The Variable

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Increased Participation and
Conversation Using Networked Devices
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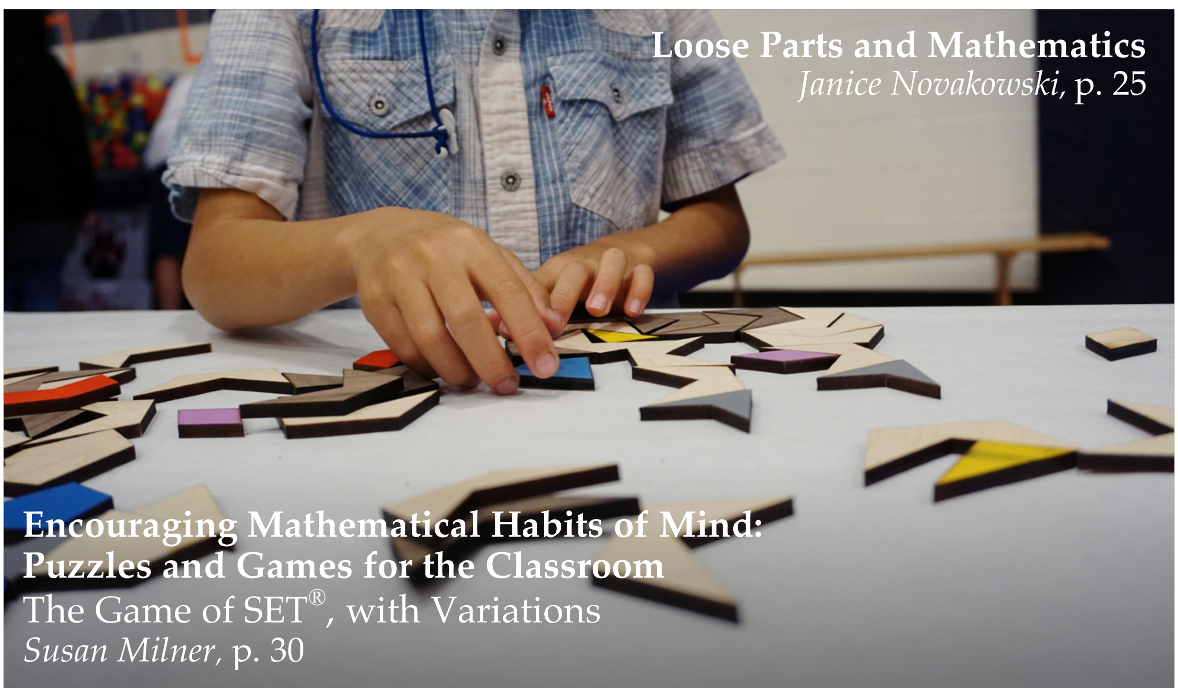

Spotlight on the Profession:
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Saskatchewan Mathematics Teachers' Society

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In this issue

Cover art

This photo was taken at the Tommy Douglas Math Fair in June 2016, where more than 300 Grade 8 students were led through a series of mathematical activities by Tommy Douglas students and staff volunteers. Students were asked to work collectively on the given tasks and become active problem posers and solvers. The fair gave students the opportunity to wonder aloud with their peers while activating curricular skills developed through elementary school instruction. For more information, see:

Math fair introduces Grade 8 students to high school. (2016, June 14). Retrieved from http://www.spsd.sk.ca/school/tommydouglas/Pages/newsitem.aspx?ItemID=29&ListID=fb208f8b-c4bb-46b3-b2d6-ad94a0532d37&TemplateID=Announcement_Item

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Notice to Contributors

The Variable welcomes a variety of submissions for consideration from all members of the mathematics education community in Canada and beyond, including classroom teachers, consultants, teacher educators, researchers, and students of all ages, although we encourage Saskatchewan teachers of mathematics as our main contributors. Submitted material is assessed for interest, presentation, and suitability for the target audience.

Please submit articles by email to thevariable@smts.ca in Microsoft Word format. Articles should be limited to 3000 words or less; authors should also include a photo and a short biographical statement of 75 words or less. Accepted contributions are subject to revision. Editors reserve the right to edit manuscripts for clarity, brevity, grammar, and style.

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For more information or to register, visit www.stf.sk.ca.
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#SUM2017
Who: K-12 teachers, coaches, consultants, coordinators, superintendents and directors
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Cost: $315 (early registration) | $375 (regular)
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Message from the President

Happy Holidays (almost)!

While the work keeps coming, I think I'm still riding high from this years' fabulous Saskatchewan Understands Math (SUM) Conference. Eight years ago, an equally great SUM Conference was my very first experience with the SMTS. Clearly it was an impactful event, as I'm still here and I feel just as passionately about SUM, if not more so, than I did then.

There were so many takeaways and wonderful conversations. I'd encourage you to hop on over to Twitter or to connect with a colleague to share yours, if you haven't done so already. For me, two ideas are at the forefront of them all, and keep rumbling around my mind, challenging my thinking. Since the Variable team is kind enough to give me a little space to write every month, I'll share mine here.

What do you need to get worse at in the short term in order to improve in the long term?

I think Peg Cagle said this much more elegantly, but it was a month ago, so forgive me. During the Panel discussion, Grace Kelemanik summed it up nicely as "dropping the glass to grab the pitcher" - in other words, abandoning practices that we are good at and that are working for us in order to embrace even more effective practices. This is such an uncomfortable space to occupy. How can we best navigate this space, as a community of learners? How do we support one another, our students, and their families during these not-so-smooth transitions? What systemic changes might we influence to make teacher growth the norm?

This leads right into the next idea from Max Ray-Riek, which is both simple and completely mind-blowing.

What does professional development that allows teachers to practice the skills of teaching and to reflect and get feedback on those skills look like?

I've lost count of how many workshops I've participated in on differentiation and problem based learning. And of course, there are always new nuances, new-to-me strategies, new things to try and to think about. However, what are the skills of differentiation in the moment? Can I get better at, more thoughtful about how I offer parallel questions to students? And is there a more engaging way of practicing this skill with other teachers than thinking up scenarios and sharing our thoughts? Max's Number Talk Karaoke strategy wasn't just engaging, it got at the root skill that teachers leverage to support students in their learning. What other skills can we target in such a deliberate and engaging way? There is just so much possibility here to explore and play with, all with the intent of making those important changes to our instructional practices.

Lastly, I'm walking away from SUM 2016 already thinking ahead to SUM 2017! We are so excited to partner with the Saskatchewan Educational Leadership Unit (SELU) and the

Saskatchewan Professional Development Unit (SPDU) for 2017 in growing this conference to include division leadership in these crucial conversations about the teaching and learning of mathematics. We look forward to showcasing all of your hard work, and to support your leadership in dropping their own glasses, so to speak, with regards to mathematics. We're always better when we learn together.

All the best this holiday season. May it be rejuvenating, in whichever way suits you!

Michelle Naidu

Problems to Ponder

Welcome to the October edition of Problems to Ponder! This month's problems have been curated by Michael Pruner, president of the British Columbia Association of Mathematics Teachers (BCAMT). The tasks are released on a weekly basis through the BCAMT listserv, and are also shared via Twitter (@BCAMT) and on the BCAMT website. This column features only a subset of the problems shared by Michael last month – head to the BCAMT website for the full set!

Have an interesting solution? Send it to thevariable@smts.ca for publication in a future issue of The Variable.

British Columbia Association of Mathematics Teachers

I am calling these problems 'competency tasks' because they seem to fit quite nicely with the curricular competencies in the British Columbia revised curriculum. They are non-content based, so that all students should be able to get started and investigate by drawing pictures, making guesses, or asking questions. When possible, extensions will be provided so that you can keep your students in flow during the activity. Although they may not fit under a specific topic for your course, the richness of the mathematics comes out when students explain their thinking or show creativity in their solution strategies.

I think it would be fun and more valuable for everyone if we shared our experiences with the tasks. Take pictures of students' work and share how the tasks worked with your class through the BCAMT listserv so that others may learn from your experiences.

I hope you and your class have fun with these tasks.

Michael Pruner

Intermediate and Secondary Tasks (Grades 4-12)

October 2, 2016

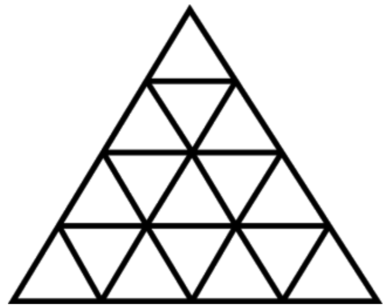
Rope Around the Earth
A rope is wrapped tight around the Earth along the equator. The rope is cut, and 1 m of rope is added to the length and then stitched back together. A big green super hero lifts the rope and throws it so hard that it enters into circular orbit the Earth (bonus for the physicists: How fast does the rope need to spin to be in orbit?). How high is the rope hovering over planet Earth? (Earth's radius is 6371 km.)

Extensions: What about Jupiter, the Sun, or a basketball? Instead of spinning the rope, the super hero lifts the rope straight up until it is tight again. How high can she lift the rope?

October 9, 2016

The Chess Board
How many squares are there on a chess board? And no, the answer is not 64.

Extensions: How many rectangles? How many triangles in the figure below?



October 16, 2016

Number Pattern
Consider the following pattern of 5 whole numbers, where each number is the sum of the previous two numbers: 3, 12, 15, 27, 42. I want the 5th number to be 100. Find all the whole "seed" numbers that will make this so (3 and 12 are the seed numbers in the above sequence).

Source: Peter Liljedahl

October 23, 2016

Cartesian Chase
This is a game for two players on a rectangular grid with a fixed number of rows and columns. Play begins in the bottom-left-hand square, where the first player puts his mark. On his turn, a player may put his mark into a square directly above, directly to the right of, or diagonally above and to the right of the last mark made by his opponent. Play continues in this fashion, and the winner is the player who gets their mark in the upper-right-hand corner first. Find a way of winning that your great aunt Maud could understand and use.

Extensions: What if you cannot move diagonally? What if the top right square means that the player loses?

Source: Mason, J., with Burton, L., & Stacey, K. (1985). *Thinking mathematically.* Reading, MA: Addison-Wesley.

Silver Coins
You have 10 silver coins in your pocket (silver means that the coins could be any of nickels, dimes, or quarters). How many different amounts of money could you have?

Primary Tasks (Grades K-3)

October 9, 2016

Materials
- 10 or more snap cubes per student

This is an activity that children can work on in groups. Each child makes a train of connecting cubes of a specified number. On the signal "Snap," children break their trains into two parts and hold one hand behind their back.

Children take turns going around the circle showing their remaining cubes. The other children work out the full number combination.

Source: Snap it. (n.d.). Retrieved from https://www.youcubed.org/task/snap-it/

October 16, 2016

Take your class outside and have students collect 5 of an object (leaves, rocks, etc..). The task is for students to work in groups and find different ways to make 5. How many ways can you make 5? How can you show all of your ways?

Extensions: How about 4? How about 6?

October 23, 2016

Give students time to explore the attributes of various 3-D shapes. Have them identify the faces, edges and vertices of the 3-D shapes. Present various problems for them to solve:

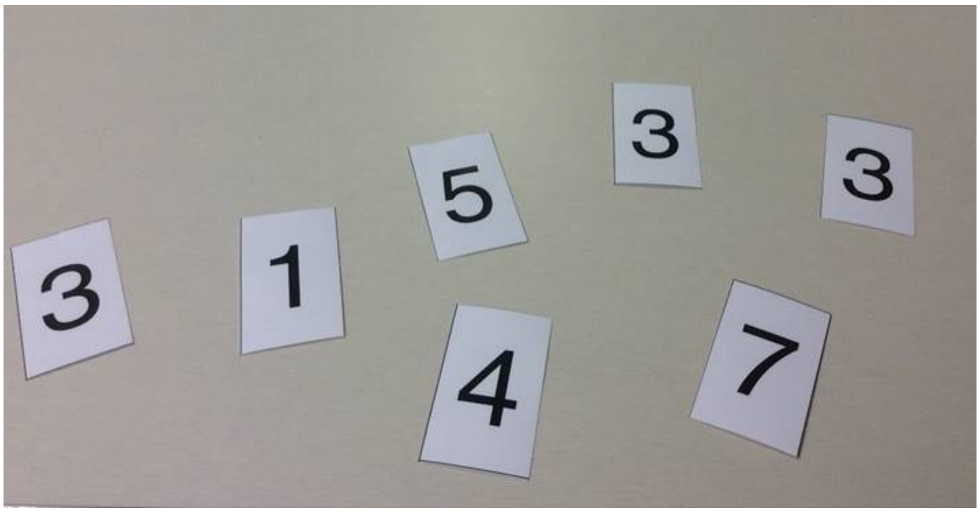
1. If you had 3 cones, 2 cylinders, and a sphere, how many faces would you have? How do you know?
2. You have 1 cube and your friend has 4 cylinders. Who has more faces? How do you know?
3. I have some objects and in total I counted 8 faces. What might the objects be? Explain your thinking.
4. I have a collection of objects that have 7 faces and a point. What shapes could they be? Explain your thinking.

