The Variable

Presented by the Saskatchewan Mathematics Teachers' Society

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Saskatchewan Mathematics Teachers' Society

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NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
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Cover art

"A student drew this perspective drawing as part of an assignment related to the Mathematics Workplace and Apprenticeship 10 Outcome WA10.9: *Demonstrate understanding of angles*. In this assignment, students have to think critically about which faces will consist of parallel and perpendicular lines and which ones will show lines sloping towards the vanishing point. Beyond understanding the meaning of parallel and perpendicular lines, they also need to have the skills to draw them accurately while having the opportunity to express their own creativity."

Zofia Gehl, St. Joseph High School, Greater Saskatoon Catholic Schools

SMTS Executive (2016-2018)

Notice to Readers

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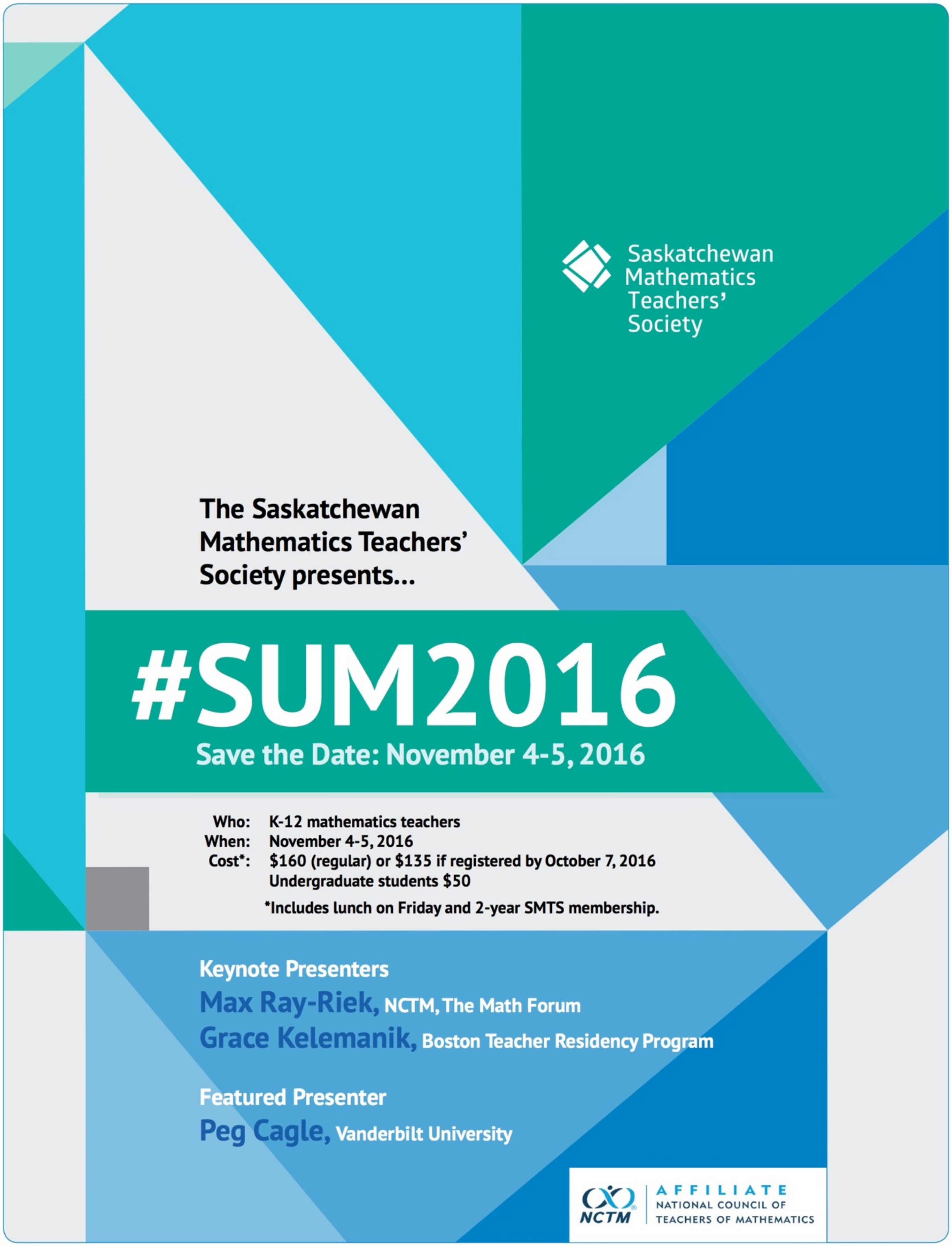
Notice to Contributors

The Variable welcomes a variety of submissions for consideration from all members of the mathematics education community in Canada and beyond, including classroom teachers, consultants, teacher educators, researchers, and students of all ages, although we encourage Saskatchewan teachers of mathematics as our main contributors. Submitted material is assessed for interest, presentation, and suitability for the target audience.

