But B.A. Better jobs
Grad But M.A. Better Jobs
?

Diversification
Right place right time

Serious implication for each individual / recognize their time line.

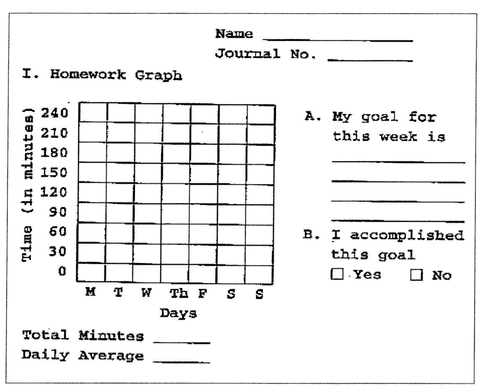
25+
Expensive / maybe at home longer.

I recently looked at some Statistics Canada income figures for people with various levels of education. The average income of a Saskatchewan resident with a university degree is $37,456. With secondary and some post-secondary education it drops to $19,432 and for those who didn't get out of grade school, it's $15,276. Some pretty good reasons for staying in school, eh?

I told the students about my father's scenario upon graduation. When he graduated from school they called it grade eight: at age 16 he wound up working at a bank. Eventually, my story would go from describing that system as K-8 to what some say is now K-16. I acknowledge using potential income to motivate students to learn mathematics or become life-long learners is subject to more discussion.

I then discussed how students would be assessed [a combination of announced and unannounced quizzes. projects...] and explained the rationale for these methods of assessment. Initially, the quizzes were structured so that if anyone watched and minimally tried she/he would have success [Attitudes and beliefs **principle**.]. The degree of difficulty of each quiz became greater as the year went on. Initially the class would have no results less than 80% and I would tell them that I think that 'downtown' will not believe how good they had become. This method of assessment also provided immediate feedback concerning students' misunderstandings in mathematics [Thanks Cec Person.].

Recognizing that many students typically have horrendous time management, homework, and study skills, I gave them handouts that included tips and graphical templates to support these skills. For example [source unknown]:



Overall, this approach has merit. However, it takes a lot more than just lip service! I admit that the lack of success with improving these skills with my students was my fault.

I used a cartoon as a lead-in to discuss with students the importance of asking questions in class and not ‘keeping secrets” [Thanks June Seaman.]. I stressed the importance of asking questions because of the greater than one pupil-teacher ratio and the importance of individualizing learning in mathematics. If students identified what they didn’t understand rather than depending on me to find it, there was a greater possibility of succeeding in mathematics. I then discussed the following *Regina LeaderPost* article, which was written about a ninth-grade mathematics student who had kept a secret [I was undoubtedly using an extreme example to encourage students to not keep secrets concerning their misunderstandings in class.]:

Break-ins lead to jail sentence

By KEVIN BLEVINS
of The Leader-Post

A Regina man — who broke into nine different homes just seven months after committing the very same crime — isn't getting a second chance this time.

Provincial court judge Harvie Allan handed a five-month jail term Tuesday despite pleas of leniency from the 19-year-old man's lawyer.

Allan said the young man had to serve time behind bars for breaking the law, even though he believed the accused is young enough to be rehabilitated.

"When is the kid going to learn that you can't thumb your nose at society? We can't just say, 'Boys will be boys and we have to rehabilitate them'," the judge said. "I think it's just such a terrific thing to do, committing these break and enters when you are out on parole for doing the very same thing seven months earlier."

pleaded guilty to nine different charges of break and enter, one count of possessing a stolen car and one count of breaching probation.

According to Crown prosecutor Dean Sinclair, and two friends, both of whom have yet to be dealt with by the courts, broke into nine different Regina homes between May 6 and May 15.

In each instance, the residents were asleep when the trio broke in, he said.

"Fortunately for them, and for Mr. none of them were awakened," he added.

In most cases, and the two others stole car keys and fled the scene in a stolen vehicle, Sinclair said.

Defence lawyer Mark Mulatz said

> **"He associates with street people, because most of the time that's where he is."**
>
> **— Mark Mulatz**

is a wayward young man, who fled home at an early age and usually finds trouble by hanging out with the wrong crowd.

"He associates with street people, because most of the time that's where he is," Mulatz said, adding the break-ins "were not idea."

He added is at a crossroads in his life and therefore deserved a second chance, especially since he had already served two-months in jail awaiting Tuesday's court date.

But Sinclair pointed to record — various terms of probation for four previous break and enters, the last being just seven months ago.

"Here we have before us someone who has committed 13 break and enters, and except for two months on remand, has not spent a single day in jail," the prosecutor said. "It's no wonder we have an epidemic of break-ins in this city.

"At some point, the message must come from the courts that this type of crime won't be tolerated."

Allan said he agreed with Sinclair's argument.

"I take into account Mr. Mulatz's submissions that just tagged along, but the good Lord gave him free will, and he chose not to use it," the judge concluded.

I would then go to the board and draw two vertical lines approximately a metre apart on the front board where students could indicate the mathematics that gave them the most difficulty before grade nine. The two vertical lines made the task appear to be a manageable and I would ask the students to “List a math weakness someone might have” [Thanks Ev Stange]. Here is a prioritized summary from four consecutive years:

Negotiating the Curriculum

"Think of a math weakness you think someone might have. We will list them on this part of the board and then rate them."

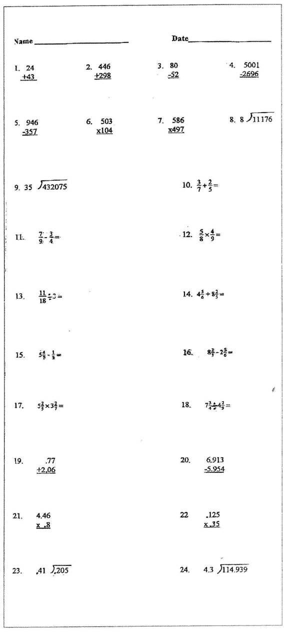
1. Problem solving – Algebra using variables.
2. Division.
3. Times tables.
4. Multiplying fractions – +/- integers.
5. Probability.
6. Graphs – Changing fractions to decimals – Dividing fractions.

1. Making an equation from a problem.
2. Geometry formulas.
3. Dividing fractions.
4. Multiplying numbers/letters (algebra).
5. Division.
6. Multiplying fractions – +/- integers – x/÷ decimals.

1. Problem solving – Integers.
2. Algebra.
3. Fractions.
4. Square Roots – Powers of 10, etc.
5. Mixed fractions – Decimals – Calculating volume.

1. Problem solving.
2. Orders of operation.
3. Exponents.
4. Integers.
5. Multiplying fractions – Geometry.
6. Division – +/- fractions.
7. Variables.
8. Decimals.

At this point I would tell the students to 'forget' the ninth-grade mathematics course and that our main emphasis would be to understand the areas they indicated that were of concern! Consulting the students about their mathematical concerns was an excellent way to start working on their attitudes and beliefs toward mathematics. This certainly beats walking into class and telling them that we are going to begin the year with a review that turns out to be the same material because of the predictable nature of students' K-8 mathematical concerns! The students had assumed some ownership for their learning by negotiating their own curriculum!

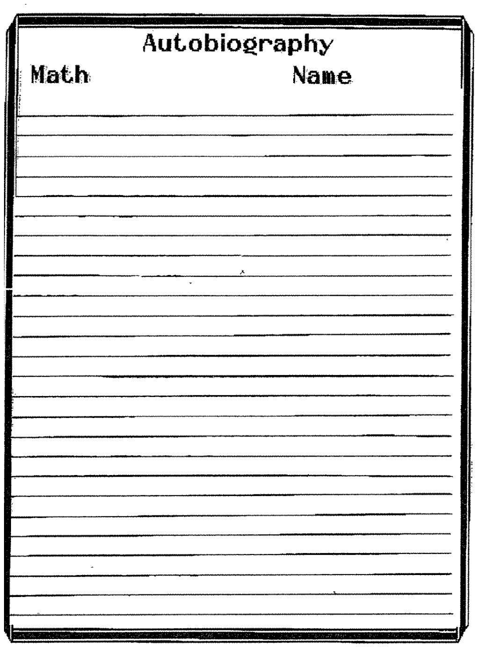


Name ____________ Date ____________

1. $24 + 43$
2. $446 + 298$
3. $80 - 52$
4. $5001 - 2696$
5. $946 - 357$
6. 503×104
7. 586×497
8. $8 \overline{)11176}$
9. $35 \overline{)432075}$
10. $\frac{3}{7}+\frac{2}{5}=$
11. $\frac{7}{9}-\frac{3}{4}=$
12. $\frac{5}{8}\times\frac{4}{9}=$
13. $\frac{11}{18}\div 3=$
14. $4\frac{5}{6}+8\frac{2}{5}=$
15. $5\frac{4}{7}-\frac{1}{3}=$
16. $8\frac{2}{3}-2\frac{5}{6}=$
17. $5\frac{2}{5}\times 3\frac{2}{7}=$
18. $7\frac{3}{4}\div 4\frac{2}{9}=$
19. $.77 + 2.06$
20. $6.913 - 5.954$
21. $4.46 \times .8$
22 $.125 \times .35$
23. $.41 \overline{).205}$
24. $4.3 \overline{)114.939}$

I had the students write a diagnostic evaluation examining their understanding of applying the four fundamental operations on the rational number system. They were not allowed to use calculators. We would begin the task in class, and normally they would finish it at home with the understanding that they were to get no outside help. I explained we would take up the diagnostic evaluation in class the next day with the idea that it would provide us' with feedback as to what they did and did not understand applying the four fundamental operations on the rational number system. When I took up the questions wit them the next day I asked the students not to erase the questions in their notebook that they got wrong. This was done so they could compare and contrast them with their solutions once they understood how to solve the questions correctly. Throughout the semester, I had the students refer back to this diagnostic and choose a question they had initially been unsuccessful with and do it again. What follows is a typical diagnostic [Faded memory suggests thank you Alex Youck.]:

As a task to also be handed in [the next day] I handed out what I called a Math Autobiography where the students on page one indicated their feeder or associate school and wrote out their mathematical history.