Have the students create their own clues to create a problem.

Source: Ball, S., & Wells, K. (2015, Spring). Spring 2015 problem sets. *Vector, 56*(1), 40-43.

October 30, 2016

I used digit cards to create a 2-digit number pattern. The wind blew the cards and mixed them up. How might you place the loose digit cards into the following arrangement to complete a pattern? (See photos below.) How do you know? How might you extend the pattern?



Source: Ball, S., & Wells, K. (2016, Spring). Mathematics problem sets for your spring classroom. *Vector*, *57*(1), 40-43.

Michael Pruner is the current president of the British Columbia Association of Mathematics Teachers (BCAMT) and a full-time mathematics teacher at Windsor Secondary School in North Vancouver. He teaches using the Thinking Classroom model where students work collaboratively on tasks to develop both their mathematical competencies and their understanding of the course content.

Reflections

Reflections *is a monthly column for teachers, by teachers on topics of interest to mathematics educators: reflections on classroom experiences, professional development opportunities, resource reviews, and more. If you are interested in sharing your own ideas with mathematics educators in the province (and beyond), consider contributing to this column! Contact us at thevariable@smts.ca.*

SUMming up SUM Conference 2016

Sharon Harvey

Well, I made it through my first Saskatchewan Understands Math (SUM) conference while being on the other side! The organizing side, that is. It is unbelievable how much work goes into making sure our SUM conference runs smoothly and delivers a fabulous experience for our attendees. So first, I want to thank the SMTS and the SUM Conference committee for organizing such a great event. And let me tell you—it was a great event!

So what makes our conference so great? (I know what you're thinking: is she really going to write a whole column about how great the conference was that she helped organize? Yes. I am.)

I've been conferencing for years, and I'm sure many of you have as well. A great conference brings together opportunities for learners that span the philosophical, practical, and everything in between. It allows attendees to interact with each other, with keynotes, and with other presenters. This is exactly what we aimed to do with SUM Conference this year, as well as in the past. Here's how I think we were able to achieve this:

1. Maximum Time with Max, Grace, and Peg

At SUM, each day's morning session is two hours long, and allows attendees to choose from four options. Each keynote and each featured presenter run one of the sessions. This means that attendees can spend up to *four hours* with a featured presenter or keynote speaker (this year's keynotes and featured speaker were Max Ray-Riek, Grace Kelemanik, and Peg Cagle)—and four hours of time to learn with an educational mastermind is nearly unheard of at a conference. These sessions were filled with great ideas and discussions, including how to learn with students, learn from students, and learn from each other.

2. Sessions for Everyone

This year, SUM offered nine learning sessions in addition to the keynote sessions. During this time, attendees played with probability, learned to code, reviewed a variety of instructional approaches, and explored what makes a great resource… And that's not even half of what was offered! Whether you came looking for an opportunity to discuss some aspect of your practice with other teachers, or something to use in your classroom on Monday morning—you found it at SUM! I spent my own time learning about the importance of communication in the math classroom, how and when to introduce coding, and how to facilitate a number talk at all grade levels.

3. The Panel

SUM runs all day Friday and until noon on Saturday (this year, it was held on November 4-5). I'm sure that you can imagine how tough it can be to get to the Saturday portion. And we get that. We really get that. But if you did, you were in for a real treat. The conference ends with a panel discussion that includes our keynotes, featured presenters, and some session presenters. This year's panel, as always, handled some deep questions and challenged our thinking with their responses. For instance, they gave us ideas for treaty education through math education, a list of books to investigate, as well as their wish list for curriculum. Lastly, they left us with a reminder that change in the classroom—especially the kind that ultimately has a positive impact on student learning—can often result in things getting worse before they get better… but to see its benefits, you just have to keep going.

Clearly, I would mark the conference as a success. But would you? If you made it to the conference, let us know how you felt about the conference by filling out the survey emailed to you last week (you can also access the survey through the following link). Remember to also let us know who you would like to see as keynotes and presenters at future SUM conferences!

If you couldn't make it this year, you can still read about some of the insights gained and see some photos of the learning in action by checking out the hashtag #SUM2016 on Twitter. Hopefully, this article has helped you see the value of SUM conference and inspires you to attend next year's installment. We promise to keep you posted as we develop plans for SUM 2017!

Sharon Harvey has been a teacher within the Saskatoon Public School Division for eight years. She has taught all secondary levels of mathematics, as well as within the resource program. She strives to create an inclusive and safe environment for her students.

Spotlight on the Profession

In conversation with Dr. Peter Liljedahl

In this monthly column, we speak with a notable member of the mathematics education community about their work and their perspectives on the teaching and learning of mathematics. This month, we had the pleasure of speaking with Dr. Peter Liljedahl.

Dr. Peter Liljedahl is an Associate Professor of Mathematics Education in the Faculty of Education and the Associate Dean Academic for the Office of Graduate Studies and Post-Doctoral Fellows at Simon Fraser University in Vancouver, Canada. Peter is a co-director of the David Wheeler Institute for Research in Mathematics Education, President of the International Group for the Psychology of Education, a senior editor for the International Journal of Science and Mathematics Education, and the coordinator of the Secondary Mathematics Master's Program in the Faculty of Education at SFU. Peter is a former high school mathematics teacher who has kept his research interest and activities close to the classroom. He consults regularly with teachers, schools, school districts, and ministries of education on issues of teaching and learning, assessment, and numeracy.

First of all, I'd like to thank you for taking the time to have this conversation. To start things off, could you discuss your current research interests and projects? How has your work kept you close to mathematics classrooms?

Almost all of my work is centered around improving the teaching and learning of mathematics. To this end, I work closely with practicing in-service mathematics teachers interested in improving their practice. At the same time, I do research on both the teaching and learning of mathematics and the professional growth of teachers of mathematics.

Some of your recent work has been centered around the notion of a "thinking classroom" (e.g., Liljedahl & Williams, 2014; Liljedahl, 2016b). How would you describe such a classroom? How can classroom norms and the classroom environment contribute, or detract from, a culture of thinking?

I define a thinking classroom as follows:

> A *thinking classroom* is a classroom that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together, and constructing knowledge and understanding through activity and discussion. It is a space wherein the teacher not only fosters thinking but also expects it, both implicitly and explicitly. (Liljedahl, 2016b, p. 362)

Institutional norms (Liu & Liljedahl, 2013) often get in the way of such goals as they are predicated on activities of sitting and watching and listening. Desks are designed for this and the classroom is set up for this. It is not surprising, therefore, that traditional teaching happens in such traditional classroom spaces. Institutional norms dictate this design and this design reproduces these institutional norms. Classroom norms are greatly informed by these institutional norms. I found that it is almost impossible to change these norms if we do not first change the physical space of the classroom.

> "What would teaching look like if students stood rather than sat, wrote on whiteboards rather than in notebooks, worked in random groups rather than individually or in self-selected groups?"

For example, what would teaching look like if students stood rather than sat, wrote on whiteboards rather than in notebooks, worked in random groups rather than individually or in self-selected groups? Teaching and learning would, by necessity, look different if this was the environment. My research has experimented with such environments and the results are revealing that student learning can be fundamentally impacted by these changes to the learning environments.

In many of your recent presentations and workshops with mathematics teachers, you have discussed the potential of vertical non-permanent surfaces and visibly random groups in contributing to the development of thinking classrooms. (Readers who attended the 2015 SUM Conference may have been introduced to these ideas by Ontario teacher Alex Overwijk, who has presented on the application of the research in his own classroom.) What advantages do these strategies offer (and why is it important that the surfaces are vertical *and that the groups are* visibly *random)?*

I know Alex well and have worked with him on several occasions in Ottawa. My research on Building Thinking Classroom emerged a set of 9 tools for transforming traditional classrooms into thinking classrooms (see Liljedahl, 2016b). These 9 tools turn out to be most effectively implemented in three groups of three tools each. In the first collection of three tools are Vertical Non-Permanent Surfaces (VNPS) and Visibly Random Groupings (VRG).

The non-permanence of the surfaces (e.g., whiteboards) somehow frees students to risk more and risk sooner. The vertical aspect increases visibility and helps mobilize knowledge around the room. Visibly random groupings help students to feel that there is no teacher strategic reason keeping them away from their friends.

I want to emphasize that these tools, including VNPS and VRG, emerged out of research. They are not simply an idea, but rather empirically proven methods of teaching that are effective for transforming non-thinking spaces into thinking classrooms.

The notion of a "thinking classroom" suggests a departure from a focus on the individual—which, as Davis, Samara, and Luce-Kapler write, has historically, overwhelmingly been seen as the "fundamental particle of knowing" (2008, p. 59)—and towards consideration of the role of the collective in teaching and learning practices. Do you see limitations in focusing exclusively on individual learners in the (mathematics) classroom?

As teachers, we tend to have our learning settings emulate our testing settings. Thinking classrooms differentiates these spaces and seeks to maximize the learning setting. My research has revealed that this is most effective when there are opportunities for collaborations and knowledge is allowed to move freely around the classroom.

Your earlier work (and thesis) focused on the "AHA! experience": "the moment of illumination on the heels of lengthy, and seemingly fruitless, intentional effort" (Liljedahl, 2005, p. 219). What effect do these kinds of experiences have on students of mathematics, and is it possible to "lead" students to such experiences?

An AHA! Experience can have profound effects on students, transforming their beliefs about mathematics and their beliefs about the teaching and learning mathematics. This is true even for students with previously very negative beliefs. However, these experiences are elusive. We can create opportunities for them to occur, but we cannot guarantee them. I like to refer to this as occasioning AHA's.

For example, my research on the AHA! showed that AHA!'s seemed to occur on the heels of a period of incubation, after having first worked on a problem in a collaborative environment where lots of different ideas had been tried and discussed. Although descriptive, these qualities can be prescribed. I was able to occasion an AHA! for some students by having them work on a problem collaboratively one class wherein I filled the space with lots of ideas, analogous problems, and manipulatives. This was followed by a forced period of incubation where we worked on something else, took a break, or sent them home at the end of class. The next class, we started with an analogous problem where I again filled the space with ideas and manipulatives relevant to the solution of the initial problem. When I then asked the students to turn their mind to the initial problem I was able to see several students having AHA!'s.

In wrapping up this interview, I'd like to touch on a somewhat tangential topic. Throughout your work, you've referenced Mihály Csíkszentmihályi's notion of "flow," a positive mental state in which a person performing an activity is completely absorbed in the action, experiencing deep enjoyment and creativity (1990/2008, 1996/2013). What connections do you draw between Csíkszentmihályi's work and the teaching and learning of mathematics?

In mathematics education, we have very few ways to think about engagement in theoretical ways. Csíkszentmihályi's notion of *flow* is one of the few ways that this can be done. I have been greatly influenced by this theory for almost 15 years both in my work on the AHA! experience and, more recently, on my work on thinking classrooms. In fact, one of the 9 tools for building thinking classrooms involves using the theory of flow to guide the way teachers can best give hints and extensions.

