
(Min + max)imize: Promoting an active stance for basic facts


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## Cover art


"The students in our Grade 7 class used motion geometry to create images like the one on the cover - translation, to be exact. The teacher could give the directions, or have the students choose the translation themselves to create their own piece of art and have other students describe the movement. The latter was the case here.

For the Lisa Simpson image (left), we gave the students the coordinates, and the students plotted the points and joined them to make the picture. Students then created their own picture and gave the coordinates to another classmate to recreate."

Jacquie Johnson and Gail Morash, Saskatchewan Rivers School Division

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## Notice to Readers

The views expressed in each article of The Variable are those of its author(s), and not necessarily those of the Saskatchewan Mathematics Teachers' Society.

## Notice to Contributors

The Variable welcomes a variety of submissions for consideration from all members of the mathematics education community in Canada and beyond, including classroom teachers, consultants and coordinators, teacher educators, researchers, and students of all ages, although we encourage Saskatchewan teachers of mathematics as our main contributors.

Submit articles by email to thevariable@smts.ca in Microsoft Word format. Articles should be limited to 3000 words or less; authors should also include a photo and a short biographical statement of 75 words or less. Editors reserve the right to edit manuscripts for clarity, grammar, and brevity.


## Message from the President



T $\mathbf{T}$ elcome to issue 2! We're so glad you came back (or that you found us)!

To be honest, I'm not sure what I was thinking when I agreed to write a monthly president's message for The Variable. In theory, it sounds completely manageable, but in reality there is this never ending pile of things that need to get done. And for me, writing is never near the top of that list.

Yet May was the month of \#MTBoS30. For those of you who aren't on Twitter, May is a month during which many math bloggers commit to blogging in some dedicated way that suits them. Some blog for 30 days consecutively, some for 30 days, and others just blog more often than their norm. Some folks blog about their classrooms and practice, while others use various prompts. The idea is to commit to some form of blogging that suits you. The end result is a lot of posting. Some is serious, some silly, some practical - but most importantly, people write. They post. In a way that most of us can't sustain without a bit of a team cheering us on, because of those things.

As I was thinking about a message for this month's Variable, it struck me: "Just write." The SMTS wants The Variable to be your journal. Which means that you too, Saskatchewan enthusiast of the teaching and learning of mathematics, you have to write. I think the secret is in the fact that it doesn't have to be special. Your everyday, normal thing that you do? That thing is someone else's game changer. That assignment you tried? Someone else is doing an internet search right now looking for it. All you have to do? Write about it.

We're looking for your voice. Which also means your writing style, whatever that may be So even if writing was "never your thing," there is a place here for you. We also happen to be hiding some really fabulous editors on our math team, so if what you do write feels too draft-y to send, send it anyway! Include a note that you'd like the editing team to do what they do best.

So with that, I wish you a wonderful June. I look forward to seeing what this amazing community of Saskatchewan educators gets up to over the coming months.

Michelle Naidu

## From the Editor

If you'll entertain me for a minute...
Envision a community of Saskatchewan mathematics teachers that are connected and supportive, where innovative ideas are easily shared from Uranium City to Maple Creek, and where the many joys and challenges of
teaching mathematics are laid bare and discussed openly as part of the continual effort to improve students' learning. Picture teachers, teacher educators, parents, and students coming together to learn from one another's perspectives about the teaching and learning of mathematics in the 21st century. Imagine a time when Saskatchewan Understands Math is not just the name of a conference, but a reality.

This publication will not accomplish these goals.
However, it can be one of the puzzle pieces that helps to complete this picture (not that it will ever be complete, as the notion of effective teaching and learning of mathematics is always changing... but again, I ask that you entertain the analogy). Over the last few months, the modest idea of facilitating the spread of timely information about professional development opportunities and the activities of the SMTS between Saskatchewan mathematics teachers has quickly evolved into a new monthly periodical that strives to serve the many and diverse needs of the mathematics education community. This publication represents one of several major efforts of the Saskatchewan Mathematics Teachers' Society to highlight innovative ideas, facilitate communication, and recognize teachers (and students!) who are doing great work in the field of mathematics education our province - in other words, to serve the community we represent.

Of course, this periodical would not exist without you - your readership and your contributions. As such, we encourage you to share your own expertise and experience with the mathematics education community in Saskatchewan (and beyond) by contributing to future issues of The Variable. In our effort to provide a forum for all voices in this community, we welcome a wide variety of submissions written for and by elementary and secondary teachers, consultants and coordinators, teacher educators, researchers, and students of all ages (to name a few).