Autobiography

Math Name

Some ideas and questions were suggested to trigger the students' memory like [source unknown]:

Math Autobiography

Write down your mathematical history. Aim for a target of two pages. As a student, you will have clear recollections of teachers, topics, experiences and feelings. Try to reconnect both successful and not-so-successful experiences and how you felt about them. Here are some ideas and questions to trigger your memory, but feel free to add anything you feel is important:

Recall any positive/negative experiences you had learning math.
Do you like math?
Are you good/bad at math?
How do you know that you are good/bad at math?
Was there any part of math learning you remember as finding particularly difficult?
Describe your feelings about math throughout elementary school?
What are the positive qualities of your favourite math teacher?
What are the negative qualitites of your least favourite math teacher?
What did people say to you when you made a mistake in math or when you did exceptionally well?
What is the best/worst part of math?
What influences do you feel past experiences have had on your ability to do well in mathematics?

It took almost a month to touch base with each student, and I used the Math Autobiography as a catalyst to initiate the conversation. I would explain to them that others experience the same difficulties that they were having with math. I shared with them that in grade four, I believed the world was over because I couldn't understand the division algorithm!

A pre- and post-metacognitive questionnaire that probed students' attitudes and beliefs toward mathematics and problem solving over five months was handed out for the students to complete and keep. It also was used to initiate conversation with the students as I met with them individually. The post-metacognitive questionnaire was useful to give feedback to parents/guardians regarding their son or daughter's attitudes and beliefs concerning mathematics on their January report cards. See both pages that follow:

Name: ______________ Date: __________

1. Do you like math?

Not at all 1	1/4 of the time 2	1/2 of the time 3	3/4 of the time 4	Always 5

2. Why or why not?

3. Describe your math skills.

Very poor 1	Poor 2	Average 3	Good 4	Very good 5

4. Describe your math grades.

Very poor 1	Poor 2	Average 3	Good 4	Very good 5

5. Math word problems are always solved in less than 5 minutes, if they are solved at all.

Strongly Disagree 1	Tend to Disagree 2	Undecided 3	Tend to Agree 4	Strongly Agree 5

6. Only geniuses are capable of discovering or creating mathematics.

Strongly Disagree 1	Tend to Disagree 2	Undecided 3	Tend to Agree 4	Strongly Agree 5

7. Knowing how to solve a problem is less important than getting the correct answer.

Strongly Disagree 1	Tend to Disagree 2	Undecided 3	Tend to Agree 4	Strongly Agree 5

8. Describe how well you solve math word problems.

Very poor 1	Poor 2	Average 3	Good 4	Very good 5

9. What questions do you ask yourself while you are reading math word problems?

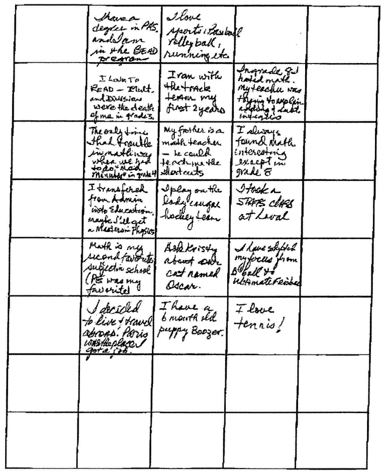
10. Do you like to solve math word problems?

Not at all 1	1/4 of the time 2	1/2 of the time 3	3/4 of the time 4	Always 5

11. Why or why not?

Classroom Environment

Finally. I asked the students to write a one-minute commercial about themselves to be handed in on Day Two. Upon receipt of their commercial, I placed information from it in a square within a grid. The students received a copy of the completed grid the next day and then proceeded to go around the classroom and ask classmates if they were the individual described. If they were, then they were asked to sign the square. This process continued until there was a signature on every square. It was a great way for students to get to know each other since not everyone had attended the same elementary school in eighth-grade. What follows is an example from a recent university class I instructed:

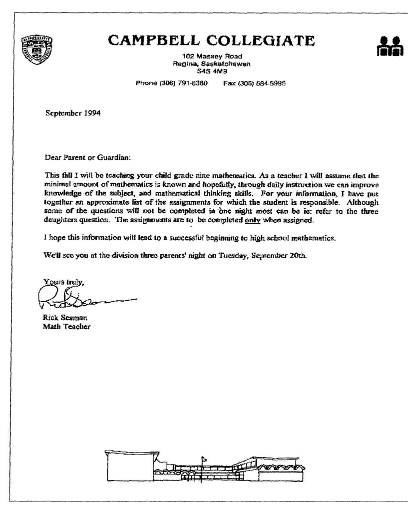


	I have a degree in PAS. and I am in the BEAD program	I love sports: baseball volleyball, running etc.		
	I love to READ – Mult. and Division were the death of me in grade 3.	I ran with the track team my first 2 years	In grade 9 I hated math. My teacher was trying to explain adding & subt. integers	
	The only time I had trouble in math was when we had to do "Mad Minutes" in grade 4	My father is a math teacher – he could teach me the shortcuts	I always found Math interesting except in grade 8	
	I transfered from Admin into Education, maybe I'll get a Masters in Physics	I play on the lady league hockey team	I took a STATS class at Laval	
	Math is my second favorite subject in school (PE was my favorite)	Ask Kristy about our cat named Oscar.	I have shifted my focus from B-ball to Ultimate Frisbee	
	I decided to live & travel abroad. Paris was the place I got a job.	I have a 6 month old puppy Boozer.	I love tennis!	

Another way to develop class cohesiveness and camaraderie is to have the students participate early in the year in a tug-o-war challenge at noon hour with the other ninth-grade homerooms. As a class we would discuss how everyone had a role from actually participating in the event to cheering on their classmates. As a homeroom teacher I would get out on the gym floor and cheer them on with as much energy as I could muster [Read: Make a fool of myself.]. They loved it!

The Home

Another principle: I believe that the involvement of the parent(s)/guardian(s) in a student's education is instrumental in achieving academic prowess. Here is a letter I sent home during the first week explaining what the pedagogic objectives were for the mathematics class, and an invitation to parent/guardians' night taking place later in September



CAMPBELL COLLEGIATE
102 Massey Road
Regina, Saskatchewan
S4S 4M9
Phone (306) 791-8380 Fax (306) 584-5995

September 1994

Dear Parent or Guardian:

This fall I will be teaching your child grade nine mathematics. As a teacher I will assume that the minimal amount of mathematics is known and hopefully, through daily instruction we can improve knowledge of the subject, and mathematical thinking skills. For your information, I have put together an approximate list of the assignments for which the student is responsible. Although some of the questions will not be completed in one night most can be ie: refer to the three daughters question. The assignments are to be completed only when assigned.

I hope this information will lead to a successful beginning to high school mathematics.

We'll see you at the division three parents' night on Tuesday, September 20th.

Yours truly,

Rick Seaman
Math Teacher

I included a tentative' copy of the assignments, which emphasized a knowledge base and problem solving dichotomy. The assignments were made up of questions that were isomorphic to what was taught in class. I would incrementally assign questions from various pages over a period of months rather than assign entire pages of questions daily. This strategy made students practice skills daily using one example of 'each type' while gaining more confidence in their mathematical abilities and also retain what they had learned. A typical daily assignment would contain four questions representing each fundamental operation and representation on the rational number system. Some students had difficulty the first day, but eventually repetition of both instruction and assignments; they began to understand how to answer the questions. These assignments were **not** overwhelming in quantity and with repetition the students were successful. [Translation: an improvement in their attitudes and beliefs toward mathematics.]

Even though I didn't assign 30 questions per assignment as John Saxon's texts suggest: I thank him for the idea. Incidentally, all lesson plans included both knowledge base objectives and thinking objectives. Accordingly the students' binders were split it into two sections: One that contained knowledge base notes and the other that contained problem solving [thinking] notes.

DO THE ASSIGNMENTS WHEN THEY ARE ASSIGNED ONLY!