> "In mathematics education, we have very few ways to think about engagement in theoretical ways. Csíkszentmihályi's notion of *flow* is one of the few ways that this can be done."

As I discuss in "Flow: A Framework for Discussing Teaching" (Liljedahl, 2016a), in the early 1970's Mihály Csíkszentmihályi became interested in studying, what he referred to as, the optimal experience (1998, 1996, 1990),

> a state in which people are so involved in an activity that nothing else seems to matter; the experience is so enjoyable that people will continue to do it even at great cost, for the sheer sake of doing it. (Csíkszentmihályi, 1990, p.4)

The optimal experience is something we are all familiar with. It is that moment where we are so focused and so absorbed in an activity that we lose all track of time, we are undistractible, and we are consumed by the enjoyment of the activity. As educators we have glimpses of this in our teaching and value it when we see it.

Csíkszentmihályi, in his pursuit to understand the optimal experience, studied this phenomenon across a wide and diverse set of contexts (1998, 1996, 1990). In particular, he looked at the phenomenon among musicians, artists, mathematicians, scientists, and athletes. Out of this research emerged a set of elements common to every such experience (Csíkszentmihályi, 1990):

1. There are clear goals every step of the way.
2. There is immediate feedback to one's actions.
3. There is a balance between challenges and skills.

4. Action and awareness are merged.
5. Distractions are excluded from consciousness.
6. There is no worry of failure.
7. Self-consciousness disappears.
8. The sense of time becomes distorted.
9. The activity becomes an end in itself.

The last six elements on this list are characteristics of the internal experience of the doer. That is, in describing an optimal experience a doer would claim that their sense of time had become distorted, that they were not easily distracted, and that they were not worried about failure. They would also describe a state in which their awareness of their actions faded from their attention and, as such, they were not self-conscious about what they were doing. Finally, they would say that the value in the process was in the doing – that the activity becomes an end in itself.

In contrast, the first three elements on this list can be seen as characteristics external to the doer, existing in the environment of the activity, and crucial to occasioning of the optimal experience. The doer must be in an environment wherein there are clear goals, immediate feedback, and there is a balance between the challenge of the activity and the abilities of the doer.

This balance between challenge and ability is central to Csíkszentmihályi's (1998, 1996, 1990) analysis of the optimal experience and comes into sharp focus when we consider the consequences of having an imbalance in this system. For example, if the challenge of the activity far exceeds a person's ability, they are likely to experience a feeling of anxiety or frustration. Conversely, if their ability far exceeds the challenge offered by the activity, they are apt to become bored. When there is a balance in this system, a state of (what Csíkszentmihályi refers to as) flow is created. Flow is, in brief, the term Csíkszentmihályi used to encapsulate the essence of optimal experience and the nine aforementioned elements into a single emotional-cognitive construct.

> "If the way we give hints and extensions is guided by trying to maintain a balance between complexity and ability, we can keep students in flow."

If the way we give hints and extensions, then, is guided by trying to maintain this balance between complexity and ability, we can keep students in flow.

Thank you, Dr. Liljedahl, for taking the time for this conversation. We look forward to your upcoming work in this area and to continuing the discussion in the future.

Ilona Vashchyshyn

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Increased Participation and Conversation Using Networked Devices[1]

Christopher Danielson and Dan Meyer

For many of us, the phrase "teaching math online" evokes a vision of teaching and learning that is not based in physical classrooms. Perhaps teachers and students are even interacting asynchronously. In math classrooms in the United States, the increasing availability of devices (e.g. laptops, Chromebooks™, smartphones, and tablets) and networks allows students to access the Internet quickly and reliably. Imagining the possibilities for classrooms is an important responsibility of curriculum developers, district and school-level curriculum supervisors, and classroom teachers.

> "For many of us, the phrase 'teaching math online' evokes a vision of teaching and learning that is not based in physical classrooms."

The authors of this article are on the teaching faculty at Desmos®, which offers a free, online graphing calculator that runs in the window of any modern web browser. In recent years, we have been extending this technology—and merging it with our pedagogical vision—by developing a suite of online classroom activities for use in secondary classrooms, with a goal of helping teachers and students maximize mathematics learning with digital tools. We currently have six dedicated activities (mostly for algebra) and two tools—Polygraph and Activity Builder—that teachers can configure to meet their curricular needs in a variety of topic areas. All of this is free to teachers for individual classroom use at teacher.desmos.com.

We begin this article by describing our vision through the principles our team has articulated in our curriculum development work. We then describe two activities we have developed that make novel use of classroom-based Internet access, including examples of the kinds of discourse and learning that these activities elicit.

The principles that guide our lesson development work include the following:

- Use technology to provide students with feedback as they work.
- Use the existing network to connect students, supporting collaboration and discourse.
- Provide information to teachers in real time during class.

Feedback

Students often receive feedback from their teachers in the form of answers marked right or wrong, or points either added or docked. When students receive written descriptive feedback, it comes at the expense both of a waiting period and of the teacher's time.

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Computers can mark answers right and wrong much faster than a teacher can. As a consequence, this quick assessment—together with hints—is the typical experience many students have of doing school math online.

We think electronic feedback should be more than hints and corrected answers. Computers can provide students with an understanding of the implications of their thinking. At a basic level, teachers who show students a graph and have them infer its symbolic form can then use computer feedback to have students check their work by using a graphing calculator. When students graph their equations, the computer shows them the implications of their thinking. In such situations, students naturally want to make changes in response to computer feedback. Maybe the parabola needs to shift left, or open a bit more slowly, or open downward. Students respond to the feedback and quickly get more feedback on their next attempt. For this reason, we refer to this process as iterative feedback.

Iterative feedback is low risk because students know they can revise their attempts—not just their first attempt, but their subsequent attempts as well. Important goals of iterative feedback are supporting students in taking intellectual risks and encouraging them to persist. When a student says or thinks, "I'll try again; I can make this better!" iterative feedback is doing its job well. Furthermore, when an online lesson is constructed to give good iterative feedback, students can respond to a prompt as simple as "Just draw (or try) anything." The teacher can trust that this entry will make the task accessible to all students while also moving students' mathematical understanding forward.

Collaboration

The increasing availability and quality of Internet-enabled devices in classrooms—and of Internet connections in those classrooms—is something we seek to harness in creative ways in our work. Rather than using the network for the purpose of connecting individual students to the teacher or to a central server, we use it to connect students with one another. In our lessons, students can share ideas, ask questions of one another, and challenge one another in rich and interesting ways. The network facilitates showing students the solutions their classmates have shared, challenging students with new tasks their classmates have designed, and sharing comments and solutions for these shared tasks.

"The increasing availability and quality of Internet-enabled devices in classrooms is something we seek to harness in creative ways in our work."

Information for Teachers

Classroom-based online instructional platforms typically come with dashboards for teachers. These dashboards give teachers at-a-glance information about the progress of individual students in their classes. In many situations, this information is broken down by content strand and by proficiency level—e.g., if José is proficient at adding fractions with common denominators but struggling to write the decimal form of fractions, then a green square and a yellow square, indicating José's proficiencies, show up on the teacher dashboard's grid.

This type of dashboard provides a view of what students have mastered, but it does not give teachers insight into how students are thinking as they work. A richer dashboard can show a teacher what students are doing as they work and allow her to move quickly between views of an individual's work and of the whole class.

When we design a teacher dashboard for a Desmos lesson, we ask ourselves these questions:

- What information will a teacher find useful while the lesson is going on?
- What information will a teacher find useful after the lesson is over, as he prepares for the next day's instruction?

We then design the dashboard to capture this information and organize it in ways that help teachers do their work.
Information that is useful for teachers during the flow of the lesson, and that we strive to make quickly accessible in our dashboards, addresses the following questions: Who seems to be guessing rather than thinking carefully? Who has one of several common wrong answers? What different correct forms of an algebraic expression has my class generated so far?

Answers to these questions help teachers decide which students to speak with, when to pause the lesson for whole-class discussion, and how to structure a summary discussion at lesson's end.

Information that is useful in planning follow-up instruction might include the full text of student responses that a teacher can skim for the big picture of the class's work or search for use of vocabulary. (We have not incorporated search as a feature, but we do display the responses of all students in a class on a single page of text, which a browser can search easily.)

Two Activities
What do these principles look like when they come into being in the form of classroom lessons? In this section, we describe two lessons and how each one relates to our principles.

Central Park
In this activity, students move from estimation to calculation to abstraction as they decide how to place dividers in a virtual parking lot so that each parking space in the lot is the same width. If one space is too wide, another will be too narrow—resulting in too few spaces and angry drivers. Students receive feedback as they watch the cars attempt to park. At each phase, students can try again by adjusting their answers and letting the cars park again.

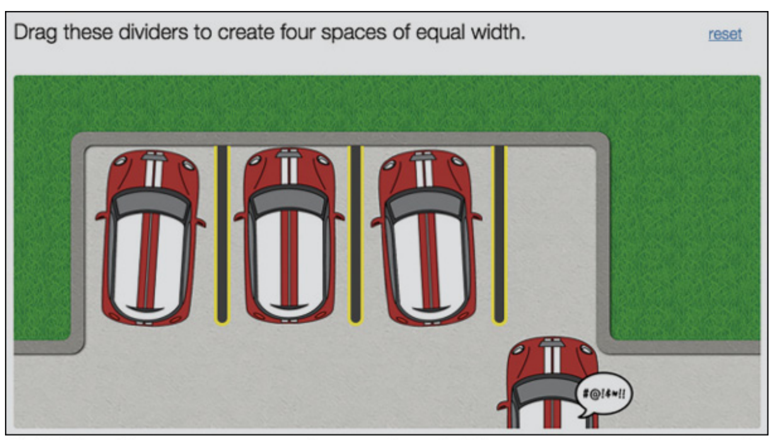

Fig. 1 When the spaces are correctly apportioned, all the cars can park; when there is trouble, as shown here, the drivers are aggravated.

Students begin by dragging dividers into place—no numbers, no computation, just estimation. In the example in figure 1, the rightmost space is too small. The student has received feedback by seeing the leftmost car park at an awkward angle to fill the large space and by seeing another car unable to park at all. Students can try again either by clicking "reset" or by moving the dividers.

In later phases of the activity, students calculate the width of the parking spaces (see fig. 2) and then use variables to describe these widths in multiple lots of different sizes and with varying divider sizes (see fig. 3). The activity is designed so that students use increasingly sophisticated tools in pursuit of expressing an algebraic relationship—and to validate the use of algebraic symbols as timesavers in doing repeated computations.

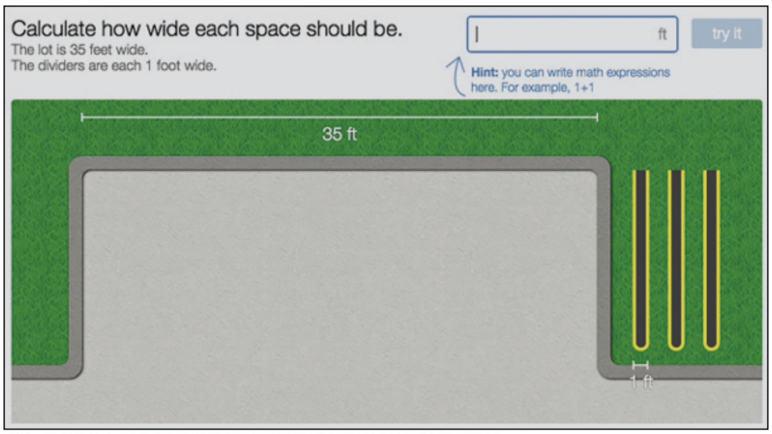

Fig. 2 The computation phase follows initial estimation.