As an ardent user of Twitter and other social media platforms for (to use the terms loosely) professional development and collaboration myself, I recognize that many teachers in our province are already connected and sharing innovative ideas with their colleagues through social media, blogs, and other means. However, in the fast-paced world of Twitter, it's easy for great ideas and valuable connections to slip by, even if one steps away from the screen for a just few minutes. The benefit of this periodical, then, is its comparatively slow pace and permanence - updated by the month and not the second, highlighting several innovative ideas rather than a torrent, those great ideas are easy to find again when you need them.

Lastly, as this is your periodical, we invite you to send us not only your conributions, but also your feedback about what you have found useful and about how we can improve. We appreciate the kind words that we have received about The Variable so far, and we hope that you continue to find it valuable for your practice or personal interest.

We look forward to hearing from you.
And as always, happy reading!


Ilona Vashchyshyn

## Problems to Ponder

Welcome to this month's edition of Problems to Ponder! Pose them in your classroom as a challenge, or try them out yourself. Have an interesting student (or teacher) solution? Send it to thevariable@smts.ca for publication in a future issue of The Variable!

## Magic decimals

Math Challenge 2016
In a magic square, the sums of the numbers in the rows, columns and diagonals are all equal. Use a $4 \times 4$ grid to make a magic square for these numbers: $0.1,0.2,0.3,0.4, \ldots 1.4,1.5$, 1.6.


## Remainders

Math Challenge 2016
a) What is the smallest positive integer that leaves a remainder of 1 when divided by 2 , remainder of 2 when divided by 3 , a remainder of 3 when divided by 4 , and so on up to a remainder of 9 when divided by 10 ?
b) Dr. Theta wants to divide his class into equal groups. When he tries to divide his students into 5 groups, there are 2 students remaining without a group. He then tries to divide the students into 7 groups, but this leaves 3 students without a group. When he tries to divide the students into 9 groups, there are 4 students remaining. What is the smallest possible number of students in Dr. Theta's class?

See solutions on page 25.

## Spotlight on the Profession

## In conversation with Diana Sproat

In this monthly column, we speak with a notable member of the Western Canadian mathematics education community about their past, present, and future work, and about their perspectives on the teaching and learning of mathematics. This month, we had the pleasure of speaking with Diana Sproat, Mathematics Consulant of Greater Saskatoon Catholic Schools.


Diana Sproat has been employed by Greater Saskatoon Catholic Schools for the past 22 years as a teacher, Teacher on Assignment, and most currently as the Mathematics Consultant. She completed her Masters of Education at the University of Saskatchewan with a focus in the area of mathematics and as a member of the Math Cohort. Diana was honored to receive the Saskatchewan Mathematics Teachers' Society Math Service Award in 2014.

First of all, thank you for taking the time to have this conversation in the full swing of the school year! Could you tell our readers a bit about the work that you do as the Mathematics Consultant for the Greater Saskatoon Catholic School Division?

I have been the Mathematics Consultant K-12 for Greater Saskatoon Catholic Schools since 2011. I believe I have the best job at the board office! This role allows me to continue to stay close to the classroom as I work with teachers to plan, model, and co-teach. I also have many opportunities to collaborate with school teams to assist in working toward setting learning improvement plan goals and monitoring strategies. Being a part of a team of consultants and coordinators allows me to learn and to expand my vision outside what is happening in my area to the broader goals of the division. The ability to collaborate with Student Services, the First Nation and Metis Education Unit, the Literacy team and many other professionals brings richness to my specific area. I am also fortunate to be a member of a Provincial Math Group, who will be developing resources and supporting teachers across the province.

In recent years, there has been much public interest in the teaching and learning of mathematics in our country and province's schools. Many parents, in particular, have expressed concern in newspapers, on the radio, and social media about recent changes in mathematics curricula and approaches to teaching. As a mathematics consultant, do you communicate with parents as well as
"The most common misconception we hear about the "new math" is that the strategies used do not make sense and that students are not learning the basic facts in school." teachers? If so, what kinds of misconceptions or misunderstandings do you hear, and what do you want parents to understand about these changes?

In my role as math consultant I do have the opportunity to communicate with both teachers and parents. The most common misconception we hear about the "new math" is that the strategies used do not make sense and that students are not learning the basic facts in school. To address the questions, concerns, and curiosity parents
have, our schools and Catholic School Community Councils offer Math Parent Information Nights, which I am often invited to lead. This is a wonderful opportunity to engage parents in "doing the math" by providing a problem solving task for them to work through collaboratively.