Assignment	Knowledge Base	Problem Solving
1		'? question'
2	Read Edward Willet 24 questions (+, x, -, ÷)	'Petals around the rose'
3	P-2 1 P-3 4 Make corrections to 24 questions (from assignment 2).	Memorize Cognitive Strategy on handout (I, II, III, IV)
4	P-5 6 P-7 8 P-9 5	Memorize Cognitive Strategy
5	P-27 6, 7	'In praise of the British' Read 'Being the problemsolver' handout.
6	P-33 11, 12	Practise your cognitive strategy on 'In praise of the British'
7	P-33 1, 3, 5	Use a problem from class to teach how to apply our cognitive strategy to someone at home. "Three boys played.."
8		"Twelve couples..." question. Write down how you'd apply the cognitive strategy I, II to this question.
9/10		Read over group behavior sheet.
11/12	Not quite minimum wage P-35 1, 2, 4 (c) (f), 7	'Bone in a glass'
13	P-37 3, 4, 6 (every second one (a) (c) (e)...). P-39 8, 10	
14	P-41 5, 6, 7, 8	1/2, 2/3, 1, 8/5, 8/3, ______
15	P-43 2, 3, 4, 5	1211, 111221, ______, ______
16	P-45 3, 4, 5	8,5,4,9,1,7,6
17	P-47 1, 2, 3, 4, 5	Find a strategy to win at Nim.
18	P-49 3, 4, 5, very important assignment	
19	P-51 4, 5, 6	'Rita's List'
20	P-191 9	Fox and Goose question
21	P-53 3, 6	What is units digit of 9^{1990}?
22	P-54,55 1, 2, 5, 7 Put brackets in 9x5 + 2 - 8x3 + 1 = 22 to make it correct.	What is units digit of 3^{1997}?

To help students gain control over their thinking, a thinking strategy was handed out to be memorized and referred to daily [Thanks Peter Hemingway and Don Kapoor.]. For example:

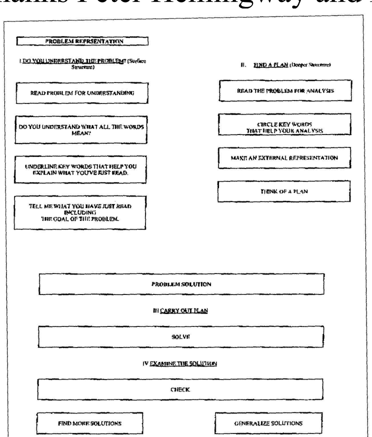
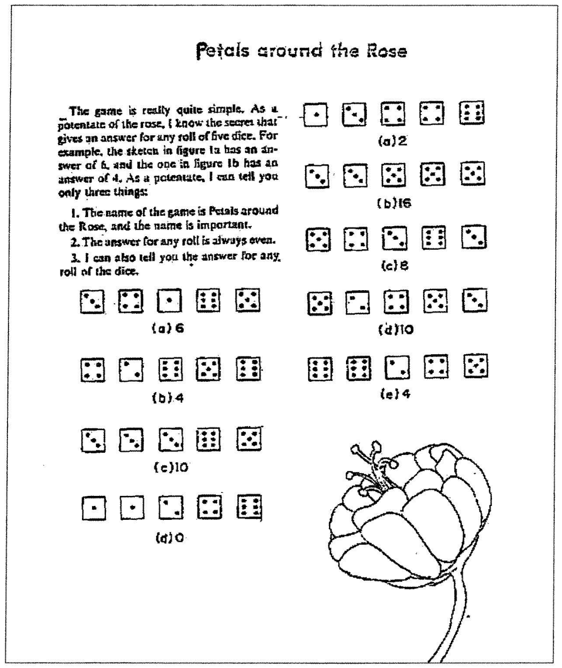


Figure 0.1

Cognitive Strategy

I followed this up by sending home problems with the students for them to work on with their parent(s)/guardian(s). The carrot dangled in front of the students for supplying a correct answer was a ‘spare’ period. One such problem taken from *The Mathematics Teacher* follows [Rose drawn by Duane Wright, a colleague.]:



Petals around the Rose

The game is really quite simple. As a potentate of the rose, I know the secret that gives an answer for any roll of five dice. For example, the sketch in figure 1a has an answer of 6, and the one in figure 1b has an answer of 4. As a potentate, I can tell you only three things:

1. The name of the game is Petals around the Rose, and the name is important.
2. The answer for any roll is always even.
3. I can also tell you the answer for any roll of the dice.

(a) 6

(b) 4

(c) 10

(d) 0

(a) 2

(b) 16

(c) 8

(d) 10

(e) 4

Without fail every parent/guardian night I would be asked for the solution to one of these problems, to which I would respond. ‘I never could get that one and I hoped one of you would get it!”

I hope that this article generates some future conversations in *The Numerator* not only on this article but also on other contributions so that this journal can become a ground swell of ideas for the teaching of mathematics.

Optimistically yours,
Rick Seaman

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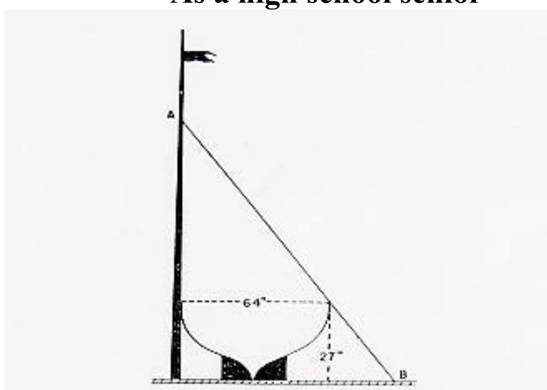
2014, Volume 4, No. 1 & 2, 68-73.

LOOKING BACK

Rick Seaman

After a 25-year career teaching mathematics in grades eight to twelve, it is interesting to look back at the pedagogic moments that led to my return to university to gain a deeper understanding about teaching mathematics: That is, beyond what I had learned from reading, attending professional development seminars, preparing lessons, teaching, reflecting on my teaching.

As a high school senior



46. The lifeboat of a sailing vessel has a 64-inch beam and a 27-inch draught. When not in use, it sits at the foot of a mast. What is the shortest guy-wire, *AB*, which will just clear the lifeboat?

I can remember thinking on the first day of twelfth-grade algebra class that if I could solve problem 46 on page 333 near the end of the textbook (Petrie, Baker, Levitt & MacLean, 1946), there would be no reason for me to take the class. Successfully solving this problem would mean that I *must* know everything in the text leading up to the problem! Otherwise, how else would I be successful? I did not really believe this hypothesis but it was still fun trying to solve the problem. Little did I realize at the time that it was an optimization problem that required some knowledge of calculus and there was a lot to learn before I could successfully solve the problem.

While sitting in my desk I used to wonder about who these 'interns' were that both visited our class and taught us mathematics. Little did I know that I was to become one of these 'interns' teaching mathematics.

Internship, mathematics education classes, and graduation

"In all, I am very pleased with Rick and confident that Rick will make an excellent teacher and has a positive contribution to make to education." Those were the words of my cooperating teacher after my three-month internship. With my Master of Arts specializing in mathematics, and a Bachelor of Education with a major in mathematics and a minor in physics completed, I was looking for a teaching position.

I recall while attending one of my mathematics education classes there was a book we were asked to purchase that we never got around to reading or discussing in class. This book was George Pólya's *How to Solve It* (1957), a book about problem solving and the strategies used by mathematicians to solve problems. At the time this book sold for $2.25, but as the commercial on television would say today, I now consider it priceless. Mathematician and mathematics educator Alan Schoenfeld also recognized the book's value when on page xi in the preface of his book *Mathematical Problem Solving* (1985) he asked, "Why wasn't I given the book when I was

Seaman, R. (2009). Looking back.
vinculum: Journal of the SMTS, 1(1), 12-17.

a freshman, to save me the trouble of discovering the strategies on my own?" So what did I do with Pólya's book when I graduated? I tucked it away in my personal collection of math books [Math students rarely throw away their math books] and promptly forgot about it!

I got the job, the first ten years

I began to teach in the 1970s and I remember teaching what might be characterized as knowledge based skills such as solving equations, factoring polynomials, and the like. If there was any problem solving or thinking, it had to wait until the end of the chapter where those sections were labeled with surface features such as work, mixture, or uniform-motion problems. I confess that I rarely taught strategies to help students with their problem solving.

Roughly ten years later I reacquainted myself with *How to Solve It*, but this time I actually opened it up and started to read it. I wished that I would have read it earlier in my teaching career but in hindsight I am not convinced that I would have pedagogically appreciated the book enough. After studying the book I decided to help students with their problem solving by discussing Pólya's "How to Solve It" list/problem-solving model with them. However, I still sensed something was still missing.

The second ten years

A few years later, I was asked to introduce an internationally recognized mathematics curriculum in the high school where I was teaching. This program included a provincial high school mathematics curriculum, which was supplemented by many first and second year university topics in mathematics. As you can imagine, time became a major factor in covering this curriculum. To offset this problem, I applied Saxon's (1982) idea of incrementally developing concepts. This allowed for a concept to be presented and practiced for more than one homework set before the next facet of the concept was introduced. In this way, students worked on concepts over a longer period of time to help them retain their understanding of these concepts. In grade twelve the calculus portion of this curriculum became a partial review of the Saskatchewan high school mathematics curriculum [Which is another story]. These students still needed to pass their regular mathematics classes to graduate provincially but wrote separate comprehensive exams to graduate internationally.