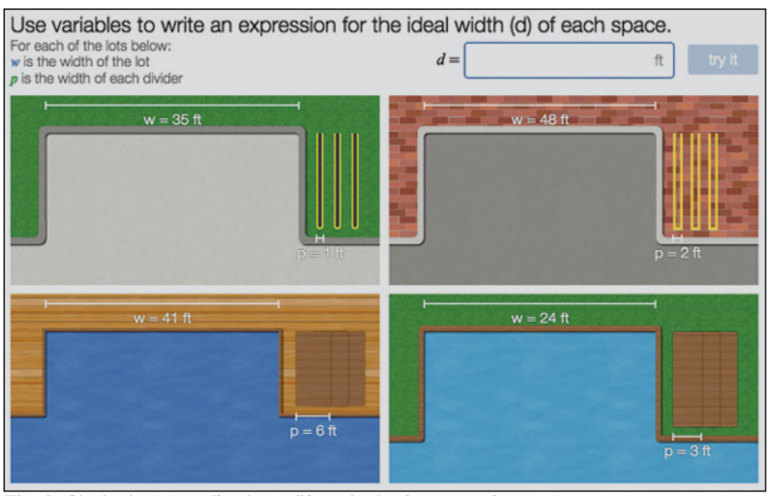

Fig. 3 Students generalize by writing algebraic expressions.

The online delivery of this activity allows students to receive feedback that goes beyond "right" or "wrong" and pre-loaded hints. Students can interpret their mistakes for themselves and adjust accordingly. Students usually will not use all necessary variables (i.e., *w* and *p*; see fig. 4). In such a case, they may successfully park cars in one scenario but fail in the others.

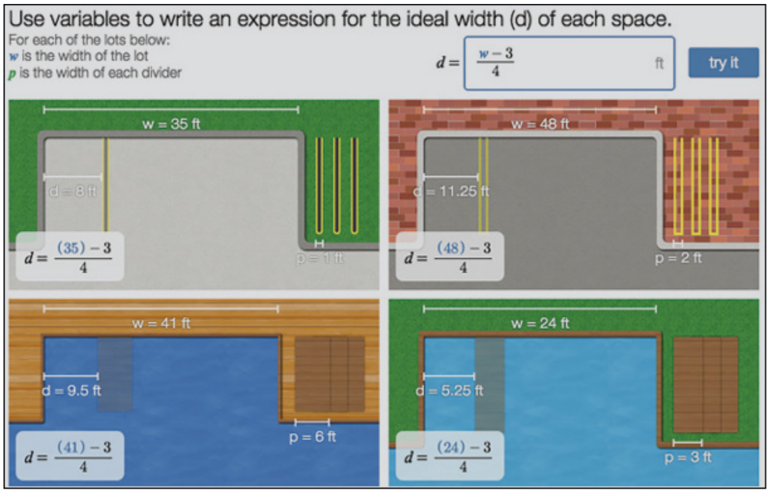

Fig. 4 The expression shown will result in an error because it does not account for the varying width of the dividers.

An important concern with iterative feedback of this nature is that students will sometimes guess-and-check their way to a solution rather than use the feedback to provoke rethinking. In the Central Park activity, a student may guess 10 feet for the width of the parking spaces in figure 2, then get feedback that this space is too wide. Such a student can guess his way to the correct value without noticing the important relationships between the given quantities. In designing this activity, we opted to leave this necessary refocusing in the hands of the classroom teacher. We give the teacher the information she needs to make an instructional decision. In this case, the guess-and-check behavior will usually result in a student being unable to write an expression using variables at the appropriate phase. With a click, the teacher can see all of the expressions her students have written, along with an icon that indicates correct and incorrect expressions (see fig. 5). We also alert teachers in our planning materials that unproductive guessing behavior is something to look for in this phase of the lesson (see "The Student Experience" at https://teacher.desmos.com/centralpark/).

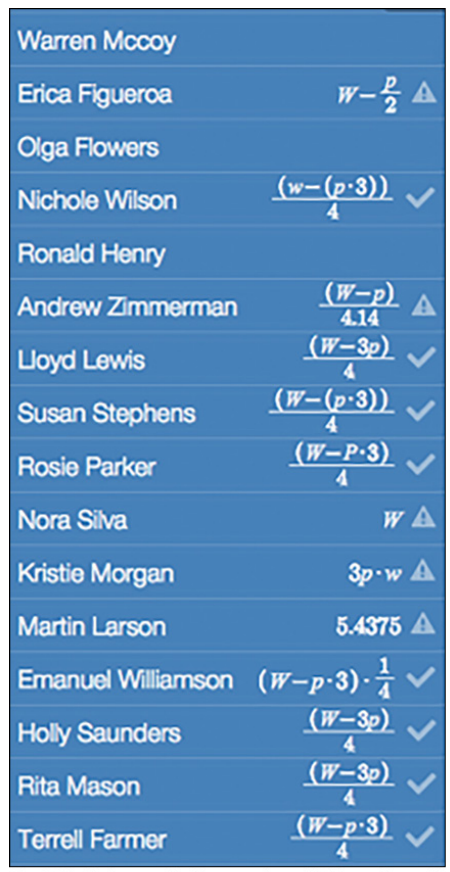

Fig. 5 Student names in this example are fictitious; the work is representative of the variety seen in actual classrooms.

Polygraph

Another activity illustrates our use of networked devices to connect students with each other. Polygraph is a question-and-answer game played in pairs. One student—the picker—selects one object out of sixteen that are displayed in a 4 × 4 array. Figure 6 shows the challenging rational functions version of the game, but there several versions, including lines, parabolas, quadrilaterals, and hexagons. The second student—the guesser—asks one question at a time, which the picker must answer by clicking yes, no, or I don't know. The goal is for the guesser to identify the correct object by asking about distinguishing features. The graphs are shuffled at the beginning of each round and appear in different places on each player's screen, in order to eliminate location in the array as a distinguishing feature.

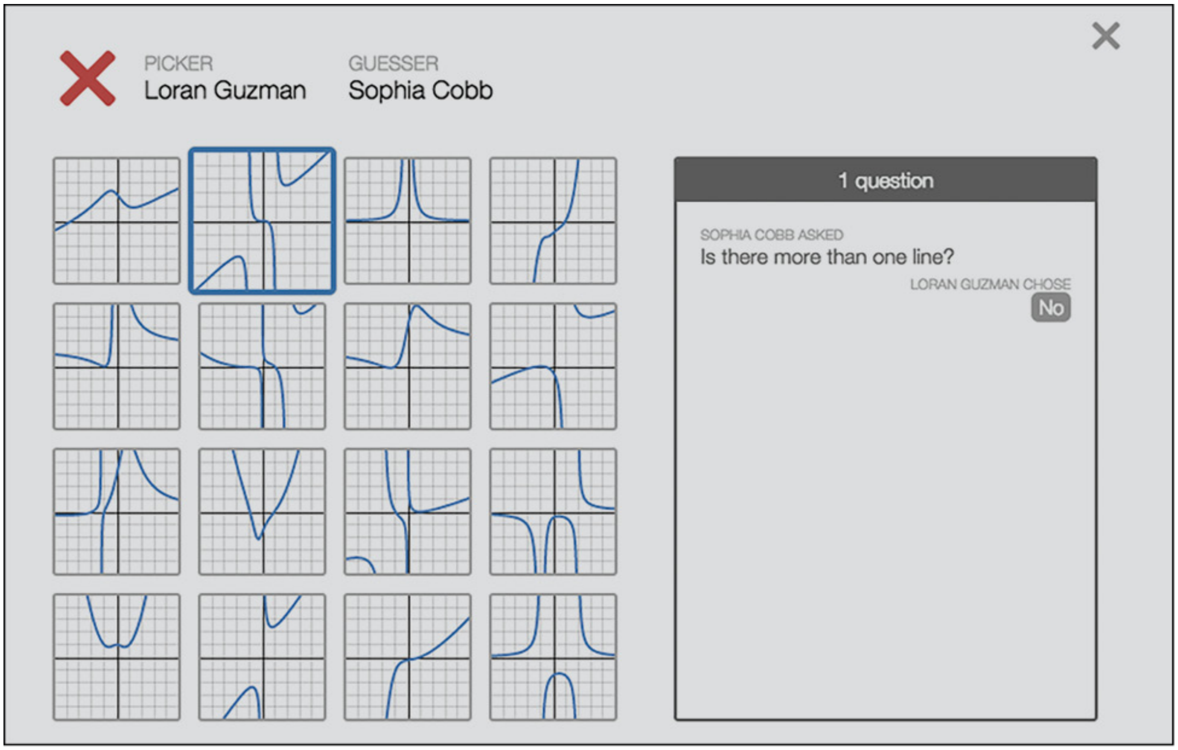

Fig. 6 In a game played in pairs, one student picks a graph and the other asks yes or no questions in an effort to figure out which graph was selected. Here, Loran's answer caused Sophia to eliminate the graph Loran had picked, and the game ended.

The activity Polygraph emphasizes collaboration. Partners work together to determine the correct object. If the guesser makes an error by deleting the object chosen by the picker, the partners are instructed to review their questions and answers and to discuss (face to face) where they went wrong.

Working together in this way creates a need for students to talk about properties of objects—properties for which they may not yet have names. This environment allows students to develop rich informal language, which is captured for teachers to use later in discussion and formalizing. Herbel-Eisenmann (2002) describes how informal "bridging languages" support students in preparing to understand and use "official mathematical language" in more meaningful ways than approaches that begin with formal mathematical vocabulary.

For example, we find students asking questions such as these:

- Is your hexagon dented?
- Does your graph have two pieces?
- Is the bottom of your graph on the x-axis?

These are informal ways of talking about concavity (of hexagons), branches (of rational functions), and vertices (of parabolas) that come from the features that students notice. The terms dented, pieces, and bottom are examples of bridging language that supports students in describing and formalizing features of these mathematical objects prior to learning the official mathematical terms for them. The question-and-answer collaborative interface built into Polygraph elicits these ideas and terms—and captures them for the teacher—from many more students than could participate in a linear, whole-class discussion.

It is worth noting also that this approach is foreign to print textbooks because they lack a built-in mechanism for progressive disclosure. Printed pages tend to put the most formal level of math thinking together with the least formal (often omitting informal ways of thinking altogether). In the cases of those print textbooks that do explicitly recognize and value student-generated informal vocabulary—such as the Connected Mathematics curriculum (Lappan et al. 1998) that Herbel-Eisenmann studied—the work of noticing, capturing, and capitalizing on this language is left up to the teacher. A carefully designed online lesson can do some of the work of capturing this language, making it easier for the teacher to capitalize on it. Further, online activities can be structured to disclose progressively. In Polygraph, we ask students to describe parabolas informally and later offer the formal vocabulary to describe the properties students have noticed.

This process can offer new insights to teachers and open new mathematical avenues for study. When a College Algebra class played the rational functions version of Polygraph, the teacher noticed students asking questions such as these:

- Does your graph have more than one piece?
- Does it have more than one line?
- Is your graph broken?
- Are there any holes in your graph?

Standard approaches to rational functions in College Algebra and similar courses focus on identifying the existence and location of vertical asymptotes. For students in this particular class, the vertical asymptotes were less immediate features of the graph than the number of branches the graph comprised. The student identified as Loran in figure 6 has chosen a rational function with three branches (names have been changed). Loran's partner Sophia has asked whether the graph has "more than one line." When Loran answered "no," Sophia eliminated that graph and the game came to an end. This miscommunication led to a face-to-face conversation in which Loran and Sophia worked out what each understood by Sophia's question about lines. In turn, this conversation prepared these students for the teacher to introduce the term branch.

To further contribute to the collaborative nature of the concept and vocabulary development in this lesson, students see questions, between rounds, that other students asked as they played (see fig. 7). This feature helps both informal and formal vocabulary be spread among students as the rounds proceed.

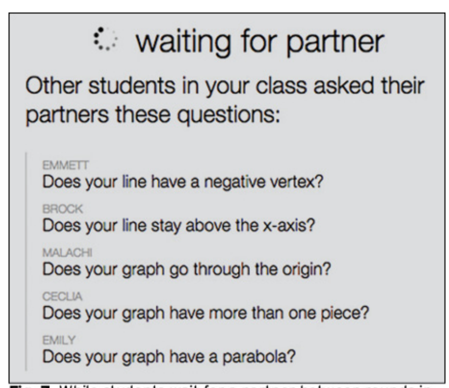

Fig. 7 While students wait for a partner between rounds in "Polygraph," they see questions that their classmates have asked while playing the game.