For example, in one task, parents were asked to build a rectangular pig pen for a pet pig using 22 meters of fencing. They were challenged to find as many ways the pen could be constructed and to identify which pen would provide the largest enclosed area. The goal, a grade 5 outcome, was to discover that different rectangles with the same perimeter have different areas. Parents were given geoboards, elastics, grid paper, and pencils. Some created the pig pens with elastics on the geoboard, others drew pens on the grid
> "These tasks highlight that not everyone solves problems in the same way, and that a variety of strategies may work
> to solve the same problem (some strategies being more efficient than others)." paper, and some worked systematically using a table to ensure all possible side lengths were considered. The beauty in this task was that in recording the side lengths and the resulting area, the formulas for calculating perimeter and area were also revealed. In another task, we ask parents to do some mental math, describing the strategies used to solve questions such as $36+48$ or $114-98$.
> "The approach and the timeline may look different, but all of the mathematical skills that parents would want their children to develop are there and continue to be components of balanced mathematics programming."

These tasks highlight that not everyone solves problems in the same way, and that a variety of strategies may work to solve the same problem (some strategies being more efficient than others). It is also important to relay to parents the balanced approach provided by the curriculum, which is designed to first build conceptual understanding followed by practice to increase procedural fluency. The approach and the timeline may look different from how some parents were taught in the past, but all of the mathematical skills that they would want their children to develop are there and continue to be components of balanced mathematics programming.

In your view, why has the public interest in mathematics education increased? Do you see an end to the "math wars," and if so, what role will teachers play?

I believe there will always be those who hold strong views on either side of the "math wars," as we all have lived experiences in the education system. Our role as teachers will be to join both camps in seeing the need for a balanced education, providing excellent mathematics programming for all students that emphasizes both conceptual understanding and procedural fluency

The public interest in mathematics education, it seems, crosses not only provincial but also national borders, and comparisons are frequently made to countries such as Japan and Singapore (who typically perform well on international mathematics assessments) during the "math wars" debates. In your 2009 article for vinculum (the journal of the Mathematics Teachers' Society), you discussed the strengths of Singapore's system of teaching mathematics. This was also a topic that was discussed at the most recent Saskatchewan Teachers' Association convention. What aspects of the approach to
the teaching and learning of mathematics do you feel can (and should) be adapted to our local context? Are there aspects that would not "cross over" well?

I first encountered Singapore Math when my son, then in grade 3, showed me a bar model that he said he could use to add and subtract. He then asked if I could figure it out! I soon realized that he was using a key component of the Singapore Math approach, Bar Model Drawing. The pictorial representation provides the bridge students need as they move from concrete to abstract understanding. Model drawing can be used by students as a powerful tool to represent, understand and solve complex problems. In our local context, this key component of the Singapore system may become part of a repertoire of personal strategies students use for part-whole calculation, comparison, rate and proportion problems.

Years later, I had further opportunity to investigate the Singapore Math through work done with the Saskatchewan Teachers' Federation. I offered a few summer courses for teachers to learn the model drawing approach to solving problems, which could be easily implemented into existing curricula. During that time, we were in the process of renewing the provincial mathematics curriculum, and some changes that occurred in the Saskatchewan renewal are reflective of the Singapore system, including a narrowing of the outcomes at each grade level to allow for deeper understanding and the introduction of algebra at an earlier age.
> "A key strength in the Singapore system is the emphasis placed on teacher education. Mathematics teachers are carefully selected, expected to demonstrate mathematical skill at a high level, and receive 100 hours of professional development each year."

Another key strength in the Singapore system that we could benefit from is the emphasis placed on teacher education. In Singapore, mathematics teachers are carefully selected, expected to demonstrate mathematical skill at a high level, and receive 100 hours of professional development each year. In contrast, one aspect that would not "cross over" well from the Singapore system would be the streaming of students at an early age into mathematics programming that suits their "ability," as measured by a public exam. New brain research around learning and the brain's ability to change, adapt and grow tells us that everyone, with the right teaching and the right attitude, can be successful at math and achieve at the highest levels.

Singapore, as many will know, has been a top performer in well-known international assessments such as the Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Studies (TIMSS). Although there have been questions about the validity and reliability of such assessments, the results (in particular, the rankings) are often taken to heart by journalists and policy makers. Do you feel that teachers and policy makers in Saskatchewan should be concerned about Canada's standing in such assessment? If so, what can they learn from the results?

Although there is information and trends that can be disseminated from large scale assessment, these assessments are only one component in a comprehensive assessment system. In measuring the achievement of our students, assessments based on provincial curriculum give us a better indication of the learning of our students and is more readily used by teachers to guide their instruction, change practice, and increase their own professional learning and outcomes for our students.