Closer examination of Saxon's (1984) *Algebra II* text revealed that each lesson and corresponding assignment could contain both knowledge-based questions and problems to solve. I decided that this would be a good way to avoid the artificiality of problems being placed at the end of the chapter while also improving the students' problem-solving skills. I decided I would try to incrementally develop concepts and assignments as Saxon had done and teach problem solving and a supporting knowledge base every class.

Around this time I read an article written by Mayer (1985) that described a cognitive strategy similar to Pólya's that was made up of two phases: problem representation and problem solution. After reading the writings of Mayer (1985), Pólya (1957), Saxon (1982, 1984), and Schoenfeld (1985) I decided to teach and assign knowledge base skills and problem solving daily under the comprehensive umbrella of a cognitive strategy (see Figure 1).

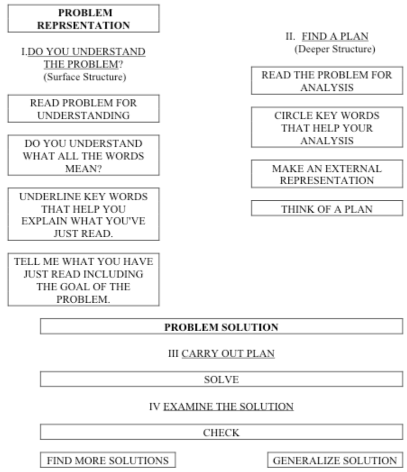


Figure 1. Cognitive Strategy

The last five years

Then as suggested in an article I read by Montague (1992) at the beginning of the term I had the grade nine students I taught memorize a cognitive strategy synthesized from reading Mayer (1985) and Pólya (1957). This provided the students with "hooks" for the problem solving process as they attempted to solve mathematical problems. Whenever a student was having trouble solving a problem, cognitive strategy gave me an entry point to answering their question. I would initially ask them to "tell me what they had just read including the goal of the problem" before I would respond. I would then make suggestions and use the cognitive strategy to guide the student through the remainder of the problem solving process. Because of the self-questioning strategies inherent in the cognitive strategy, students gained control over their thinking (Schoenfeld, 1985), utilized more representational strategies (Montague & Appelgate, 1993a, 1993b) and also slowed down their thinking process by reading for understanding before reading for analysis. Yes this was frustrating for some of the students!

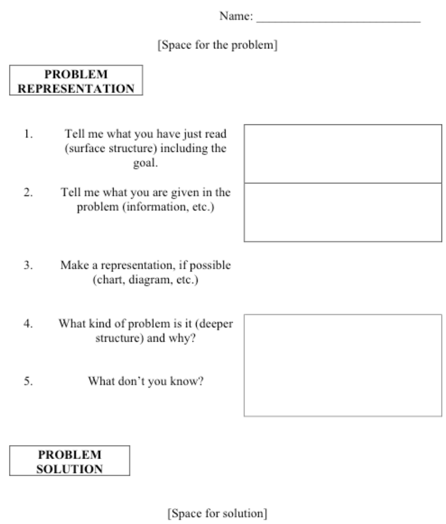
To gain further insights into the students' understanding of the problem solving process I would give them an open-ended activity where they were asked the following question, "You have just been chosen to write an article in a national supermarket tabloid. Your headline for the article is: 'What my cognitive strategy for problem solving has taught me and what it means to me personally?'" A typical response was:

> Problem solving is very important in life. Without it, we wouldn't be where we are today. It has taught me to slow down my thinking. Before I hardly read the problem before I started calculating numbers, which never came out to be the right answer. Now, I've slowed down and underline, circle, and find the surface structure (what the problem is all about) before I even begin to analyze. Now I actually get the correct answers without having to ask someone. I think this theory will really help me in life. Not only in mathematics, but everyday life, other subjects and other things I usually wouldn't have the answers to. Maybe someday I'll be able to solve one of the world's great mysteries! [Student's response to the tabloid question]

I found that the more students were asked to classify a problem the more it became evident that they didn't classify a problem according to what was most influential in helping them solve the problem (deeper structure). Their categorizations contained superficial features such as question form, contextual details and quantity measured (Gliner 1989; Silver, 1977, 1979). These students, who made decisions based on surface features, were then instructed to perceive problems on the basis of their deeper structure (Schoenfeld & Hermann, 1982). In order to reinforce their choice of deeper structure, they had to justify their decision (Hutchinson, 1986, 1993), while acknowledging that deeper structures are not unique for each problem but depend on how one 'chunks' their knowledge for problem solving success. As a result, when assessing students' work I assessed each student's choice and justification of deeper structure, and when taking up these quizzes I reinforced that there could be more than one way to represent and solve a problem.

> Well, problem solving has taught me many things. That problems must be represented before they are solved, and that categorizing problems according to deeper structure makes solving problems quite easier because you have formed a plan. The nice thing is that it makes problem solving understandable, and it helps out my grade nine year. [Student's response to the tabloid question]

Assessing students' problem solving quizzes was facilitated by using structured worksheets (see Figure 2) that contained room for one word problem at the top and spaces for each aspect of the cognitive strategy (Hutchinson, 1986, 1993).


Name: ____________________

[Space for the problem]

PROBLEM REPRESENTATION

1. Tell me what you have just read (surface structure) including the goal.
2. Tell me what you are given in the problem (information, etc.)
3. Make a representation, if possible (chart, diagram, etc.)
4. What kind of problem is it (deeper structure) and why?
5. What don't you know?

PROBLEM SOLUTION

[Space for solution]

Figure 2. Structured worksheet

The structured worksheets were scored analytically (Hutchinson, 1986), with each component of problem representation and solution rated 2, 1, or 0, depending on the degree of understanding demonstrated by the student.

Lesson plans evolved to support the objectives of developing students' mathematical thinking and teaching the supporting knowledge base. The questions in their assignments were then divided into two categories: knowledge base and problem solving. In order to minimize any redundancy in the questions assigned, they were chosen according to their deeper structure. This allowed for questions with similar methods of solution to be incrementally assigned.

In the meantime

In the eighties and early nineties I was also a sessional lecturer at the University of Regina teaching Mathematics 101, a class that is required for a Bachelor of Education degree in elementary education and satisfies a degree requirement in the Faculty of Arts. I soon learned that most of the students in the class were math anxious and typically left taking this class until the end of their respective programs. I taught the class applying the ideas previously described.

Students began to demonstrate expert-like retrieval strategies for solving a problem by stating, for example: "it was just a proportional reasoning problem". Others indicated they had transferred the application of the cognitive strategy to their other classes with success.

> Well, problem solving, to me has taught me to look at things (problems, especially) from a different point of view. I used to just give up on a problem after just reading it and not knowing how to attack it. Now, I can say that I don't give [up] so easily and I know of some ways to attack a problem. Personally I didn't feel that problem solving had anything to do with reality since the only time I used it was in math. I figured that we would never need this, but it seems as though I was wrong because it not only helps your thinking in math, but it affects the way you also handle things in life. [Student's response to the tabloid question]

At the end of one class one student in a 'thank you' card said, "I surprised myself by actually enjoying a math class!" I found that students I taught began to like mathematics and solving problems a lot more.

Reflection

Looking back on my years of research, thinking about and teaching mathematics suggested that instruction occur under a problem-solving umbrella supported by a cognitive strategy: a cognitive strategy that has students among other things classifying with justification problems according to deeper structure, while acknowledging the use of multiple representations when solving problems. Lessons and assignments would be supported by the incremental development of knowledge base and problem solving with students' assignments in problem solving scored analytically. I found teaching mathematics to students in this manner lead to students' improvement in their attitudes and beliefs toward mathematics and problem solving.

Return to university

It was time to read Pólya's book again, this time to prepare myself to return to graduate school and to research how I approached teaching mathematics. A year after my graduation, I was awarded a National Doctoral Thesis Award (Seaman 1995) for "...the evaluation and dissemination of new ideas in education...and assist in improving the quality of education." Today, as an associate professor of mathematics education at the University of Regina, this research has become the basis of what I employ to expand preservice mathematics teachers' ways of thinking about and teaching mathematics. My hope is that they will begin their teaching career with the pedagogical experience I have gathered in my teaching career.

> Hey Rick,
>
> Just wanted to "drop you a line" and let you know that I am loving this job. Every day just gets better and more exciting than the last. I'm sure that sounds naive and idealistic, but I've really had that good of a start to my semester. I'm super busy with lots of extra curricular stuff (SRC, volleyball, Outdoor Ed Club), but it has really helped me get to know the staff and the kids.
>
> I'm always trying to incorporate some of your EMTH ideas into my lessons... I often go back to my notes and cartoons we got from you in those classes. The longer I'm here, the more I understand what you were saying when I jotted down those notes in class.
>
> Anyway, I had a few minutes this morning so I thought I'd let you know that I really couldn't have made a better choice professionally!
>
> Thanks for all the advice. [Graduate e-mail]

> Hi Rick,
> Just wanted to send a little "thank you" note for the training you gave us! I just had a meeting with one of my vice-principals and she has been so happy with how I'm teaching "thinking" skills that she's asked a director

to come and watch me teach. I can't believe how much I'm learning the more I'm in the classroom. I love this job! Hope things are going well. Keep educating your flock... we end up really appreciating it. Thanks. [Graduate e-mail]

Hi Rick,
… I was just thinking of you and your fantastic class a couple of days ago. I was answering some questions on a state job application and one of them was about classes or experience that had helped develop my problem solving skills. I had to talk about your class, and how it changed the way I look at problems. No matter what the question, I still look for the deeper structure. Thank you for sharing your wisdom. [Graduate e-mail]

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THE 'ETHANOL' ACTIVITY

Tyler Pokoyway, Stacy Renneberg and Rick Seaman

With the push for renewable resources and cleaner energy sources two secondary mathematics education interns in the fall of 2007 developed an activity investigating the use of ethanol as an energy source. After the first intern had taught the lesson at their school both interns met to discuss how the lesson could be improved upon. The 'new and improved' lesson was then taught at the second intern's school. The reader will notice how mathematics an "inch deep" based on; in this case, proportional reasoning can become a cross subject area focus on real-world issues which are a "mile deep".