Summary

Our principles for online instruction may be quite different from current orthodoxy, which calls for individualization and atomization of skills. We believe in the power of combining quality provocations, robust tools to connect students, and skilled teachers to help students build mathematical understanding, vocabulary, and skill.

We hope that these ideas are infectious and inspirational and that they help to improve the conversation about the possibilities of educational technology in mathematics classrooms. Our development work continues, with information necessary for getting started at teacher.desmos.com and learn.desmos.com.

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Lappan, G., Fey, J. T., Fitzgerald, W., Friel, S. N., & Phillips, E. D. (1998). *Connected Mathematics*. Upper Saddle River, NJ: Prentice Hall.

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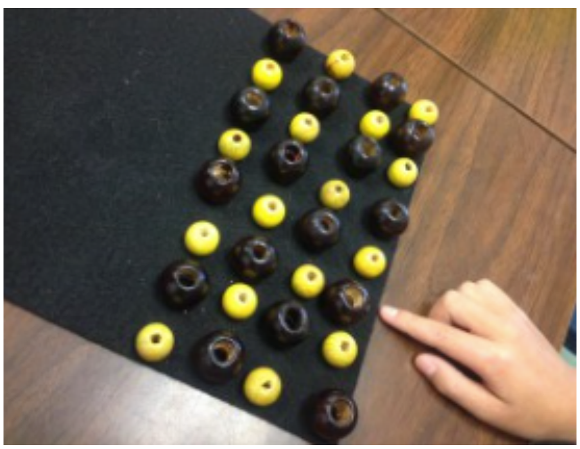
Loose Parts and Mathematics[2]

Janice Novakowski

In January, classroom teacher Michelle Hikida and I introduced the Reggio-inspired patterning kit to her Grade 2-3 class and considered the affordances of different materials to support mathematical thinking and inspire inquiry. The students used both mathematically structured materials, like pattern blocks and Cuisenaire rods, as well as loose parts. Loose parts are collections of materials such as rocks, glass gems, and buttons. The term loose parts originated within the field of landscape architecture, with Simon Nicholson expressing the "theory of loose parts" in the 1970s. Loose parts are materials that can be moved, combined, used to create with, and then taken apart and re-used in different ways. Many of our elementary students have been using loose parts in their classrooms across different curricular areas. For instance, as Michelle's students were invited to investigate the question, "What makes a pattern a pattern?", they used a variety of materials to explore patterning. Some examples of using loose parts for patterning experiences from the Grades 2 & 3 classroom appear below:



In March, Michelle approached the concept of fractions in the same way, laying out a variety of materials and asking students to show what they knew about fractions. What happened in response to this provocation surprised her and caused some reflection. Instead of representing their understanding of fractions with the loose parts and math materials, they represented the symbolic notation of fractions. In discussion with her students, Michelle realized that this is what they knew about fractions – they didn't understand the concept, they were only familiar with the symbolic notation.

[2] With thanks to teachers Michelle Hikida and Andrew Livingston and their students.

A prior version of this article was published on June 14, 2015 on Janice's blog at http://blogs.sd38.bc.ca/sd38mathandscience/2015/06/14/loose-parts-and-mathematics/.
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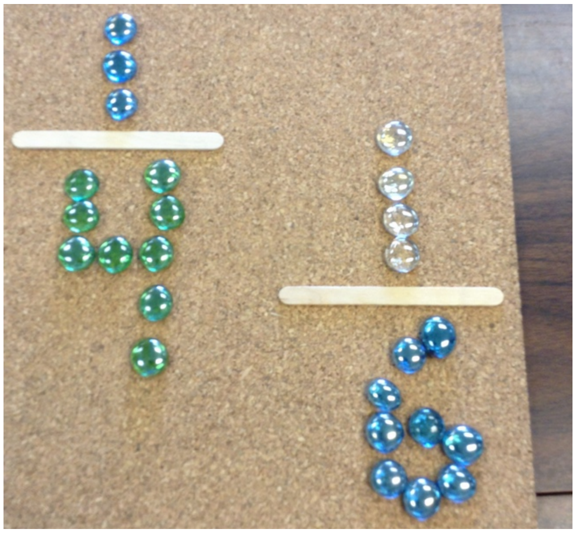
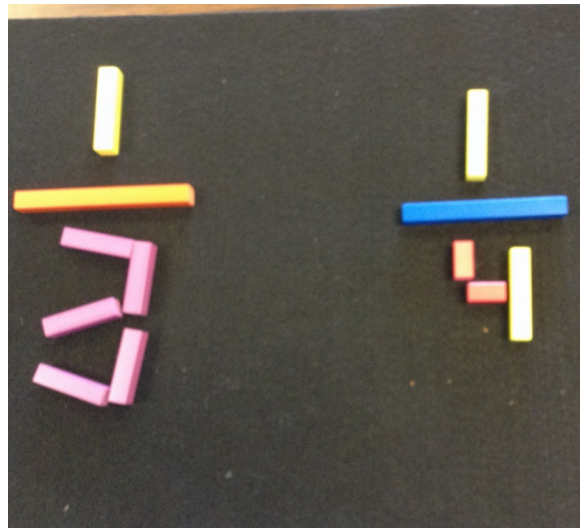
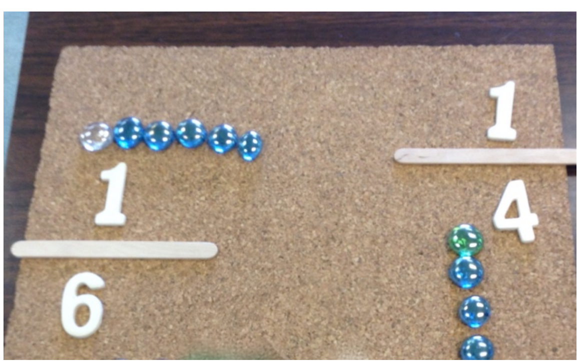
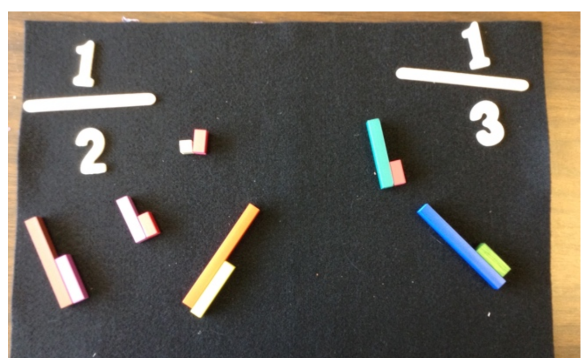
For example, students initially represented fractions this way:



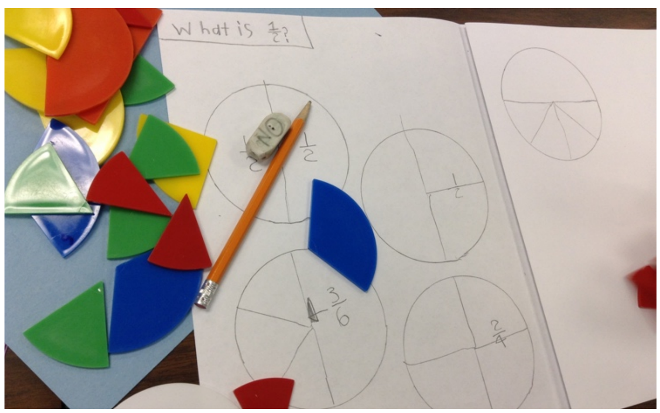


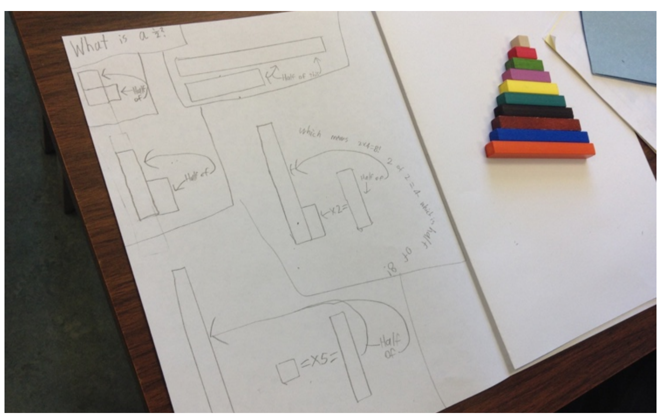
This made Michelle think back to the experience of when we introduced patterning. Students in Grades 2 and 3 have had previous school experiences with patterning and therefore have a place to start when demonstrating their understanding. However, when it comes to fractions, although students may have had informal experiences with the concept at home and at school, fractions are not formally introduced until late primary in our curriculum. After this experience, Michelle had her students spend some time working with loose parts and math materials such as pattern blocks and Cuisenaire rods, using an inquiry approach to help them develop understanding of fractions. Working with the loose parts and other materials, and guided by questions like *"What is a half?"* and *"How could you show what 3/4 means?"*, over time the students were able to develop and represent a conceptual understanding of fractions.



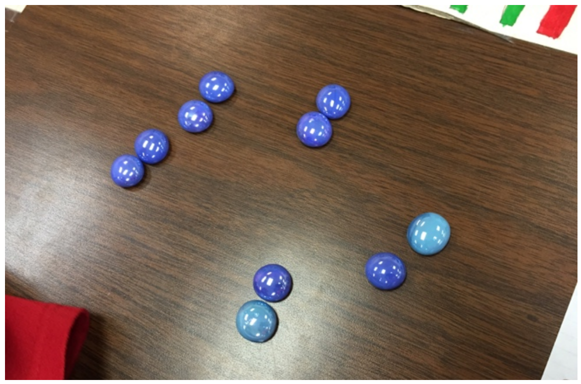


I visited the class a few weeks after this experience and observed that the students had made big jumps in their conceptual understanding and were able to represent fractions both concretely and pictorially, connecting to the symbolic notation. As the first photograph above shows, students were able to show an understanding of fractions as part of set (e.g., 1/6 is represented as 1 glass bead out of 6). In the photograph with Cuisenaire rods, the students are using a linear model to show their understanding of fractions. In the photographs below, the students are working with pattern blocks and fraction circles to compare fractions and represent them as part of a whole/area. Using loose parts and mathematically structured materials, young children are able to investigate different models and ways of representing fractions.





Another example of representing mathematical thinking with loose parts comes from Andrew Livingston's grade 3 class. Although we had previously looked at creating representations of what multiplication and division meant and the relationship between those two operations, for this particular lesson students were given loose parts to represent multiplication equations. The following photo shows that the student understands that 5 × 2 = 10, as the student has shown five groups of 2. If the student had used the loose parts to represent the equation by making a 5 and then a 2 and adding "symbols" made of other materials, it would not show evidence of conceptual understanding, just a representation of the equation, similar to what Michelle's students had initially done in their representation of fractions with loose parts.



In general, as educators, we need to be keen "noticers" when students are using materials, and consider the following questions in particular:

How are students using the materials?
What are the materials offering the students (or not)?
Do some materials have more affordances than others when it comes to learning and representing understanding of specific concepts?
Are the materials supporting students' thinking and understanding?
Are our questions or provocations supporting students' thinking and understanding?
What do students need in order to use loose parts successfully? What do we need to do as educators?

I believe this is a matter of responsiveness and awareness. To be responsive to what we notice in our students' actions and products, we need to take time to observe, notice, and be curious about their learning, but we also need to be aware and knowledgeable about the mathematics that the students are investigating so that we can respond and provoke their thinking.

Loose parts, along with mathematically structured materials, both provide different ways for students to think about mathematics and to represent their mathematical understanding of concepts and relationships. For more information about using loose parts and mathematically structured materials in the math classroom, see the following resources:

- *Loose Parts: Inspiring Play in Young Children* (2014) by Lisa Daly and Miriam Beloglovsky
- *Mathematics with Manipulatives*, 6-DVD Set by Math Solutions, featuring Marilyn Burns (features base ten blocks, pattern blocks, Cuisenaire rods, colour tiles, geoboards, and more)
- *20 Thinking Questions* series (primary and intermediate), available through McGraw-

Hill/Creative Publications (features pattern blocks, base ten blocks, fraction circles, rainbow cubes, attribute blocks, geoboards, and sorting treasures)

Janice Novakowski is a teacher consultant for the Richmond School District in BC where she learns alongside teachers and students in the areas of K-12 mathematics and science. She has taught preschool through university students and is currently a PhD candidate at the University of British Columbia where her doctoral studies have focused on the problem posing and inquiry practices of primary students.