Switching gears for my last question. As a mathematics consultant, you must be an expert in the resources that are available for mathematics teachers at the elementary and secondary level who are looking to improve their practice. Could you share your top few resources with our readers, perhaps a few from each level, and say a little about how they can help improve teachers' practice?

I do have some favorites that have made it onto my bookshelf in the past year or two.

- Number Talks, Grades K-5: Helping Children Build Mental Math and Computation Strategies by Sherry Parish and Making Number Talks Matter: Developing Mathematical Practices and Deepening Understanding, Grades 4-10 by Cathy Humphreys \& Ruth Parker - I love the simple and highly effective framework that teachers can use to build students' number sense and computations skills while they mentally solve problems and discuss their strategies.
- Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching by Jo Boaler - Incredibly interesting research applied to create strategies for use in the classroom that can improve students' mindsets and learning outcomes.
- High Yield Routines by Ann McCoy, Joann Barnett, Emily Combs - This book features easy-to-implement activities to infuse math into daily routines and to enhance students' mathematical understanding.
- Math Work Stations: Independent Learning You Can Count On by Debbie Diller - I found this to be a very practical book to help teachers in all elementary grades (although it is meant for Grades K-2) set up their classrooms to give students opportunities to work on instructional materials that further their mathematical understanding and to allow the teacher to provide differentiated and small group instruction.
- Solving for Why: Understanding, Assessing, and Teaching Students Who Struggle with Math, Grades K-8 by John Tapper - This book is a valuable resource for all educators who look for ways to close the gaps for students who struggle.

Two recently purchased, but still unfinished:

- Principles to Actions: Ensuring Mathematical Success for All by the National Council of Teachers of Mathematics - I picked this book up while attending the NCTM Annual Meeting and Exposition in San Francisco in April. Many of the conference sessions focused on one or more of the 8 specific teaching practices that, according to the book, are essential for a high-quality mathematics education for all students.
- Intentional Talk: How to Structure and Lead Productive Mathematical Discussions by Elham Kazemi - The title says it all!

Thank you, Diana, for taking the time to discuss your work and expertise with our readers. We look forward to continuing the discussion in the future.

If you would like to get in touch with Diana Sproat, please email DSproat@gscs.sk.ca.
Ilona Vashchyshyn

## Student Art

Briannah Cook


"This drawing represents the divide between the rich and the poor. Even when the rich can clearly see the terrible conditions that the poor are living in, they often don't reach out to help."

Briannah, a Grade 10 student, applied the concepts of parallel lines and vanishing points to create this perspective drawing in her math class.

## Reflections

Reflections is a monthly column for teachers, by teachers on topics of interest to mathematics educators: lesson plans, book/resource reviews, reflections on classroom experiences, and more. If you are interested in sharing your own ideas with mathematics educators in the province (and beyond), consider contributing to this column! Contact us at thevariable@smts.ca.

## 10 things I learned at \#NCTMannual <br> Amanda Culver

Last month, I was lucky enough to be able to spend a few days in San Francisco for the National Council of Teachers of Mathematics (NCTM) Annual Meeting and Exposition. This was my first experience at an NCTM Conference, and it won't be my last! My days were full of sessions and I overbooked myself, because there were multiple sessions I wanted to go to that were scheduled at the same time. However, all of the sessions that I ended up attending were awesome (well, with the exception of one, which I walked out of because there weren't enough materials for all participants; no big deal - I went teacher crazy at Target!). The amount of swag at the conference was ridiculous my suitcase was bursting with all of the notepads, bags, pencils, pens, ribbons, pins, and t-shirts that I picked up. So many t-shirts!


Teachers love free stuff!

Of course, the swag isn't the only take-away from a conference like this. I got to meet passionate math teachers from around the world, and I got a closer look at math education in the United States. If I had to summarize it in one word: interesting. For example, it's interesting that in the United States, geometry and algebra are separated into separate subjects (while they are integrated here in Saskatchewan) and that there seems to be more of a focus on statistics. I found this quite exciting!

A lot of the sessions that I chose to attend to were sessions about problem solving, as I work with a lot of really great students who are always looking at ways to improve their problemsolving skills. I tried out one of the problems I grabbed from a session on my first day back to class, which I'll share below (you can find the answer at the end of the article). I'll admit that, initially, I only got half of the answer, and I thus had the same experience as my students did when they later tried the problem.