Background

The following is a cross-discipline learning activity focused on the production of ethanol for fuel in the province of Saskatchewan, Canada (http://www.gov.sk.ca/). With the environment being a 'hot' topic all over the world, there is a push for renewable resources and cleaner energy sources. One such energy source is ethanol and because some of the ingredients used in making ethanol are corn and grain sorghum, wheat, and barley there has been a push to use Saskatchewan's agricultural resources for its production (Enterprise Saskatchewan 2009).

During the fall of 2007 semester two secondary mathematics education interns in the Faculty of Education at the University of Regina (http://education.uregina.ca/) developed a mathematics lesson concerning the use of ethanol blended gasoline that involved other subject areas such as English, science, and social studies. Both interned at the high school level at different schools (http://www.rbe.sk.ca/) in Regina (http://www.regina.ca/) the capital of Saskatchewan. The idea was that after the first intern had taught the lesson they would meet and discuss how the lesson went so as to make improvements to it. The intern who had not taught the lesson yet would then take the 'new and improved' lesson and teach it to their class. What follows describes the mathematics lesson developed called the 'Ethanol' activity and the collaborative efforts of the two interns.

'Ethanol' Activity

In 2007 Saskatchewan gas pumps were required to blend 1% ethanol into their total gasoline sales today the figure has been increased to 7.5% (Enterprise Saskatchewan 2009). The government claims that this renewable fuel will not only improve the economy but also protect the environment (Saskatchewan Ministry of Enterprise and Innovation 2007). However, there are many critics against the use of ethanol as a way to protect our environment and claim that ethanol is more of a hassle than a benefit. The debate has begun and there are many players on both sides of the issue in both Canada (Coxworth 2006) and the United States (HybridCars.com 2007). Other sources of energy such as wind power, solar and geothermal energy are also encouraged in Saskatchewan.

First School

The lesson began with a mock scenario that had students acting as candidates running for the position of Premier of Saskatchewan, with voters wanting to know where they stood as candidates on mandating the use of ethanol-blended fuel. The task was for them to sit down with their advisors (their group) and go through the numbers and read the research on the issue. Then they were to make an informed decision, on whether or not they supported the move to use the provincial agricultural resources for the production of ethanol. This scenario served as a 'hook'

Pokoyway, T., Renneberg, S. & Seaman, R. (2010). The 'ethanol' activity.
vinculum: Journal of the SMTS, *2*(1), 37-44.

to engage students into the 'Ethanol' activity. The objective was to introduce the social, environmental, and ethical issues surrounding the ethanol debate during English class (see fig. 1). Then students in their mathematics class were to apply proportional reasoning to interpret related data and then make a list of pros and cons to help them substantiate their decision of whether or not they supported the production of ethanol for fuel.

1. If ethanol is made from wheat and corn what will that do to the selling price of wheat and corn crops? Will this affect you and your family at the grocery store? What if your family was growing these types of crops?

2. How could the province provide the cropland needed for ethanol production while still producing the amount of wheat needed for human consumption (food for people) and feedstock (food for farm animals)? Is there enough land area for the ethanol crops?

3. Are there any international issues/effects as a result of producing crops specifically for ethanol production? Explain.

4. If our demand for/consumption of oil/gasoline is increasing and considering the amount of CO_2 from the producing ethanol, does using ethanol as a fuel source actually reduce CO_2 emissions? Explain.

5. Instead of producing more environmentally friendly fuel, what other environmentally friendly alternatives could we do to meet our transportation/energy needs?

Fig. 1. Issues/Dilemmas

English Class. English class began with finding out what the students already knew about ethanol and then explaining the 'Ethanol' activity to the students. The English component consisted of a resource package of Issues/Dilemmas discussion questions (see fig. 1) and five articles from the local newspaper (http://www.canada.com/reginaleaderpost/index.html). One article by the financial editor, Bruce Johnstone (2006), discusses the pros and cons of the renewable fuels industry and whether tax dollars should be used to subsidize the industry. The second article written for the Associated Press (2007) describes the impact a corn-based ethanol boom in prices would have on the corn and related industries in the United States. A third article written by an agrologist consultant and Saskatchewan farmer, Kevin Hursh (2007), discusses the impact the renewable fuels industry will have not only on the environment but the grain industry and rural development. The last two articles are letters to the editor one in response to production of ethanol for fuel (Kurtenbach 2007) and the other (Leader 2007) questioning what drivers are willing to do to reduce greenhouse emissions and what is spent on gas.

Students were placed in groups of five and with each group member taking a different article in the resource package to read and point out what they thought were the main ideas in the article. Then each group member discussed the article they read within their group so that everyone could understand the issues contained in the articles. Then the group responded to the Issues/Dilemmas questions (see fig. 1). A class discussion about these questions followed ending with a brainstorming session about the possible alternatives to using automobiles for transportation and burning fuel for energy. The English component took place over two, one-hour class periods.

Math Class. The students continued the 'Ethanol' activity in their mathematics class, in the same groups, with the previous resource package supplemented with: Two more articles from the local newspaper that discussed the construction of ethanol production plants in Saskatchewan and supply side concerns (MacAfee 2006; Johnstone 2007); a page which compared the benefits and environmental impacts of ethanol fuel, in Brazil and in the United States (de Oliveria et al 1987); a map of Saskatchewan Crop Districts and Rural Municipalities (Saskatchewan

Agriculture 2006); information on how ethanol is made and where it comes from (Integrated Grain Processors Co-operative Inc. 2005) and agricultural data from Statistics Canada (2006; 2008). The students used their supplemented resource package to find the facts necessary to answer the questions on the DATA sheet (see fig. 2). After every group had completed their investigations they discussed the implications of their findings and how they tied into the issues discussed previously during English class. The groups then created a list of pros and cons to help them support their position. Each group member was to come up with their own decision as to whether or not they supported the province's decision to produce and blend ethanol into gasoline. As an exit slip each student was to answer the questions: "What do you know now about ethanol and the ethanol debate in Saskatchewan that you didn't know before? As an informed citizen what can you do about this issue to express any concerns or issues you have?" Interestingly one of the students responded with: "I learned that ethanol might not be good for our environment because it takes a lot of crop land and it makes carbon dioxide."

1. Find the ratio of the amount of CO_2 emissions from the production and consumption of ethanol (in tonnes) to the amount of ethanol produced (in litres), if the average amount of CO_2 emissions is 1.43 tonnes for every 1000 litres of ethanol produced.

2. How many cars could drive for a year and emit the same amount of CO_2 emissions as that which would be emitted by producing the required amount of ethanol that Husky Energy and Terra Grain Fuels claim they will produce in a year? Note: The average passenger car produces ≈ 5.20 tonnes of CO_2/year (US Environmental Protection Agency: Office of Transportation and Air Quality 2007).

3. If Saskatchewan produced 18.6 million tonnes of wheat last year. Find the ratio of the amount of wheat needed for ethanol (in tonnes) for one year to the amount of total wheat produced (in tonnes) in 2006. Note: 1 bushel ≈ .027 tonnes

4. What is the percentage of wheat needed for ethanol of the total amount of wheat produced last year in Saskatchewan?

5. How much crop area would be needed to produce the wheat/corn for ethanol production?

Fig. 2. Data

Second School

Improvements were made to the 'Ethanol' activity at the first school plus adaptations were made considering the differences in student abilities in the updated lesson plan for the second school. Improvements/adaptations made include:

- adding a science portion to the activity,
- this time the math lesson would take place before the English lesson in order to accommodate the debate portion at the end of the activity,
- inclusion of additional resources (a YouTube™ video, SMART Board™ demonstration),
- a guest speaker from the provincial government,
- and a pro-ethanol and anti-ethanol class debate.

There were also instructional adjustments needed to accommodate the learning needs of each group of students as the lesson was transferred from the first school. A Revised Question Worksheet was included to help the students answer the mathematics related questions. For a sample question see fig. 3.

Fig. 3. Sample: Revised Question Worksheet.

Science Class. A large Nimbus® water bottle and a flame were used to demonstrate the complete combustion property of ethanol to the science 10 class (Metacafe, Inc. 2007; Saskatchewan Learning 2005). It served as an introduction to ethanol and its combustion process. Since the students knew very little about ethanol and its applications it turned out to be a great way to introduce the topic. One student stated "I learned that ethanol is an alcohol that we use for fuel." The students became absorbed in the topic and asked questions about other options for reducing pollution and why our government would mandate ethanol if there were other 'greener' options to research. Everything from hydrogen fuel cells to wind power was discussed with one student suggesting that "we recycle older vehicles and replace them with hybrid vehicles."