Please submit articles by email to thevariable@smts.ca in Microsoft Word format. Articles should be limited to 3000 words or less; authors should also include a photo and a short biographical statement of 75 words or less. Accepted contributions are subject to revision. Editors reserve the right to edit manuscripts for clarity, brevity, grammar, and style.


Saskatchewan Mathematics Teachers' Society
The Saskatchewan Mathematics Teachers' Society presents...
#SUM2016
Save the Date: November 4-5, 2016
Who: K-12 mathematics teachers
When: November 4-5, 2016
Cost*: $160 (regular) or $135 if registered by October 7, 2016
Undergraduate students $50
*Includes lunch on Friday and 2-year SMTS membership.
Keynote Presenters
Max Ray-Riek, NCTM, The Math Forum
Grace Kelemanik, Boston Teacher Residency Program
Featured Presenter
Peg Cagle, Vanderbilt University
NCTM
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Message from the President

Welcome back, readers new and old! I hope September has seen you comfortably getting back into routine, whatever that might look like for you.

There is no denying that fall is in the air - which means that it's perfectly acceptable for me to have nothing but the Saskatchewan Understands Math (SUM) Conference on the brain. For those of you who might be new to the SUM Conference, consider this your official invitation to join us for two days of what just might be one of the *very best* conferences for mathematics teachers in North America! I don't say this flippantly, as I have been actively attending conferences highly recommended by trusted members of my network to branch out and learn; and yet, to date, SUM conference consistently comes out on top, tied with Twitter Math Camp. (Readers, please don't make me pick a favorite between SUM and TMC.)

Why SUM? I've spent a lot of time reflecting on what makes this conference a unique experience, and why you should join us this year. Here are just a few good reasons:

- **Extended morning session blocks**. Participants can spend up to *4 hours* working deeply with keynotes and featured presenters. This extended time is so important, as it gives you the time you need to do the math, to learn, and to meaningfully reflect on how the ideas apply to your own personal teaching situation.
- **A commitment to explicitly focus on supporting Indigenous students in mathematics by improving our personal teaching practice**. We ensure participants have multiple opportunities to engage with authentic ways of honouring cultural diversity in the classroom that go beyond rewording word problems.
- **High quality keynotes and featured speakers.** We work hard to bring you presenters that fill huge auditoriums at other conferences, and then let you work with them in a small group! (By the way, don't miss our conversation with Max Ray-Riek, one of our two SUM 2016 keynotes, in this issue's Spotlight on the Profession column; see p. 21. If you missed our conversation with keynote Grace Kelemanik, see the August edition of *The Variable*.)
- **High quality local facilitators.** Does anyone have a time turner? I always desperately want one during break-out sessions. The advantages of learning with someone who knows your curriculum, teaches in a similar context, and who you can email next week can't be underestimated.

I could go on, but I think there are space limitations to this column. Obviously, I really hope to see you all there (head to our website to register). That being said, if you are perhaps choosing to use your precious and limited professional development funds to attend a different conference or experience, maybe you'll consider joining us next year and sharing what you've learned in a breakout session?

Michelle Naidu

Problems to Ponder

Welcome to this month's edition of Problems to Ponder! Have an interesting solution? Send it to thevariable@smts.ca for publication in a future issue of The Variable!

Practice need not be mindless. This month's problems were chosen for their potential to engage students in practice of basic skills while at the same time encouraging the mathematical practices of pattern-seeking, working systematically, generalizing, posing interesting questions, and more. Several of the problems have a very high ceiling!

Keep in mind that the particular numbers used in the problems can be changed to suit students' skill levels.

Hundred-dollar Nim [1]

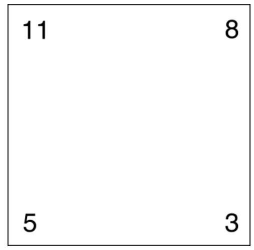
This is a game for two players. Imagine that you have a pile of $100, and on your turn you can remove $1, $5, $10, or $25. Players alternate turns; the player to reduce the amount to 0 cents is the winner (and gets to keep the $100). What's your strategy?