The three side lengths of a triangle are all integers. If two of the sides are 6 and 8 , what is the ratio of the number of possible obtuse triangles to the total number of possible triangles? (Problem courtesy of McKendry Marano)

Through the \#MTBoS (MathTwitterBlogosphere), I discovered that many educators were sharing their lists of things that they learned at NCTM. So, I figured I'd try my hand at it, too!

## 10 Things I learned at \# NCTMannual, in no particular order

1. Teachers love free stuff.

Seriously. Teachers were grabbing HANDFULS of free merch.

2. Dan Meyer is a wonderful giant. And I have a selfie to prove it!
3. The history of logarithms interests math teachers. This particular session (presented by Michael Manganello and David R. Miller) was so packed, they had to turn teachers away! I was lucky enough to get a spot and revisit the idea that logarithms are used to simplify operations, rather than complicate them.
4. Never say anything a kid can say.

You're supposed to challenge them! Don't give them answers - make them work. Or, use it as an opportunity to use (new) math vocabulary.

Yes, Dan Meyer really is that tall!
5. Courage $=$ heart-ful; it takes courage to try new things. Latin "cor" = heart. Etymology is pretty neat. Thanks, Hill Harper, for sharing this!

## 6. Math still challenges me, and I love it.

I don't often challenge myself with math, which I need to start doing more frequently! I love that it makes me think and get out of my comfort zone.
7. Students are a lot smarter than we give them credit for.

Like, really. They can do some pretty neat stuff! Have you ever needed help solving a tech issue? Or want to see different ways of approaching a math problem, which perhaps you didn't think of? Ask a student! For example, that triangle problem I shared? Teachers went straight to calculations and "math-y" answers. Some students just drew it out!
8. American math is a dichotomy between Algebra and Geometry, which seems like it would create gaps in knowledge.
It seems like a good idea to separate the two, as more time could be used to build and practice those skills, but it's like separating verbs from nouns when trying to learn a language - you need them both, working together, to really become fluent.
9. American math curricula include things I wish we'd cover in Canada, such as more in-depth geometry and statistics.
When I saw that matrices were taught in America, I remembered that I learned matrices in grade 12, too. But we don't teach it anymore (at least, not in Saskatchewan).
10. Textbook questions are so wordy - rewrite questions! It's easier to add than to subtract.

Word problems are where many students get lost... heck, sometimes I even skim over them because there's too much to process! If we take away the text and leave the image, we can develop the questions. Or better yet, the kids can start to develop the questions. This takes a lot of practice, yes, but it is a simple and quick way to build great question sets. (Yes, I did listen to what Dan Meyer was saying... I wasn't being a complete fangirl!)

Answer to problem: 6:11


Amanda Culver has been a French and mathematics secondary teacher within the province of Saskatchewan for four years. She aims to make her classroom a safe and supportive space to be and to learn mathematics. Amanda's closet is full of math $t$-shirts, and she got a "pi" tattoo on Ultimate Pi Day. Needless to say, she loves math!

Do you have a lesson plan, reflection, book review, or other thoughts or ideas that you'd like to share with the Saskatchewan mathematics education community - and beyond? Consider contributing to The Variable! See page 23 for more information.

# (Min + max)imize: Promoting an active stance for basic facts 

Nat Banting

## Prologue

In recent years, the teaching and learning of mathematics has polarized itself into two stereotypical camps. On one side is the traditional camp. Here, students sit while their teacher presents a set of predetermined skills that even he or she does not dare to question. On the opposing side is the reform camp. Here, students are free to pursue methods regardless of how fruitless or inefficient they may be, and teachers run alongside naming their inventions as if they didn't exist before. At the risk of sounding excessively preachy, I may suggest that the obsession with classification of certain pedagogies, philosophies, and classroom practices has caused us to lose track of what matters most: the teachers and learners of mathematics. Both of these descriptions are purposely flamboyant; they are described in this way to highlight that the debate has left both sides undesirable and inaccessible for practicing teachers. This polarization has left the daily actions in mathematics classrooms (and thus, math teachers and learners) in the resultant chasm.

The void is periodically filled with a cycle (or re-cycle) of trends, each promising to reconcile the two fabricated camps or, at least, to provide morsels from each buffet. Each round of professional development brings in reform, which often contains too many bells and whistles, or tradition, which seems bland and disinteresting. While words are exchanged through media outlets (both social and mainstream), teachers are left treading water. One lightning rod has emerged through the haze of this educational cosplay: the multiplication facts.
> "Multiplication facts represent a larger group of basic facts that have fueled endless argument. In the process, the facts have become the target when the focus should be on how students utilize them."