Math Class. This time the math lesson took place before the English lesson in order to enhance the debate portion at the end of the activity. Both teachers noticed that the students from the first school were anxious about the proportional reasoning associated with the assignment and that they had some trouble with the wording of the questions on the questionnaire (see fig. 2), so a guided answer sheet was created with helpful hints to help students solve the problems (see sample fig. 3). To guide the students through the math questions the first question was read together and the teacher showed them how to use the resources provided. It was hoped that by working out the questions with the students it would reduce their math anxiety and help them stay focused on the activity and the debate preparation. The map that showed the crop districts and rural municipalities in 2004 (Saskatchewan Agriculture 2006) was used with SMART Board™ technology to suggest what might happen if there were areas of drought, flooding, or fire and how this could affect the land area available for farming. This helped the students see how using crops for fuel might affect our available food resources.

English Class. Following math class the students discussed the Issues/Dilemmas questions (see fig. 1) in English class. This class began with the YouTube™ video (halcollins0 2006) that illustrated the process of ethanol production and how some race-cars now use 100% ethanol fuel. This turned out to be a great starting point for the discussion. For example, one student asked, "Why can't all cars run on 100% ethanol?" Added another, "We should all use ethanol, or stop driving so much." The format of the English lesson was the same at both schools except this time the civil servant who wrote the ethanol mandate for Saskatchewan was the guest speaker. He spoke on the benefits and faults of using ethanol as a fuel source, which added a dynamic feeling to the classroom. The students became comfortable enough to ask him questions like, "How much land would we need to run our vehicles on 100% ethanol?" They asked him his opinion concerning the questions on the Issues/Dilemmas sheet (see fig. 1).

The students were then separated into pro-ethanol and anti-ethanol groups, based on their stance on the issue. Because the students were unfamiliar with how to run a debate, a package was created called How to Run a Debate Information that included a rubric created by one of the interns (see fig.4), debate guidelines (LifeBytes 2003), and a debate guide which asked for the following information:

- •The student's position,
- •Main points for your argument plus the source,
- •Main points against your argument plus the source,
- •Interesting facts to consider when time to vote … things I haven't thought about.
- •Do I agree with the position I took? Why or why not?
- •Would I like to investigate further?

Name:
Group Viewpoint:
Comment:
Final Score:

	100%	80%	60%
View point	Viewpoints are clear and organized	Most viewpoints are clear.	Viewpoints are unclear and disorganized
Use of facts and examples	Arguments are supported with facts and examples	Most arguments are supported with facts and examples	Arguments lack factual support.
Relevance of supporting arguments	All supporting arguments are relevant.	Many, but not all, supporting arguments are relevant	Few supporting arguments are relevant.
Strength of arguments	All arguments are strong and convincing	Some arguments are convincing	Arguments are not convincing
Speaking voice	Voice can always be heard	Voice is heard most of the time	Voice is difficult to hear.
Preparation	Student is well prepared	Student needs more preparation	Student is unprepared to defend argument.

Fig. 4. Debate Rubric

To further facilitate the debate the guest speaker was invited to help out each side of the argument with the two interns helping the students think about different arguments the other side might present in order to prepare a rebuttal. The students took 15 minutes to go over their pros and cons, and to write their opening comments.

The debate lasted 30 minutes. Each student had a role to play, some were the speakers, and others gathered information or pointed out something from an article that could help their side of the argument. One student had the "role" of asking the other side to "PROVE IT".

After the debate the students had an idea of where they stood and worked independently to finish the questions on fig. 1. After the debate one student concluded "I love arguing, that was fun."

Additional Student Comments

The same exit slip was handed out again asking students to respond to the questions: “What do you know now about ethanol and the ethanol debate in Saskatchewan that you didn’t know before? As an informed citizen what can you do about this issue to express any concerns or issues you have?” Some typical responses:

- “Ethanol pollutes just as much as regular gasoline, when it comes to carbon, we should try other energy methods.”
- “It is good that we can use ethanol for fuel, because we are using all of our fossil fuels up”
- “We will need a lot of farm land to make ethanol, so I don’t know if Saskatchewan can support ethanol as a fuel. Ethanol is good because plants absorb carbon emissions, but we need to drive tractors to get the crops for ethanol anyway, so I don’t know if it is a good or bad idea.”
- “We can just invade [the province of] Manitoba if we need more land!”
- “Why don’t we just make hybrid cars mandatory? Then we wouldn’t need as much fuel for transportation.”

Reflections

The students were not well informed on the ‘ethanol debate' prior to this activity. As the ethanol topic was explored, and the students began reading the articles and discussing the issues they became involved and asked many questions. Many of the students not only wanted to know about the use of ethanol but asked about other options for renewable fuels. As a result of this activity an excellent discussion ensued regarding renewable energy and other alternatives for transportation and energy sources.

At the first school, it was thought that the time lapse between English and math lessons was a little long. It would have been more beneficial if the lessons had occurred within the same week or the next day as happened at the second school. Also when the ‘Ethanol’ activity was revamped at the second school for the science and math classes, the students were able to stay focused on the activity during two back-to-back periods.

The science curriculum was a good fit for this activity because the lessons involving hydrocarbons, finding formulas, making chemical equations and the properties of different substances lent itself to using ethanol. As the debate and discussions developed it became apparent that the ‘Ethanol’ activity could easily involve not only mathematics, science and English classes but also social studies, and physical education classes.

Looking back we found that bringing in a government official to address the provincial ethanol mandate into the English class at the second school was beneficial. After reading the articles and having a class discussion the students were well prepared to ask more relevant questions of the speaker. Bringing the speaker government official into the debate also brought another opinion and new information for the students to consider. As interns we also benefited from the presentation probably as much as the students did, as we learned that the use of ethanol-blended fuel was mandated in Saskatchewan for economic reasons and not just environmental reasons. The debate also added an exciting dynamic to the lesson, as a student wondered if ethanol was the best option (see fig. 5).

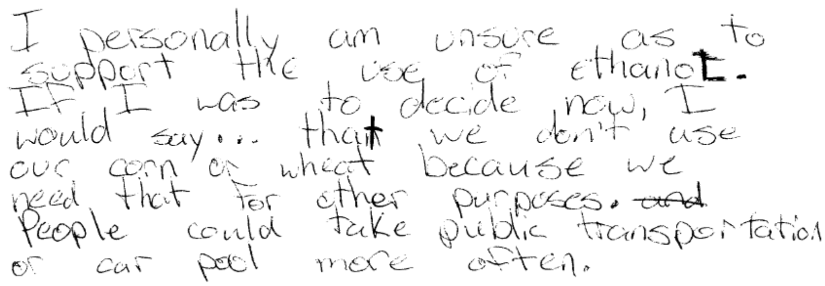
I personally am unsure as to support the use of ethanol. If I was to decide now, I would say... that we don't use our corn or wheat because we need that for other purposes. ~~and~~ People could take public transportation or car pool more often.

Fig. 5. Sample student response.

The 'ethanol' activity gives suggests how mathematical content an "inch deep" involving, in this case, proportional reasoning can be utilized to connect a variety of subject areas while making learning not only relevant but a "mile deep".

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The Eighties: Celebrating 10 years (1980-1989) of the Journal of the SMTS

Rick Seaman

Preamble

It is a worthwhile exercise to look back and learn from the past. As educators, we often think that our present concerns in education are unique. However, it is not uncommon to find out that someone else has also encountered the same issue. If you have been teaching long enough, chances are you will have experienced the same concern before but labelled it in a different way. As your experience accumulates, these same concerns tend to reappear, as though on a cycle.

With these thoughts in mind I began reading the ten articles that were chosen from the *Journal of the Saskatchewan Mathematics Teachers' Society* (JSMTS) to celebrate 'The Eighties'. The articles' authors sought to make a contribution to improve the teaching of mathematics. It is my hope to retain their message through the liberal use of quotes and to exercise my license to change the chronological order of the articles according to themes and include some 'present day' commentary.

The Origin and History of the Saskatchewan Mathematics Teachers' Society

Garth Thomas (1987), a mathematician at the University of Saskatchewan who was involved in formally organizing the Saskatchewan Mathematics Teachers' Society (SMTS), wrote on the first 25 years of the society's existence. In the article Thomas explained how the SMTS came into existence in 1962 through the assistance of multiple partners involved with education in Saskatchewan. The SMTS decided that every year or two a convention would be held that focused on the teaching of mathematics. Thomas noted that 'recently' the SMTS had joined with the Saskatchewan Science Teachers' Society to have a joint conference every two years fittingly named SCIEMATICS. He also pointed out that sponsoring mathematics contests was an important part of the SMTS's mandate.

Thomas mentioned that "one of the strengths of the society has been the importance it has attached to publications" (p.8) which reinforces the present undertaking of celebrating 50 years (1961 – 2011) with 50 articles from the JSMTS (now *viniculum*). On an optimistic note he stated "We hope that 'the best is yet to come' and we invite your continued support so that we can accomplish even more in the future" (p.8).

"Look up, look way up." Leading up to 'The Eighties'

I am not referring to the 'Friendly Giant'(MacLaughlin, May 21, 2008) a favourite children's television program on CBC Television from September 1958 through to March 1985 but rather to an experience that took mere seconds but had major ramifications on education. It was in the fall of 1957 and Sputnik, a Soviet Satellite, crossed the night sky.