Encouraging Mathematical Habits of Mind: Puzzles and Games for the Classroom

The Game of SET®, with Variations

Susan Milner
Professor Emerita, Department of Mathematics & Statistics, University of the Fraser Valley, British Columbia
www.susansmathgames.ca

I've shared Set with students of all ages, from Kindergarten to Grade 12, as well as with many adults. Children often pick it up more quickly than adults, which is something that children love to hear!

This isn't the first game based on attributes (see my previous article in Vol. 1, Issue 5 of *The Variable* on Latin squares for a much earlier one), but it is the grandmother[3] of more than a few popular commercial and educational games. It's not just about having fun, either, as considerably complex mathematics can be developed from the game of Set – but that comes later, for those who want to pursue more serious mathematics. Everyone can enjoy and benefit from playing with patterns, however, so let's focus on that.

There is essentially one rule for this game, and once students understand it completely, they are ready to play quite a wide variety of solo, cooperative, and competitive games. Rather than simply telling a class what it's all about, however, I encourage them to observe the cards and then let them figure out the rule by completing my examples.

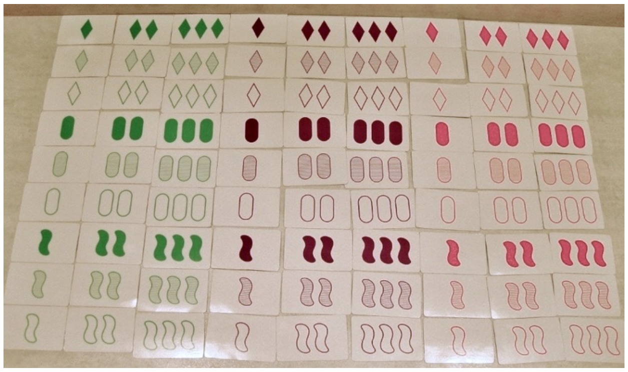
Here is the approach I've found most successful in introducing Set to students of all ages. I modify it for the youngest grades by using only one-third of the deck of cards, which I'll discuss later.

Observe first

I give a deck of cards to each group of students. It works best if only two or three students share a deck, so that everyone can get involved. I ask the students to spread out the cards so that everyone in the group can see them all.

Some students show a strong desire to sort the cards, which is in itself interesting and instructive.

[3] Marsha Jean Falco, a population geneticist who was studying inherited traits in German shepherds, created the game in 1974 for her own amusement and that of her family. She started marketing it in 1990, since which time it has become very popular. Find out more about the game, its rules, history, and on-line versions, as well as several articles for and by teachers at http://puzzles.setgame.com/set/main_page.htm



The photo on the left shows one way of sorting the cards. I don't like to disturb the momentum of the introduction by waiting for students to complete their sorting, but some just can't help themselves! They'll sort concurrently with the official classroom activities.

In a Grade 4/5 classroom where the teacher bought decks of Set and allowed students to explore them during free time, sorting was very popular.

Back to the class activity. I ask the students to raise their hands if they notice something about the cards. Here are some of the most common (and useful) answers, which I write on the board:

- 3 colours – red, purple, and green
- 3 shapes – diamond, oval, and squiggle (or toothpaste, or peanut)
- 3 shadings – solid, striped, and open (or hollow)
- 3 numbers – 1,2, 3
- all the cards are different

Already, it feels as though 3 is an important number here!

Some students will try to express the idea that each type of card comes in all colours, shapes, and so on, which is a very good observation but more complicated than we need at the moment, so I acknowledge this thought but don't pursue it right away.

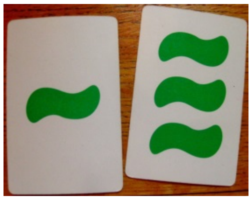
Complete the pattern

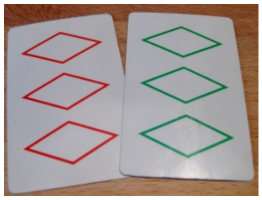
Now it's time for students to start completing some patterns. I'll hold up two cards and ask them to find the one that's missing. I usually ask them to hold the missing card up for me to see, but if I notice that some groups are looking to others for their answer, I ask students to put their card face down until the whole class is ready to show me at once. Once I've checked every group's answer, I show the class the completed Set.

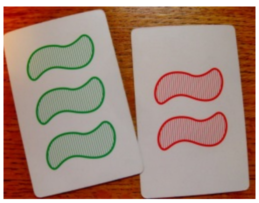
This process is a lot of fun and can be quite noisy, as the students become more and more excited by their ability to identify increasingly complicated patterns. Only after I am sure that most of the class can complete my patterns by feeling their way will I write an analysis on the board. The analysis often helps the few who might still be uncertain.

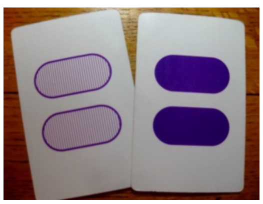
Below is a collection of examples, in the order I would present them. If one type of example seems to cause difficulties, I'll do a few more like it. You can't get the full effect by reading this article, as you will probably see the full Set at almost the same time as you see the incomplete one. Try to figure out what the missing card on the left side of the page should be before you look at the right side (cover it up, if necessary)!

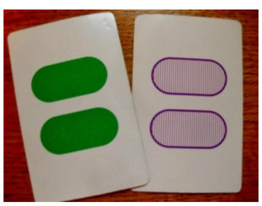
Incomplete Set

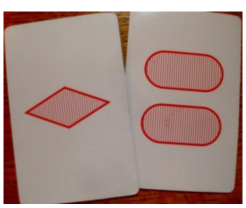


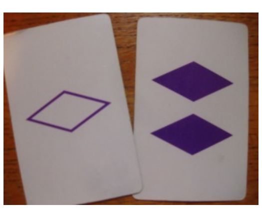




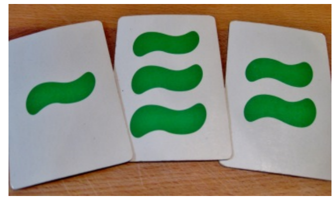


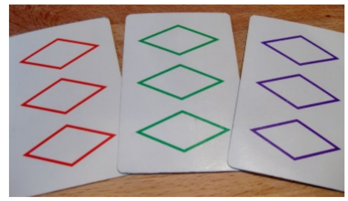


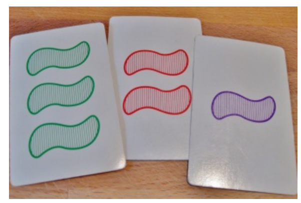


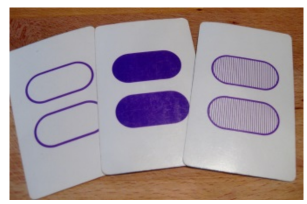


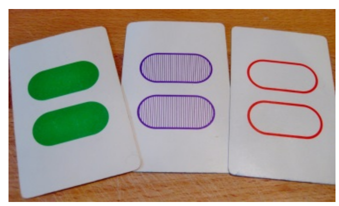
Completed Set

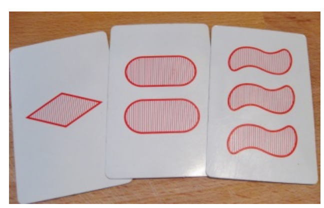


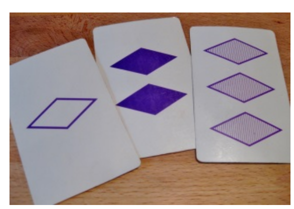


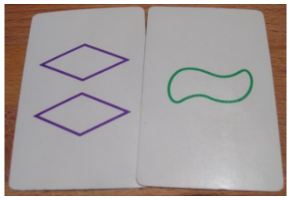


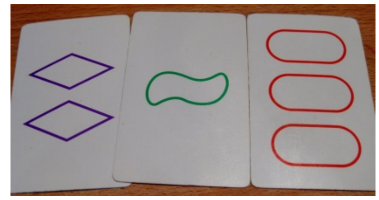


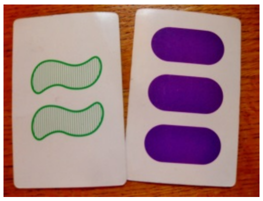


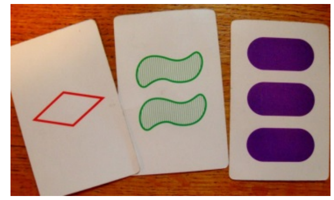


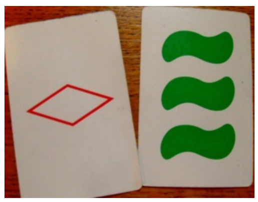


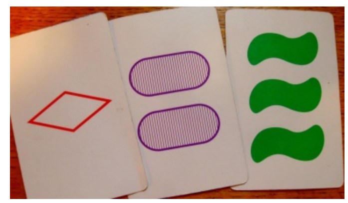










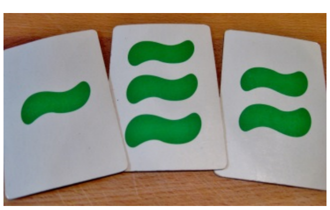


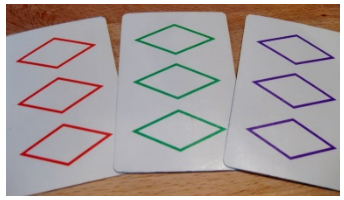
The last two examples are of the same complexity. This is as complicated as it gets, and many students like to try it twice before we move on.

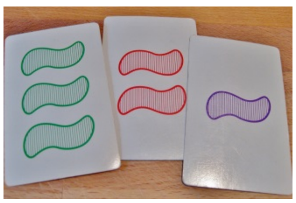
It's worth noting that *every* pair of cards has exactly one card to complete its Set. I'll ask two students from one group to each choose a card at random, without looking at what the other student chooses. Then I'll hold up both and ask the class to find the third card. If there is enough time, we do this several times.

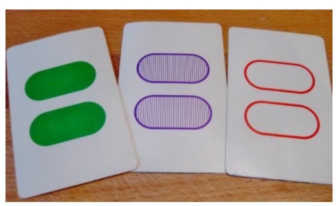
Some analysis

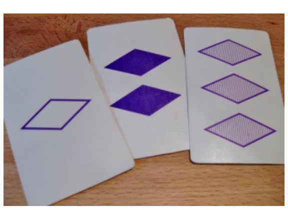
Now, it's time to go back to the list on the board and turn part of it into a chart, which we'll use to analyze several Sets. Here I will use some of the Sets in the above examples, but in class I also create new ones.

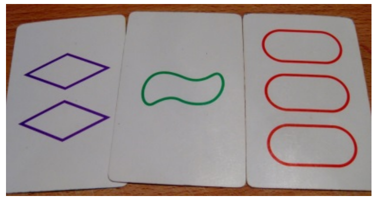
Set	attribute	same	different
	colour	✓	
	shape	✓	
	shading	✓	
	number		✓

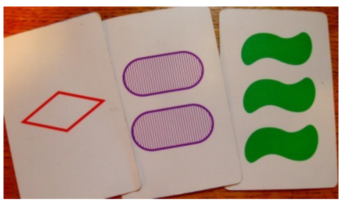
Set	attribute	same	different
	colour		✓
	shape	✓	
	shading	✓	
	number	✓	

Set	attribute	same	different
	colour		✓
	shape	✓	
	shading	✓	
	number		✓

Set	attribute	same	different
	colour		✓
	shape	✓	
	shading		✓
	number	✓	

Set	attribute	same	different
	colour	✓	
	shape	✓	
	shading		✓
	number		✓

Set	attribute	same	different
	colour		✓
	shape		✓
	shading	✓	
	number		✓

Set	attribute	same	different
	colour		✓
	shape		✓
	shading		✓
	number		✓

Is it possible to create a Set where all four attributes get a check-mark in the "same" column? It doesn't take long for students to figure out why it is *not* possible.