Multiplication facts (most commonly known as mad minutes) represent a larger group of basic facts that have fueled endless argument. In the process, the facts themselves have become the target when the focus should be on how students utilize the facts. Teachers need to provide opportunities for basic facts to become the foundation (or base, if you will) of mathematical reasoning (NCTM, 2014). In order to elicit student reasoning in real time, students need the opportunity to make choices from an active stance of critical decision making. The simple game of (min + max)imize is provided to illustrate the idea that the artefacts of mathematics (like multiplication tables) are not, in and of themselves, harmful. What needs to be re-thought is the stance in which the students employ them.

## Game mechanism

(Min + max)imize is a dice rolling game. The particular die used determines which numbers the students are going to work with, but all of the variations of the game detailed here likely work best with a single 10 -sided die. This is the only material students will need (besides pen and paper) to play.

Each round begins with a given structure: a series of blanks (for the results of dice rolls) and operations that link the blanks together in various ways. The students can clearly see how many rolls of the die will occur (one per blank) and copy the structure onto their playing surface before the round begins. When all students are ready, the die is rolled. After each roll, the value must be placed in a blank. It is illegal to wait until all of the rolls are complete to make choices; it is also illegal to switch a choice after it is made. Imposing these restraints forces students to think not only about which numbers might produce desired results, but also how likely a better option is to appear.

The objective chosen for a particular round of the game affects which blanks may be desirable locations for certain numbers. Before the first roll, the class can decide whether the goal will be to minimize the result, maximize the result, or to try to get the result as close to zero as possible. As they become more comfortable, they may choose a specific target number to aim for. For example, the winner of a round may be the one who gets a final answer which is the closest to 50 . If 50 was not obtained, a fantastic extension problem can be posed: Was it possible to get exactly 50 ?

The role of the teacher is to circulate the room and roll the die. There will be times when students will make comments about which results they would like. Quick conversations about strategy fill the void between rolls, and audible groans often occur when the last roll is an undesirable result. Even a result very distant from the target provides an opportunity for a discussion. What number were you hoping for? How much closer would that have made your result? If you could swap two numbers, would you? Which ones? Embedded in all of this
> "Embedded in all of this action are basic facts, barely recognizable in the chaos. They are the substance of each decision, but far from the focal point of the game." action are basic facts, barely recognizable in the chaos. They are the substance of each decision, but far from the focal point of the game.

## Game variations

## Place value / addition

This is a great place to introduce students to the mechanism of the game. The structure consists of five blanks, each representing a digit of a 5-digit number (Figure 1). Decide whether to aim for the largest or smallest number, and then roll the die five times. Students need to use their knowledge of place value to build a large or small number. Hesitations arise when students need to gamble on whether or not better options will be rolled or if they should play it safe.

Figure 1: The five blanks of a five-digit number
In a slight extension of this variation, students create ten blanks that represent the digits of two 5-digit numbers (Figure 2). The goal is then to create the largest or smallest sum of the two.


Figure 2: A structure for the sum of two 5-digit numbers

## Subtraction

In this variation, the students change the addition structure from Figure 2 into a subtraction problem. Students can attempt to make the smallest difference or largest difference, but the most interesting strategies emerges when the goal is to create a difference the closest to zero. Students require pairs of numbers that are very close together in order to obtain a small difference, but the stakes get higher as the place value gets larger. A large difference in the thousands place hurts much more than a large difference in the tens place.

If matching numbers are not rolled on the dice, students may encounter a situation where the setup of borrowing actually results in a difference close to zero. Hope is not lost if the digits in respective place values are no more than one away from each other.

## Order of operations

This variation provides the greatest amount of flexibility for the structure. Any number of blanks can be interconnected by a series of operations (although I've found four to five blanks to be an effective number). Students then need to decide how to make the rolled numbers work either maximally or minimally for them. Strategy is dependent on the given structure of the blanks for the round. If the goal is to get a result as small as possible, students may choose to use a subtraction operation to achieve a negative result (Figure 3).


Figure 3: The student uses subtraction to obtain a negative result
Discussions of strategy pepper the room as students question their neighbour's placements. Vindicated smiles emerge after a perfect roll matches their strategy. The teacher can choose the intricacies of the structure based on the needs of their students. Figure 4 provides a variety of structures to get started.


Figure 4: Three possible structures for the order of operations game

## Division / simple factors

Including the operation of division provides an interesting twist. This variation contains another stipulation: The result of an operation must always yield an integer. This means that certain numbers are better options for numerators. Of course, every placement is viable, because a one could always be rolled for the denominator, but students must focus on creating numerators with a large variety of factors to maximize the chances of a suitable answer (Figure 5). Any students who do not get an integral result are disqualified.