Biggest product [2]

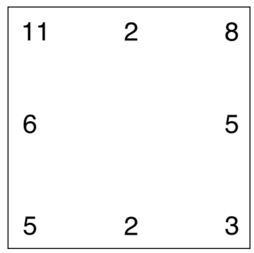
Pick a number: say, 25. Now break it up into as many pieces as you want: 10, 10, and 5, maybe. Or 2 and 23. Twenty-five ones would also work. Now multiply all those pieces together. What's the biggest product you can make? Pick another. What's your strategy? Will it always work?

Squares of differences [3]

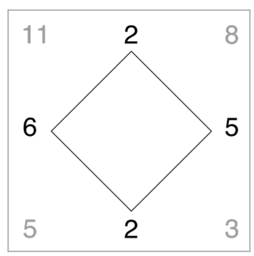
Draw a square, and pick four positive integers to go in each of the corners. For example:



Then, at the midpoint of each side, write the (positive) difference of the numbers at the two adjacent vertices:



Now connect the midpoints to form a rotated square inside the original square:



Repeat. What do you notice? Try with different sets of numbers; explore. Will this always happen? What if you started with a triangle instead of a square?

BONUS: Twenty-nine

Find the most interesting property, not related to size, that the number 29 has and that 27 does not have.

Sources

[1]*Adapted from* Vennebush, P. (2011, July 11). 5 math strategy games to practice basic skills [Web log post]. Retrieved from https://mathjokes4mathyfolks.wordpress.com/2011/07/11/5-math-strategy-games-to-practice-basic-skills/

[2]*Adapted from* Swan M., as cited in Meyer, D. (2013, April 16). [Confab] Tiny math games [Web log post]. Retrieved from http://blog.mrmeyer.com/2013/tiny-math-games/

[3]*Adapted from* Squares of differences: Subtraction practice toward a greater purpose [Web log post]. (2011, April 27). Retrieved from http://mathforlove.com/2011/04/squares_of_differences/

Problems to Ponder, May-June: The Extended Cut

Edward Doolittle

This month, I'm going to give solutions to some of the additional questions I posed in my solutions of the June 2016 and May 2016 Problems to Ponder [*which appeared in the June 2016 and July 2016 issues of* The Variable*, respectively. –Ed.*].

Flipping Coins (June 2016)

There are 100 coins on a table. Each coin is numbered, and they are all arranged heads up.

First, you turn over all of the coins. Then, you turn over only the even numbered coins. Then every third coin, every fourth, every fifth, and so on. You do this until, on the very last turn, you turn over only the hundredth coin.

When you finish, which coins will be heads up? Which will be tails up?

I gave a variation on the flipping coins problem, sometimes called the locker problem. The locker problem is well known, but the variation I gave is interesting, surprising, and unusual. Let's recap the problem, in terms of the locker framework. The first problem is when I have students numbered 1 to 100 and send them down a row of closed lockers also numbered 1 to 100. In the first variation, I send all the students down the row. The student numbered 1 toggles the state of each locker. The student numbered 2 toggles the state of all lockers which are divisible by her number 2, i.e., lockers 2, 4, 6, …. Student 3 toggles the state of all lockers which are divisible by 3, i.e., lockers 3, 6, 9, …. And so on for all 100 students. It turns out that at the end of this procedure, only the perfect-square-numbered lockers, i.e., lockers 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, are open.

I described an alternate variation. Instead of 100 students numbered 1 to 100, I have a smaller group. I send all the squareful-numbered students home before they even touch a locker. That is, I send home every student whose number is divisible by a perfect square larger than 1; I send home students with number 4, 8, 9, 12, 16, 18, 20, 24, 25, 27, 28, … . The students I am left with are the students with squarefree numbers: 1, 2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17, 19, 21, 22, 23, … . My question is: What happens when we send just the squarefree-numbered students down the hall to toggle the lockers?

Let's get our hands dirty and try it out with a smaller collection of lockers, since 100 lockers don't fit well across the page. Student 1 opens every locker.

```
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
O O O O O O O O O  O  O  O  O  O  O  O  O  O  O  O  O  O  O  O  O
```

Student 2 closes every second locker.

```
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
O C O C O C O C O  C  O  C  O  C  O  C  O  C  O  C  O  C  O  C  O
```

Student 3 toggles the state of every third locker.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
O	C	C	C	O	O	O	C	C	C	O	O	O	C	C	C	O	O	O	C	C	C	O	O	O

Student 4 was sent home, so doesn't do anything. Student 5 toggles the state of every fifth locker:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
O	C	C	C	C	O	O	C	C	O	O	O	O	C	O	C	O	O	O	O	C	C	O	O	C