At this point, I've found it very helpful to ask students to create their own Sets according the description I give on the board. We usually do three or four, until they are confident that they understand and can describe what they may have felt intuitively earlier. Each student can create a Set, or they may work together if they prefer.

Here's an example of what I might ask for:

attribute	same	different
colour	✓	
shape		✓
shading	✓	
number		✓

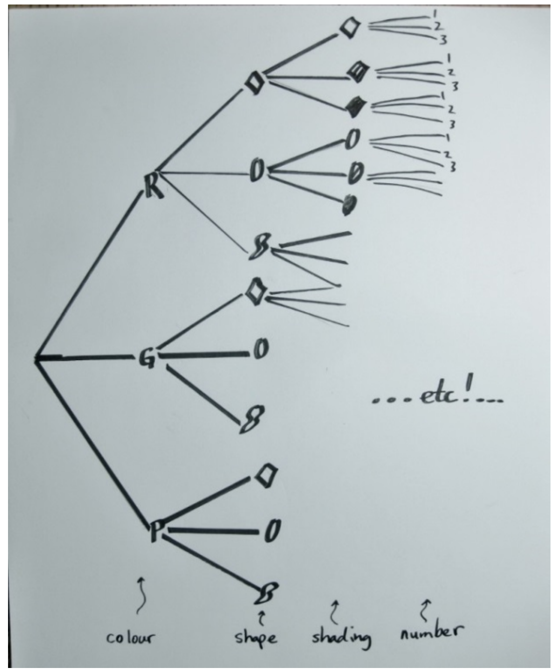
The rule
Finally, once we have spent quite a lot of time ensuring that everyone knows what a Set looks like, I'll write the definition on the board. It's very succinct:

A Set is a collection of three cards such that each attribute is either <u>all</u> the same or <u>all</u> different.

I've found that most people agree that they'd far rather start with examples than with the definition!

Number of cards

If I am working with a group that has studied or is about to study probability, I like to spend a few minutes thinking about how many cards there must be in a deck, without actually counting them. I ask the students what they think and usually get a wide variety of guesses. Often, a student will come up with the correct answer and even be able to explain how it arises.



A (partial) tree diagram is a good way to picture things.

There are three colours, each of which has three shapes, each of which comes in three shadings, and each of which comes in three numbers. Whew…

So the total number of cards is:

$$\begin{aligned} 3\cdot 3\cdot 3\cdot 3 &= (3\cdot 3)\cdot (3\cdot 3) \\ &= 9\cdot 9 \\ &= 81 \end{aligned}$$

Introducing Set to the younger set

While I've worked with individual children as young as five or six who have been able to pick up the idea from the full deck of cards, I've found it much more effective to reduce the complexity when I introduce Set to whole classrooms from Kindergarten through Grades 2 or 3. I go through the decks beforehand, separating out all the solid shapes. This comes to 27 cards in each deck.

We develop the idea of a Set in exactly the same way I've described above, but without the attribute of shading. We can then play modified versions of the games below.

If your group really enjoys this over a couple of sessions, then you might like to try the games with the full deck. Or, as an intermediate level, divide up the cards by shape rather than colour. That is, one student will get all the diamonds, one all the ovals, and one all the squiggles from a full deck. This way, you can introduce shadings without students having to work with all 81 cards.

Games

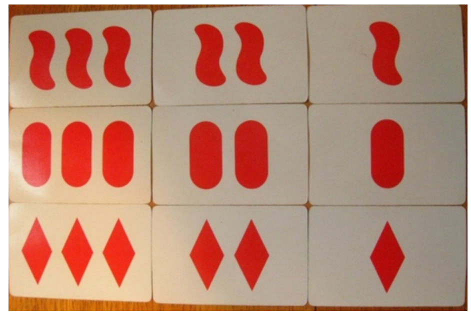
Once we understand how to create a Set, there is usually time (and energy) to play one or two games. I find that people get tired after 30 to 40 minutes of learning how to play Set, so it is best not to try to do too much during the first session.

The competitive version of the game that's described in the instructions accompanying the official deck of cards can be fun, but tends to get both irritating and discouraging if someone who is playing can spot Sets much more quickly than the rest of the group. I don't recommend it for general classroom play, although some students will enjoy it on their own time.

Here are some of my favourite variations, in order of increasing complexity.

1. Magic Set Squares

Arrange 9 cards in a 3x3 grid so that each row, each column, and the two main diagonals are Sets.

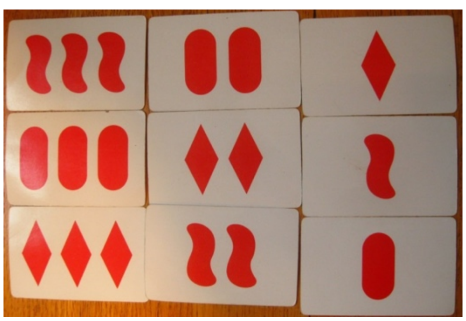


There are thousands of solutions. Each student can try it on their own or they can work with their group.

Any uncertainties as to what constitutes a Set will become evident, allowing teacher or peers to clarify the concept in a positive way. Sometimes it is the teacher who receives clarification from a student, which is always fun.

I really enjoy watching people wrestle with the problem, internalise it, form an approach, and create a magic square. Many will reduce the complexity of the problem (deliberately or accidentally) by working with a single colour and shading, or even a single colour, shape and shading.

That is great problem solving. Once they have succeeded, I suggest that they make it harder by adding in more colours!

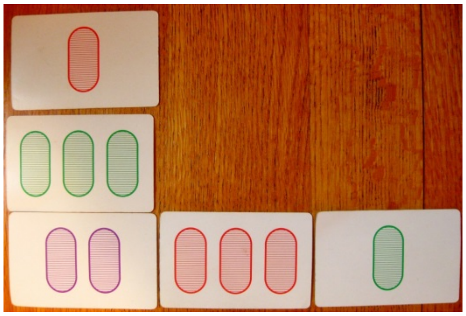


Sometimes, a student will notice that the same cards can be used to make a different magic square.

Here, the first column is identical to the original, but the first row was altered, forcing a rearrangement of every other column.

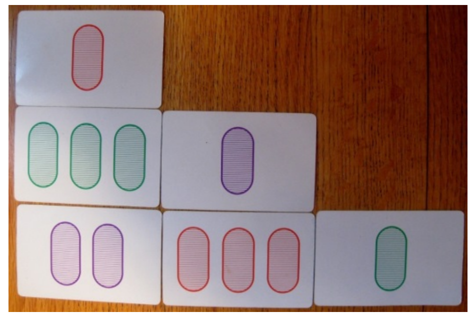
Students will happily play this game for quite a while and, with encouragement, will create ever more complicated squares.

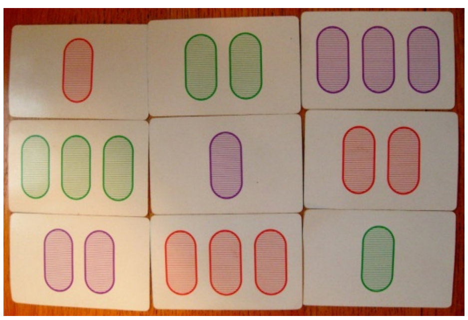
Sometimes, individuals or groups will run into difficulties if they use the following approach: put a Set in the first row, then a Set in the second row, then a Set in the third row. It's often possible to keep track of the columns, but the diagonals tend to be forgotten or quickly become problematic. If I see a group starting to get too frustrated, I'll give them a hint, starting with part of the structure they have created. Here's an example:



Let's use the first column and bottom row, as they are definitely Sets. Now think about the diagonal from top left to bottom right. We have two out of three cards, so we know for sure what has to go between them.

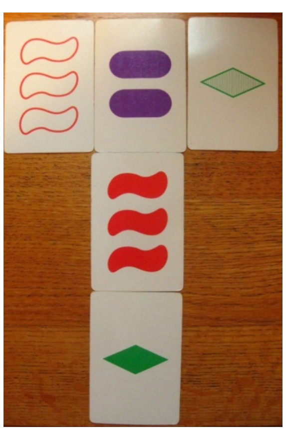
Now the students can finish the square on their own.

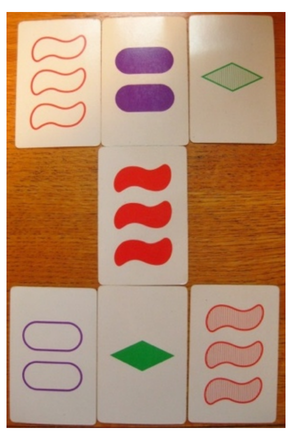


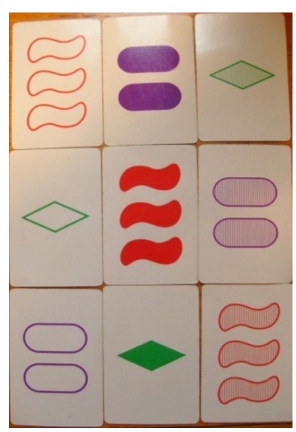


Remarkably, this approach will *always* generate a magic square. Why should we get something consistent? Why should the forced "choice" of that third card in the diagonal Set mean that we will get Sets in all the other directions? Good questions.

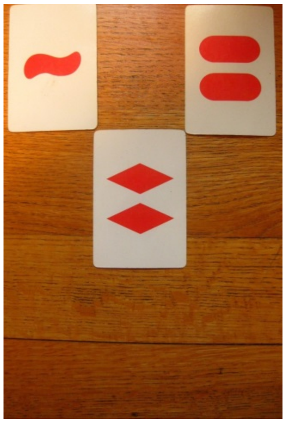
Here's another example, using a T-shape as the starting place. Note that we have two out of three of the Set in each diagonal, so we can complete the diagonals next. Finally, we fill in the columns.

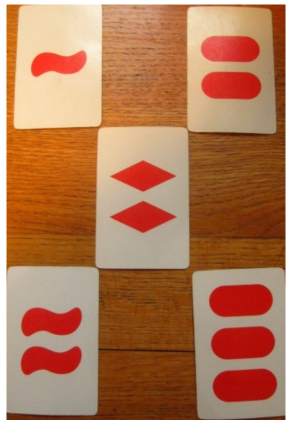


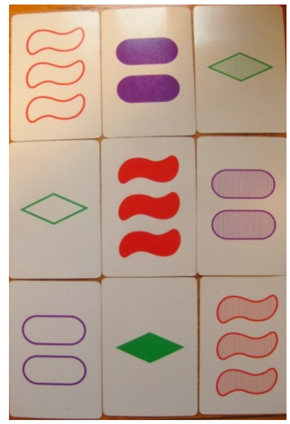




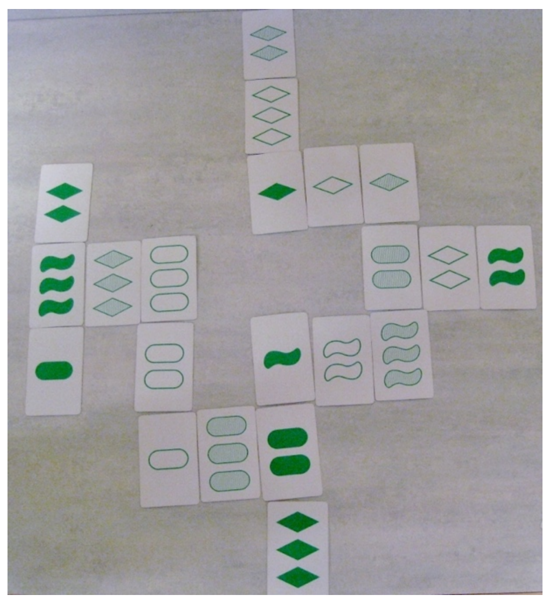
In fact, we needn't start with two complete Sets. It's enough to start with any three cards which do not themselves form a Set, as in the next example. Once you have chosen those three cards, the rest of the magic square is completely determined.