Figure 5: Two (min + max) imize structures that include the operation of division

## Integers

This variation is an addendum to the previous two games. Here, the die is rolled and a coin is also flipped to determine if the number will be negative or positive. The decision making becomes immediately more complex. For example, if students want to create the largest result, they may opt to multiply two very large, negative numbers (Figure 6).

$$
(-6+-3) \times(2-7)+5
$$

Figure 6: A student chooses to create two large negative numbers to multiply to a large positive

After the round is complete, it is useful to allow students to try again, this time knowing the available numbers beforehand. This eliminates the pressures of time and uncertainty and fosters further discussion.

## Epilogue

> "We chase the phantoms of EdTech and real world in the hopes that great teaching and learning will emerge from a classroom tool or a great task, but teaching and learning are not pre-packaged-they are acted out."

The game is a simple one. It doesn't involve any fancy technology; it doesn't contain a contextual base. Too often, we chase the phantoms of EdTech and real world in the hopes that great teaching and learning will emerge from a classroom tool or a great task, but teaching and learning are not pre-packaged-they are acted out.

The act of teaching is one of anticipation and improvised decision making. The action of the group occasions responses from peers, interventions from the teacher, and shifts in the mathematical environment. The teacher may choose to initiate further conversation (say, around the idea of borrowing in the subtraction game) or level the action at the understanding of place value. Through careful preparation, the classroom action can be anticipated to a point (Smith \& Stein, 2011). Examples of these anticipations are outlined in the Game Variations section of this article. However, amplifying the anticipated student thinking is a process of flexible teacher intervention-teaching occurs in real time.

The game of ( $\min +\max$ )imize is meant to show that our goal as math teachers is not to eradicate the practice of basic skills any more than it is to focus entirely on their passive digestion and isolated use. Rather, the goal is to get students to engage their basic skills as pieces of reasoned mathematics, thus placing the teaching and learning of mathematics in an active stance.

## References

National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA: NCTM.

Smith. M. S., \& Stein, M. (2011). 5 practices for orchestrating productive mathematics discussions. Reston, VA: NCTM.


Nat Banting is a high school mathematics teacher in Saskatoon and graduate student at the University of Alberta. His work and studies revolve around the advancement of learning collectives in mathematics class.

Outside of the classroom, he is the curator of fractiontalks.com and founder of the \#FreeScalene movement. He blogs his thoughts concerning the teaching and learning of mathematics at musingmathematically.blogspot.ca.

## Intersections

In this monthly column, you'll find information about upcoming math (education)-related workshps, conferences, and other events that will take place in Saskatchewan and beyond. If travel is not an option at this time or if you prefer learning from the comfort of your own home, see the Online workshops and Continuous learning online sections. Some events fill up fast, so don't delay signing up!

For more information about a particular event or to register, follow the link provided below the description.

## Within Saskatchewan

## Conferences



## SUM Conference

November 4th - 5th, Saskatoon, SK
Presented by the SMTS
Our own annual conference! Join us for two days packed with learning opportunities, featuring keynotes Max Ray-Riek and Grace Kelemanik and featured presenter Peg Cagle. This conference is for math educators teaching in $\mathrm{K}-12$, and registration includes lunch on Friday and a two-year SMTS membership. See the poster on page 5 for more information, and keep checking the SMTS website (www.smts.ca) in the coming months for registration details.

Presenters: Are you interested in presenting at SUM 2016? The SMTS is now accepting proposals for one-hour sessions focused on improving the teaching and learning of mathematics. Presenters receive one complimentary registration (includes lunch and a 2year membership). Head to www.smts.ca/sum-conference/sum-call-for-proposals/ to submit your proposal.

## Workshops

## Comprehension Strategies in All Subject Areas

July 27th, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit
Students are faced with increasingly complex texts in every subject area. Research literature confirms the importance of explicitly teaching comprehension strategies to students to support their understanding. By explicitly teaching comprehension strategies in subject areas such as science and math, teachers can help students develop deeper understanding of these and other subject areas. This workshop will have participants experience a number of practical strategies that they can connect back to the subjects that they teach.

See https: / / www.stf.sk.ca/professional-resources / professional-growth/eventscalendar / comprehension-strategies-all-subject

# Structures for Differentiating Elementary Mathematics 

July 28th, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit
We know through formative assessments that our elementary students are at different places in their understanding of mathematics, but how do we structure our classrooms to meet their individual needs? This workshop will provide the opportunity for participants to design their classroom structure so that it allows children to move flexibly among large groups, small groups and individual instruction. By having a structure in place, teachers can create a differentiated learning experience without creating individualized learning programs for every child.