There was Sputnik excitement in the United States, too, but then concerns at the political, military and educational level surfaced! In the spirit of one-upmanship, the United States launched scientific 'information gathering' satellites and on July 20th, 1969 the first manned mission landed on the Moon. The Space Race with the Soviet Union was on. To further meet the challenges presented by engineers and mathematicians in the Soviet Union the United States introduced a new Science curriculum and the study of mathematical structure in Mathematics called the 'new math'. "Characteristic of the reform was not the new subject-matter but the emphasis on the axiomatic, logical structure of mathematics, even at the earliest levels, and a consequent downgrading (by many enthusiastic teachers and textbooks) of exercise in routine manipulations" Ralph A. Raimi (August 26, 2005).

In Saskatchewan high school mathematics teachers returned to university to learn about set theory, groups, rings, integral domains, fields, mathematical induction and the like. Professor K. G. Toews at the University of Regina worked hard teaching Mathematics 205, a class that would bring them up to speed with the 'new math'.

It is no wonder that with this abstract approach to teaching mathematics that in the 70's parents would demand a 'return to the basics'. This movement was further encouraged by books such as, *Why Johnny Can't Add: The Failure of the New Math* written by Morris Kline in 1973.

So let's begin the celebration!

'The Eighties' in the JSMTS

Mathematical Deficiencies in Incoming Freshman Students

Garth Thomas (1980) because of "deficiencies in the mathematical backgrounds possessed by incoming freshmen" (p.16) wrote that in the mid-seventies the University of Saskatchewan Mathematics Department developed and administered a High School Mathematics Review Test to them. The initial tests took place in 1976, 1977 and 1979 with the Mathematics Department acknowledging that there were "limitations and shortcomings" (p.16) in these tests. However, Thomas emphasized that the objective of the review tests was "not to assess, blame or criticize individuals or particular schools, but to try to see if something can be done to improve the situation" (p.17).

In order to minimize incoming student's mathematical 'deficiencies', some possible solutions suggested were to "make students work harder; more emphasis on comprehensive exams; higher standards for passing exams; inform students of our expectations; ..." (p.17). Also a longitudinal study was suggested to give the High School Mathematics Review Test to "High School classes; give it in middle of university year; determine correlation with final mark in university classes in mathematics; determine correlation with High School mark; etc." (p.17)

By the end of the seventies National Teachers of Mathematics recommended in *An Agenda for Action* (NCTM, January 13, 2009a) the following recommendations for School Mathematics of the 1980s (NCTM, January 13, 2009b):

1. Problem solving be the focus of school mathematics in the 1980s;
2. Basic skills in mathematics be defined to encompass more than computational facility;
3. Mathematics programs take full advantage of the power of calculators and computers at all grade levels;
4. Stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. The success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
6. More mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;
7. Mathematics teachers demand of themselves and their colleagues a high level of professionalism;
8. Public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.

In my commentary about the SMTS articles from 'the eighties' I will touch on a few of these recommendations while acknowledging that the authors were aware of the other recommendations in their articles.

Marlow Ediger, an education professor from Northeast Missouri State University in his article *Psychology in Teaching Mathematics* (1987/88) added that the "structure of knowledge approach has much to recommend itself" (p.13) and suggested that:

1. teachers emphasize structural ideas in a minimally repetitious, spiral curriculum with built in review at more complex levels in the mathematics curriculum.
2. induction receive adequate attention in teaching-learning situations with meaningful explanations injected when necessary.
3. creative teaching in using diverse methodologies be emphasized thoroughly and methods and subject matter have to be adjusted to the present achievement level of each student (p.14).

Problem Solving and Microcomputers

In the forward of the book *The Road Ahead* (1996) Bill Gates described how 20 years earlier as a college sophomore, he and Paul Allen read the feature article in *Popular Electronics* by Roberts and Yates (1975) about the Altair 8800 Minicomputer. Gates wrote:

> As we read excitedly about the first truly personal computer, Paul and I didn't know exactly how it would be used, but we were sure it would change us and the world of computing. We were right. The personal computer revolution has affected millions of lives. It has led us to places we barely imagined then (p.xiii).

As I read the articles written by Gary Bitter (1982) of Arizona State University and Anthony Ralston (1983) of State University of New York at Buffalo, Bill Gates' candid comments gave me a deeper appreciation of their submissions to the JSMTS.

Bitter (1982), while recognizing the third recommendation (NCTM, January 13, 2009b) to make use of computers in K – 12 classrooms, acknowledged that computer use in education remained low. He cited possible reasons for this being a lack of sufficient funding, an overall "lack of knowledge about computers, and the outright hostility" (p.31) toward them by teachers. Taking implementation inertia into consideration, some suggestions/models were offered that computer education could be initiated in pre-service teachers' education: students should "interact with computers on a daily basis" (p.32); special courses should be created "to teach students the use and functions of a microcomputer" (p.32) or the computer should be integrated into existing courses for pre-service teaching (p.32).

Bitter wrote how the microcomputer could contribute to a student's education in three areas:

1. Computer awareness: Being able "to identify a computer and its role in society, its application to the world around us, and to use it reasonably well" (p.32). Some topics that still warrant our attention are Computer Misuses; Computer Experiences (i.e., Programming) and Computer Careers (i.e., Programmer, Artificial Intelligence).
2. Computer managed instruction: For attendance, grades...
3. Computer assisted instruction (CAI): This can be Drill and Practice, Tutorial, Problem Solving and Simulations but what caught my attention was the following application: "The ability to <u>retrieve information</u> from networks, cable television or telecommunications via satellite utilizing a microcomputer has unlimited potential for education. Cable television is growing very rapidly. There were one million users in 1980, a number which is expected to grow to 30 million by 1982. Interactive video disk and video text capabilities

make this an option with a lot of exciting potential for education" (p.35). Think about this last comment by Bitter in today's world of Google and Social Media!

Bitter stated "Problem solving with the microcomputer involves the ability to control the machine. Therefore, programming a microcomputer in its own language is essential" (p.35). This CAI application supported the first recommendation of *An Agenda for Action* (NCTM, January 13, 2009b) that problem solving be the focus of school mathematics in the 80's.

Approximately 30 years later Douglas Rushkoff in his book *Program or Be Programmed: Ten Commands for a Digital Age* (2011) also emphasized the importance of computer programming:

> The difference between a computer programmer and a user is much less like that between a mechanic and a driver than it is like the difference between a driver and a passenger. If you choose to be a passenger, then you must trust that your driver is taking you where you want to go. Or that he's even telling you the truth about what's out there. You're like Miss Daisy, getting driven from place to place. Only the car has no windows and if the driver tells you there's only one supermarket in the country, you have to believe him. The more you live like that, the more dependent on the driver you become, and the more tempting it is for the driver to exploit his advantage (p.9).

Bitter (1982) noted school boards were seriously considering "the purchase of microcomputers inasmuch as the drill-and-practice capabilities of CAI are in perfect accord with the 'back-to-basics' movement" (p.31) and *An Agenda for Action* (NCTM, January 13, 2009b). Bitter hoped that "education can take full advantage of this microelectronic revolution and actively utilize the potential in the classroom to teach children and adults" (p.38).

Alec Couros (February 19, 2009) of the Faculty of Education at the University of Regina, suggested in his blog a direction where the microelectronic revolution could be headed in education:

> Recently, I have been conceptualizing/personalizing the concept of open teaching as informed by my facilitation of EC&I 831 and ECMP 455. In my view, open teaching goes well beyond the parameters of the Free and Open Source Software movement, beyond the advocacy of open content and copyleft licenses, and beyond open access. For open teaching, these are the important mechanisms, processes, and residuals, but they should not be viewed as the end goals in themselves. Rather, open teaching may facilitate our approach to social, collaborative, self-determined, and sustained, life-long learning.
>
> My working definition of open teaching (focused on the above areas) follows: Open teaching is described as the facilitation of learning experiences that are open, transparent, collaborative, and social. Open teachers are advocates of a free and open knowledge society, and support their students in the critical consumption, production, connection, and synthesis of knowledge through the shared development of learning networks...
>
> Through interactions with current and former students, the resulting practice has lead to a learning environment where the walls are appropriately thinned.
>
> Through the guiding principles of open teaching, students are able to gain requisite skills, self-efficacy, and knowledge as they develop their own personal learning networks (PLNs). Educators guide the process using their own PLNs, with a variety of teaching/learning experiences, and via (distributed) scaffolding. Knowledge is negotiated, managed, and exchanged. A gift economy may be developed through the paying-forward of interactions and meaningful collaborations.

Computer Science and Mathematics

Ralston (1983) presented a case for changing university mathematics classes for computer science students. He called for a balance between what has usually been taught in social and management sciences mathematics and discrete mathematics.