2. Set Crosswords
This game is a bit like Scrabble, except that each "word" must have exactly three cards. It's easy to get started – the tricky part is to use all of the cards. Here only 21 of the 27 green cards are shown.



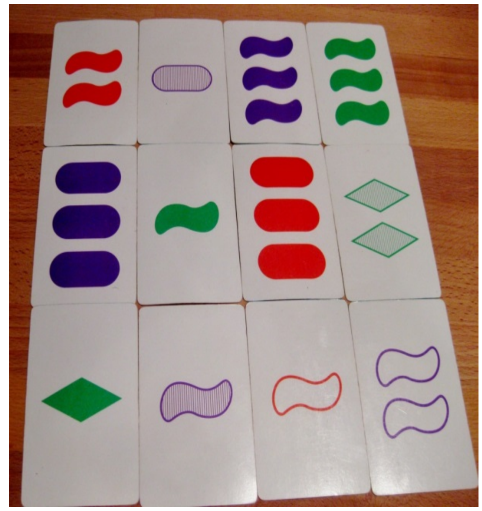
I start by asking students to use only one colour, or only one shading, or only one shape, or only one number.

This can be treated as a game of solitaire, as a cooperative venture, as a game where people take turns to place a new "word" using cards they have been dealt, or as a competition between teams.

If students can use up all of the 27 cards which share a single attribute, then I challenge them to try to use all 81 cards. That takes serious concentration and time!

3. Find all the Sets

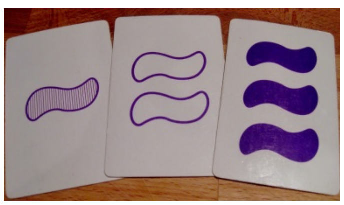
Six Sets can be made from these cards. Can you find all of them? Some cards will be used more than once.

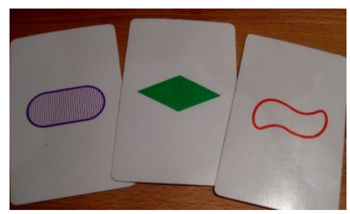


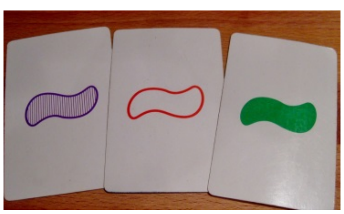
This variation can be played online at http://www.setgame.com/set/puzzle, which publishes a new puzzle every day, and at http://www.nytimes.com/crosswords/game/set/, which publishes puzzles at two levels of difficulty every day.

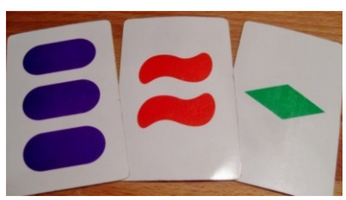
I've found that the easiest way to play this variation in the classroom is to assign each card in the collection a number from 1 to 12, then to have students write down each set of three numbers as they discover a Set. Then it is easy to put the triplets on the board.

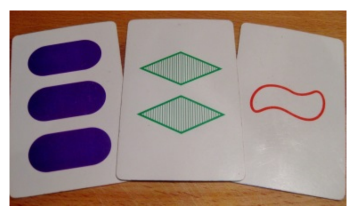
The answers? Here are the six Sets that I found in the above collection:











4. No-Set

What is the largest collection of cards that you can make which does not include any Sets? Answers vary from 16 to 20, and seem to depend on where you start.

This is a fascinating puzzle, in part because of the way it turns the hunt for Sets inside out, but also for the way it encourages players to develop a systematic approach. I recommend it only after students have been playing Set for a little while and can easily recognise Sets.

Other variations

More variations are described on the official Set website, http://www.setgame.com/teachers-corner/other-ways-to-play. Your students will probably come up with others if they are encouraged to spend some time with the cards.

Sources of Set cards

The official decks cost about $15 CAD each and are worth it for a school to purchase, especially if several teachers are keen on the game. These cards hold up very well to repeated use. You can usually find them at Indigo, Amazon, Toys R Us, and many chain and independent toy/game stores, especially before Christmas.

If your budget this year doesn't run to buying the official decks, however, you can make your own. I've posted a template on my website with simplified shapes: http://susansmathgamesca.ipage.com/wp-content/uploads/2014/09/DIY-Set-cards.pdf

If you have more than one deck of Set cards, whether you buy them or make them, I *strongly* recommend that you mark the backs, so that any stray cards can easily be returned to their own deck.

At right are the markings from five of my decks.

In conclusion

Playing with Set cards involves a great deal of mathematical thinking, including analysing and producing patterns, developing strategies for solving problems, and playing with spatial relationships. It also encourages mathematical habits of mind, such as attention to detail, persistence, logical thinking, inventiveness, the willingness to start over, and curiosity. Probably most important from the point of view of the students – there is a tremendous amount of fun to be had!

Susan Milner taught post-secondary mathematics in British Columbia for 29 years. For 11 years, she organised the University of the Fraser Valley's secondary math contest – her favourite part was coming up with post-contest activities for the participants. In 2009 she started Math Mania evenings for local youngsters, parents, and teachers. This was so much fun that she devoted her sabbatical year to adapting math/logic puzzles and taking them into K-12 classrooms. Now retired and living in Nelson, BC, she is still busy travelling to classrooms and giving professional development workshops. In 2014 she was awarded the Pacific Institute for the Mathematical Sciences (PIMS) Education Prize.

Intersections

In this monthly column, you'll find information about upcoming math education-related workshops, conferences, and other events. Some events fill up fast, so don't delay signing up!

For more information about a particular event or to register, follow the link provided below the description. If you know about an event that should be on our list, please contact us at thevariable@smts.ca.

Within Saskatchewan

Number Talks and Beyond: Building Math Communities Through Classroom Conversation
January 17th, Regina, SK
Presented by the Saskatchewan Professional Development Unit

Classroom discussion is a powerful tool for supporting student communication, sense-making and mathematical understanding. Curating productive math talk communities requires teachers to plan for and recognize opportunities in the live action of teaching. Come experience a variety of classroom numeracy routines including number talks, counting circles, quick images and more. Take math conversations to the next level by strengthening your skills as a facilitator of classroom discourse and student thinking.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/number-talks-and-beyond-building-0

Technology Integration for Differentiation in Mathematics
January 19th, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

Are you interested in using technology to help differentiate your mathematics classroom? Workshop participants will be introduced to various blended learning structures, then focus on the station rotation and flipped classroom models. Whether you have one device or a classroom of devices, these two classroom structures are beneficial to increasing student engagement and to providing opportunity for teachers to have individual and small group instruction. The idea of using technology to create differentiated opportunities through adaptive instructional websites and math and presentation-related apps will be explored and connected to curricular outcomes, student learning progressions and assessment.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/technology-integration-differentiation

Early Learning With Block Play – Numeracy, Science, Literacy and So Much More!
January 25th, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

This is a one-day workshop for early learning educators from prekindergarten, kindergarten and Grade 1 to join together, discover and deepen their understandings around the many foundational skills that children develop during block play. Through concrete, hands-on activities, participants will experience and examine the many connections between block play and curricular outcomes, and the current research on the topic. Participants will have opportunity for reflection on their current practice, planning for block play and for creating a network of support.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/early-learning-block-play-numeracy-2

Extending Early Learning Block Play into Project-Based Inquiry
January 27th, Yorkton, SK
Presented by the Saskatchewan Professional Development Unit

This one-day workshop for early learning educators from prekindergarten, kindergarten and Grade 1 will deepen understanding around the foundational skills that children develop during block play and extend that understanding into project-based learning in early years. Through concrete, hands-on activities participants will experience and examine the many connections between block play, curricular outcomes and project-based inquiry in early years.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/extending-early-learning-block-play

Using Tasks in High School Mathematics
February 8th, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

Using tasks in a high school mathematics classroom can provide rich opportunities for differentiated learning and authentic assessment. How do we choose tasks that meet both curricular outcomes and student needs? Tasks allow students to enter mathematics where they are at and extend their learning. In this workshop we will look at a variety of resources for finding good high school tasks. We will also reflect and discuss what planning and teaching moves can assist in maximizing student learning through mathematics tasks.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/using-tasks-high-school-mathematics

Technology in Math Foundations and Pre-Calculus
February 9th, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

Technology is a tool that allows students to understand senior mathematics in a deeper way. This workshop is designed to have math foundations and pre-calculus teachers experience a variety of technology tools that allow students to represent and visualize mathematics concepts. Tools highlighted are useful for students to explore, learn, communicate, collaborate and practice, in order to enhance their understanding of mathematics in secondary mathematics.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/technology-math-foundations-and-pre

Beyond Saskatchewan

NCTM Annual Meeting and Exposition
April 5-8, 2017, San Antonio, TX
Presented by the National Council of Teachers of Mathematics

Join more than 9,000 of your mathematics education peers at the premier math education event of the year! NCTM's Annual Meeting & Exposition is a great opportunity to expand both your local and national networks and can help you find the information you need to help prepare your pre-K–Grade 12 students for college and career success. Classroom teachers, administrators, math coaches, supervisors, college professors, and preservice teachers can all benefit from the sessions and learning at this event. Improve your knowledge and skills with high quality professional development and hands on activities; gain insights by connecting and sharing with like-minded educators; collect free activities that will keep students engaged and excited to learn; and learn from industry leaders and test the latest educational resources.

Note: Did you know that the Saskatchewan Mathematics Teachers' Society is an NCTM Affiliate? When registering for an NCTM membership, be sure to support the SMTS by noting your affiliation during registration.

See http://www.nctm.org/Conferences-and-Professional-Development/Annual-Meeting-and-Exposition/

OAME Annual Conference: Now for Something Completely Different
May 11-13, Kinston, ON
Presented by the Ontario Association for Mathematics Education

This year's keynote speakers are Dan Meyer, well-known for his work integrating multimedia into an inquiry-based math curriculum, and Gail Vaz Oxlade, host of the Canadian television series *Til Debt Do Us Part*, *Princess* and, most recently, *Money Moron.* Featured speakers are George Gadanidis, Marian Small, Ruth Beatty, and Cathy Bruce.

See http://oame2017.weebly.com/; follow @oame2017 on Twitter for updates

Online Workshops

Education Week Math Webinars
Presented by Education Week

Once a month, Education Weekly has a webinar focusing on math. They also host their previous webinars on this site. Previous webinars include Formative Assessment, Dynamic vs. Static Assessment, Productive Struggling and Differentiation.

Past webinars: http://www.edweek.org/ew/webinars/math-webinars.html
Upcoming webinars:
http://www.edweek.org/ew/marketplace/webinars/webinars.html

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NCTM | AFFILIATE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Call for Contributions

Did you just deliver a great lesson? Or maybe it didn't go as planned, but you learned something new about the complexities of teaching and learning mathematics. Maybe you just read a book or attended a workshop that gave you great ideas for presenting a topic your students have always found difficult, or that changed your perspective about some aspect of teaching. **Why not share your ideas with other teachers in the province – and beyond?**

The Variable is looking for a wide variety of contributions from all members of the mathematics education community in Canada and beyond, including classroom teachers, consultants and coordinators, teacher educators, and researchers. Consider sharing a favorite lesson plan, a reflection, an essay, a book review, or any other article or other work of interest to mathematics teachers in Saskatchewan. If accepted for publication, your piece will be shared, as part of this periodical, with a wide audience of mathematics teachers, consultants, and researchers across the province, as well as posted on our website.

We are also looking for student contributions, whether in the form of artwork, stories, poems, interesting problem solutions, or articles. This is a great opportunity for students to share their work with an audience beyond that of their classroom and their school, and for teachers to recognize their students' efforts during their journey of learning mathematics.

All work will be published under a Creative Commons license. If you are interested in contributing your own or (with permission) your students' work, please contact us at thevariable@smts.ca.

We look forward to hearing from you!