## See https:/ / www.stf.sk.ca/ professional-resources / professional-growth/events-calendar/structures-differentiating-elementary

## Beyond Saskatchewan

## Waterloo Math Teachers' Conference

August 23rd - 25th, Waterloo, ON
Presented by the Centre for Education in Mathematics and Computing
A conference for teachers of grade 7-12. While the Grade 9-12 sessions are directed towards university preparation and mainly Ontario teachers, teachers from any province or country will benefit as well. Registration is now open and spots fill up fast, so sign up early! Participation is restricted to two teachers per school until the May 31st deadline.

## See http: / / www.cemc.uwaterloo.ca/events / mathteachers.html

## Online Workshops

## Math Daily 3

July 3th-July 30th
Presented by the Daily CAFÉ
Learn how to help your students achieve mathematics mastery through the Math Daily 3 structure, which comprises Math by Myself, Math with Someone, and Math Writing. Allison Behne covers the underlying brain research, teaching, and learning motivators; classroom design; how to create focused lessons that develop student independence; organizing student data; and differentiated math instruction. Daily CAFE online seminars combine guided instruction with additional resources you explore on your own, and are perfect for those who prefer short bursts of information combined with independent learning.

## The seminar includes:

- online access to videos, articles, and downloadable materials
- access to an exclusive online discussion board with colleagues
- a certificate of attendance for 15 contact hours

See https: / / www.thedailycafe.com/workshops / 10000

## Continuous Learning Online

Education Week Math Webinars

Presented by Education Week
Once a month, Education Weekly has a webinar focusing on math. They also host their previous webinars on this site. Previous webinars include Formative Assessment, Dynamic vs. Static Assessment, Productive Struggling and Differentiation.

Past webinars: http:/ / www.edweek.org/ew/webinars/math-webinars.html
Upcoming webinars:
http: / / www.edweek.org/ew / marketplace / webinars/ webinars.html

## ShadowCon 2016

Hosted by the NCTM
FREE
ShadowCon is a teacher-led mini-conference hosted by the NCTM and organized by Zak Champagne, Mike Flynn, and Dan Meyer. This year, ShadowCon was held on April 15th at NCTM Annual in San Francisco, and featured speakers Gail Burrill, Brian Bushart, Graham Fletcher, Rochelle Gutierrez, Robert Kaplinsky, and Kaneka Turner. Missed seeing them live in San Francisco? You can still watch their talks and join the conversation online, for free!

See http:/ / meetings.nctm.org/tag/shadowcon2016/

## Call for Contributions

Did you just deliver a great lesson? Or maybe it didn't go as planned, but you learned something new about the complexities of teaching and learning mathematics. Maybe you just read a book or attended a workshop that gave you great ideas for presenting a topic your students have always found difficult, or that changed your perspective about some aspect of teaching. Why not share your ideas with other teachers in the province - and beyond?

The Variable is looking for a wide variety of contributions from all members of the mathematics education community in Canada and beyond, including classroom teachers, consultants and coordinators, teacher educators, researchers, and students of all ages. Consider sharing a favorite lesson plan, a reflection, an essay, a book review, or any other article or other work of interest to mathematics teachers. If accepted for publication, your piece will be shared, as part of this periodical, with a wide audience of mathematics teachers, consultants, and researchers across the province, as well as posted on our website.

We are also looking for student contributions, whether in the form of artwork, stories, poems, interesting problem solutions, or articles. This is a great opportunity for students to share their work with an audience beyond that of their classroom and their school, and for teachers to recognize their students' efforts during their journey of learning mathematics.

All work will be published under a Creative Commons license. If you are interested in contributing your own or (with permission) your students' work, please contact us at thevariable@smts.ca.

We look forward to hearing from you!

## Problem Solutions

Grade 7-8 Problem: Magic decimals<br>Math Challenge 2016

One possible solution:

## Grade 9-10 Problem: Remainders Math Challenge 2016

a) 2519
b) 157

| 0.1 | 1.5 | 1.4 | 0.4 |
| :--- | :--- | :--- | :--- |
| 1.0 | 1.1 | 0.8 | 0.5 |
| 0.7 | 0.6 | 0.9 | 1.2 |
| 1.6 | 0.2 | 0.3 | 1.3 |

In the spirit of promoting productive struggle, we are considering eliminating the answer key section of The Variable. However, we still encourage readers to send in interesting solutions to thevariable@smts.ca for publication in a future issue of the periodical. We welcome your feedback on this matter!