Ralston reasoned that Computer Science lends itself to "discrete analysis and mathematics which contain the overwhelming majority of the tools needed in dealing with that quintessentially discrete device, the computer" (p. 6). He also recognized that because of computers "we are living in the beginnings of a Second Industrial Revolution whose focus is not the material but the immaterial--information, knowledge, communication" (p. 7). Taken together, he envisioned a union between discrete mathematics and the computer to solve future problems in mathematics. So as a mathematical tool of the First Industrial Revolution "the position of calculus and, more generally, classical continuous analysis, is overdue for reassessment and revision" (p. 7). Ralston made the point that "not to do this—or to fail—would mean that we shall continue to instill yesterday's knowledge and skills in tomorrow's professionals" (p. 7). He further wrote that "my proposals are, I think, evolutionary and not revolutionary" (p. 7).

Ralston hoped that a conference being planned would become the vehicle to "provide the momentum for effecting the desirable changes" (p. 10). So at the end of June 1983 a small conference of 25 invitees was held consisting mainly of mathematicians who shared "to some degree a belief in the possibility and desirability of a new two-year mathematics curriculum balanced between the traditional calculus-linear algebra material and material from discrete mathematics" (p. 10). Ralston knew his computer science colleagues would for the most part support him, but was forewarned that the mathematical establishment would resist his ideas with whatever force necessary. He admitted that he "should believe—that the eventual aim of a new undergraduate mathematics curriculum was, at best, many years away" (p. 11). "The (really) new mathematics" (p. 7) had to wait!

Twenty One years later, *Discrete Algorithmic Mathematics* [3rd ed.] (2004) coauthored by Stephen B. Maurer and Anthony Ralston was available on Amazon with the following book description (November, 25, 2012):

> Thoroughly revised for a one-semester course, this well-known and highly regarded book is an outstanding text for undergraduate discrete mathematics. It has been updated with new or extended discussions of order notation, generating functions, chaos, aspects of statistics, and computational biology. Written in a lively, clear style that talks to the reader, the book is unique for its emphasis on algorithmics and the inductive and recursive paradigms as central mathematical themes. It includes a broad variety of applications, not just to mathematics and computer science, but to natural and social science as well.

Today, Anthony Ralston is Professor Emeritus in Computer Science and Engineering, University at Buffalo, *The State University of New York*.

Curricular Reform

Hellmut Lang (1984) of the Faculty of Education at the University of Regina in his article draws further on the concern for needed curricular reform at the high school level in Saskatchewan. The motivation for his article emanated from his many years teaching undergraduate courses in education, the "1981 Minister's Mini-Conference on Curriculum for the '80s" and the "Report of the Task Force on Secondary Teacher Education, University of Regina, 1980" (p. 17).

Scanning the section headings in Lang's article presents an overview of his concerns for needed curriculum revisions for Saskatchewan High Schools. For example, he acknowledged an existing curriculum where: individual needs were not being met; was irrelevant; was determined by university and technical institute entrance requirements; had little emphasis on thinking skills, concept development processes and problem solving; contained minimal attention to life and vocational skills, and provided minimal preparation for an increasingly technological world. With respect to the last concern, he stated that: "Technology is the increasing fact of current and future living. ... Surely the school has a responsibility for preparing youth in the tools and processes of today and tomorrow" (p.19). Lang was particularly concerned with the preparation of teachers "to handle or teach the new technologies and how to handle the potential dehumanizing effects of technology" (p.20).

As I read Bitter (1982), Ralston (1983) and Lang (1984), I am again reminded of what Douglas Ruskoff's (2011) words:

> In the emerging, highly programmed landscape ahead, you will either create the software or you will be the software. It's really that simple. Program, or be programmed. Choose the former, and you gain access to the control panel of civilization (p.13).

Is today the time for another 'convention/task force' to discuss the possibility of a content/curricular reboot - a reboot that would focus on programming, technology and digital literacy to facilitate students' search for knowledge under the guidance of teacher expertise?

Mathematics: Basics Versus an Activity Centered Curriculum

Marlow Ediger (1984) wrote about what mathematics teachers who emphasized the 'basics' in the mathematics curriculum might abide by, versus what those advocating an activity centered curriculum might emphasize. Ediger stated that "there generally are essential learnings for all pupils to achieve" (p.10) and that "basic learnings can be emphasized within the framework of activity centered methods of teaching and learning" (p.10). He concluded that "the teacher needs to make adequate provision for each pupil to achieve optimally" (p.10) reinforcing recommendation 6 of *An Agenda for Action* (NCTM, January 13, 2009b) for "a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population."

The Importance of Teaching What is Yet Unknown

Don Faust (1985) of Northern Michigan University, Marquette, also supported recommendation 6 of *An Agenda for Action* (NCTM, January 13, 2009b) in his article by suggesting that mathematics teachers find time for their students to experience mathematics not just "as an already completed body of knowledge" (p.17) but also to "see its exciting and open-ended nature" (p.17). Some of Faust's examples might be considered when the high school mathematics teacher believes there is a readiness or as a natural extension to what is being taught. For instance, *Goldbach's Conjecture* is an easily understood problem but yet still unsolved, the topic of infinite cardinalities and the recently proven *Fermat's "Last Theorem"*. Being a mathematics teacher I would get a copy of John Lynch and Simon Singh's documentary, which first aired January 26, 1996 on BBC Two – Horizon, regarding Andrew Wile's journey to prove *Fermat's "Last Theorem"*. My teaching objective would be for the students to observe Andrew Wiles' mind 'in action' applying the mathematical processes in his relentless effort to prove *Fermat's "Last Theorem"*. Reading Faust's article was an "echo chamber" experience for me.

Mathematics for the Gifted Learner

As the decade came to an end Verda Petry (1989), a well-known Regina high school mathematics teacher and former Chancellor of the University of Regina, weighed in on the need for special programs for gifted learners in mathematics. Petry's article focussed on recommendation six in *An Agenda for Action* (NCTM, January 13, 2009c) where it was stated that: "The student most neglected, in terms of realizing full potential, is the gifted student of mathematics. Outstanding mathematical ability is a precious societal resource, sorely needed to maintain leadership in a technological world." Petry researched the cognitive and affective characteristics and needs of gifted learners and went on to say that the "identification of students for special classes should be multi-dimensional and should include *task commitment* and creativity as well as high level cognitive ability" (p.11). With due respect to the author, I italicized 'task commitment' as an important understanding that gifted learners and parents/guardians should address before deciding to become part of a special program. Without it, there is a temptation for parents/guardians to negotiate to 'normalize' special programs.

Sex-Related Differences in Mathematics Achievement at the Division IV Level

I have left Verda Petry's 1985 article about sex-related differences in grades 10 – 12 in mathematical achievement to last. Petry was motivated by a quote "from the April, 1981 issue of *Discover* magazine: 'The most recent study deals with the long-established fact that skill in mathematics is far more common among men than women'" (Weintraub, 1981, p.15). Petry attempted to verify this claim by conducting research with students at the high school where she taught mathematics.

Inferred in this article is the importance of classroom teachers to regard themselves as professionals who not only recognize such statements in their readings but also-question them. The inference is that along with other educational partners classroom teachers could establish a meaningful 'bottom up' approach to classroom research. In today's context teacher educators would establish blogs, personal learning networks and set up hashtags such as #classroomresearch to generate classroom research.

Concluding "The Eighties"

Reading these ten articles make you feel like you have gone through a 1980's "Back to the Future" moment of concerns in pedagogy, assessment, enrichment, equal opportunity, technology and even the history of the SMTS. I wonder how these concerns and ideas of 'the eighties' were addressed in the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989)...?

I would like to thank Dr. Egan Chernoff, the editor of *viniculum*, for the opportunity to look back and comment on these ten articles that appeared from 'The Eighties' in the JSMTS.

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Dear Rick, Thank you for your commitment to the teaching and learning of mathematics in the province of Saskatchewan – found both inside the pages of vinculum: Journal of the Saskatchewan Mathematics Teachers' Society (clearly!) and beyond.

~Egan J Chernoff

Congratulations, Rick, on your retirement! This issue of vinculum is like a snapshot of the contributions you've made to mathematics teaching and learning in Saskatchewan and Canada. Your articles and ideas have opened spaces for teachers, professors, and sessional instructors to try new ideas and have spurred mathematical conversations. These conversations always invite teachers of all levels to broaden their own understandings of mathematics, of teaching, and of learning. Enjoy your well-deserved retirement.

~Florence Glanfield

In Our Circles, In Our Circles.

Although I cannot remember the story behind the line, every time I went to see your classes, Rick, and you detailed how our lives were moving in parallel circles, I could hear Carol Burnett and Madeline Kahn saying "In our circles, in our circles," and I imagined them twisting their raised hands around. In that one moment, in that one story of our circles, you made me feel welcome and safe. That is my most poignant memory of being around you, Rick – that you always make people feel welcome and safe to be themselves. I have always admired the way you encouraged and cared about the endeavours of your students: the future mathematics teachers and continuing citizens who entered your classrooms. Of course, I will also never forget, nor stop being influenced by, our deep and rich conversations around curriculum renewal, representation, proportional reasoning, and real-world problem-solving. Those too are circles into which you kindly invited me, and which I will continue to revisit for many years to come. I wish you well, in whatever circles you will continue to create, be part of, and contribute to.

~Gale L. Russell

Congratulations on your retirement, Rick! The U of R mathematics education subject area will not be the same without you... and neither will my impromptu hallway conversations, OCRE, Faculty council meetings, the Rams, and a host of other experiences you've been a part of! Wishing you the very best.

~Your math ed colleague, Kathleen Nolan

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