Student 6 toggles the state of every sixth locker:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
O	C	C	C	C	C	O	C	C	O	O	C	O	C	O	C	O	C	O	O	C	C	O	C	C

Student 7:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
O	C	C	C	C	C	C	C	C	O	O	C	O	O	O	C	O	C	O	O	O	C	O	C	C

Student 10:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
O	C	C	C	C	C	C	C	C	C	O	C	O	O	O	C	O	C	O	C	O	C	O	C	C

Student 11:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
O	C	C	C	C	C	C	C	C	C	C	C	O	O	O	C	O	C	O	C	O	O	O	C	C

To save some space, let's do students 13, 14, 15, 17, 19, 21, 22, 23 all at once, since they only affect one locker each and all affect different lockers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
O	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C

As you can see, after the squarefree students have finished walking down the hall toggling the state of the 25 lockers, only locker 1 remains open. It seems likely that is the case if we have any other number of lockers (100, 500, or whatever), but in order to be sure, we need to think about the situation.

Let's go back to our original problem, where we sent all the students down the row of lockers which resulted in exactly the square numbered lockers being open. Suppose we want to fix the situation so that locker 4 becomes closed instead. That means we have to send home student 4 so that locker 4 remains closed. But then locker 8, which was properly closed, is now open. To fix that, we have to send home student 8 as well. Locker 12 is still wrong, so we have to send home student 12 as well. Now since we sent home students 4 and 8, locker 16, which was open, is still open, so we have to send home student 16 to fix that. It looks like we'll have to send home all the multiples of 4.

We have a similar problem with locker 9. We send student 9 home so that locker 9 ends up closed instead of open. Then locker 18 is open, so we send home student 18 as well. Locker 27 would also be open, so we have to send home student 27. It looks like we'll have to send home all the multiples of 9.

Now, things get a little hairy when we think about locker 36. Being a perfect square, in the original problem it would end up open. When we send home all the multiples of 4, locker 36 would end up closed. But then we send home all the multiples of 9, and it seems that 36 would end up opened again. The key here is noticing that 36 is both a multiple of 4 and a multiple of 9, but student 36 can't be sent home twice, so we've double counted the number of times 36 is sent home. We correct that, and find that locker 36 ends up closed too.

The above is just a sketch of a proof. You should try to fill in the details. You can find an interesting, difficult, but very general discussion of the problem at https://cms.math.ca/crux/v33/n4/page232-236.pdf.

Patchwork (June 2016) [1]

Take square and draw a straight line right across it. Draw several more lines in any arrangement so that the lines all cross the square, and the square is divided into several regions. The task is to color the regions in such a way that adjacent regions are never colored the same. (Regions having only one point in common are not considered adjacent.) What is the fewest number of different colors you need to color any such arrangement?

I gave a variation of the patchwork problem using simple closed curves (sometimes called Jordan curves) instead of straight lines[2]. You should be able to solve this easily by noting that the same strategy works when we substitute the left or the right side of a line with the inside or the outside of a Jordan curve. We can even mix lines and Jordan curves.

There is one surprising twist to this problem if you think about it very carefully, as mathematicians have. My solution leaves one important question unanswered: Does a Jordan curve really separate the plane into exactly two pieces, an inside and an outside? It seems obvious, and it is true, but it is actually very difficult to prove. You can find a discussion at https://en.wikipedia.org/wiki/Jordan_curve_theorem. For adventurers, the complete, original proof of the theorem can be found at http://www.maths.ed.ac.uk/~aar/papers/hales1.pdf. Modern proofs are much shorter and cleaner, but use advanced machinery such as algebraic topology.

Magic Decimals

In a magic square, the sums of the numbers in the rows, columns and diagonals are all equal. Use a 4x4 grid to make a magic square for these numbers: 0.1, 0.2, 0.3, 0.4, … 1.4, 1.5, 1.6.

---

[1] Mason, J., Burton, L, & Stacey, K. (1985). *Thinking mathematically.* Essex, England: Prentice Hall.

[2] Recall that a simple closed curve is a curve that ends at its starting point but otherwise doesn't intersect itself. For example, 0 is a simple, closed curve, while 8 is not. The shape of a simple closed curve can otherwise be arbitrarily complicated.

Recall that in solving this problem, we first found an arrangement of 16 face cards into a 4x4 square such that each row, column, and major diagonal contained all four face values and all four suits. The solution was as follows:

A♠	K♥	Q♦	J♣
Q♣	J♦	A♥	K♠
J♥	Q♠	K♣	A♦
K♦	A♣	J♠	Q♥

Recall that we had a lot of choice for the first row. We could permute the faces A, K, Q, and J in any way, say for example by changing A → J, J → Q, Q → A, and K → K. Similarly, we could permute the suits in any way. Those two permutations together would allow us to change our starting row to any other starting row. The magic property of the square (all four faces and all four suits in every row, column, and major diagonal) is preserved if we carry out those permutations consistently throughout the square.

However, there is another set of solutions that aren't accessible through such permutations. A representative member of this second set can be found by making the choice "spade" instead of "ace" in the cell occupied by the Ace of Hearts (AH) in the above, and then filling in the rest of the cells by "propagating the constraints":

A♠	K♥	Q♦	J♣
J♦	Q♣	K♠	A♥
K♣	A♦	J♥	Q♠
Q♥	J♠	A♣	K♦

Now, I have said that no permutation of faces followed by a permutation of suits can take the first solution into the second, yet I asked you to find a permutation taking one to the other. Hmm.

The answer is that the permutation which takes the first solution to the second is A ↔ S, K ↔ H, Q ↔ D, and J ↔ C! We are permuting faces with suits (S = spades, H = hearts, D = diamonds, and C = clubs). Note that that permutation leaves the first row unchanged, but changes AH → SK = KS in "position 3" and, furthermore, correctly changes every cell of the first solution into the corresponding cell of the second solution. With that permutation, and the other 4! x 4! permutations we discussed earlier, we have shown how all 4! x 4! x 2 solutions to the problem can be derived from the first solution.

My next question was for you to improve the construction procedure of the numerical magic square so that it was "more magical", i.e., so that all 2x2 sub-squares added up to the magic number 34 as well. Let's use the first solution again and think more carefully about what happens when we turn it into a number square:

A♠	K♥	Q♦	J♣
Q♣	J♦	A♥	K♠
J♥	Q♠	K♣	A♦
K♦	A♣	J♠	Q♥

I suggested letting J=1, Q=2, K=3, A=4 and C=0, D=4, H=8, S=12. Those choices ensured that our numerical magic square was magic in every row, column, and major diagonal because each row, column, and major diagonal contains exactly one of each face and one of each suit. But the bottom center 2x2 subsquare doesn't contain every suit, only the black suits C and S. And the middle left 2x2 subsquare doesn't contain every face, just Q and J. Note that the bottom center 2x2 subsquare is magic in my first attempt at a numeric magic square, but the middle left 2x2 subsquare is not. Aha! This gives us an idea of what we might try to fix the problem.

I just picked the correspondence between faces and numbers arbitrarily (from the choices 1, 2, 3, 4). If I pick more carefully, I might be able to arrange it so that Q + J + Q + J = 1 + 2 + 3 + 4; and in fact, this expression is true if J = 1 and Q = 4! Then, if I set A = 2 and K = 3, I get the right center 2x2 subsquare to be magic as well. The resulting number square is

14	11	8	1
4	5	10	15
9	16	3	6
7	2	13	12

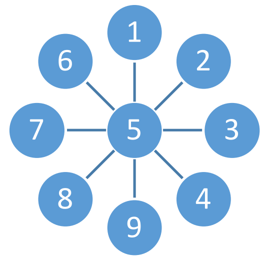
This square is even *more* magical than an ordinary magic square, and indeed, it is sometimes called a *panmagic* or a *most-perfect* magic square. There are many other patterns of numbers that add up to the magic constant 34 in our panmagic square; see how many you can find. You should also be able to find pairs of numbers in patterns that add up to half of the magic constant; e.g., 14 + 3 along the main diagonal add up to 17.

For the final question I gave in the Magic Decimals solution, I explained a game in which players alternately take numbers from a pile of the numbers 1 to 9. The first player with three (different) numbers adding up to 15 is the winner. To understand this problem better, let's write down all the ways of making a sum of 15 from three numbers in the set 1 to 9:

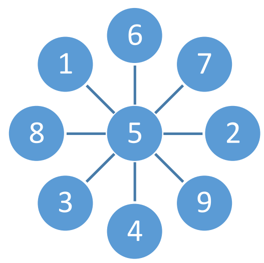
1 + 5 + 9 = 15	2 + 4 + 9 = 15	2 + 6 + 7 = 15	3 + 5 + 7 = 15
1 + 6 + 8 = 15	2 + 5 + 8 = 15	3 + 4 + 8 = 15	4 + 5 + 6 = 15

No clear pattern emerges, but you can see that the number 5 appears most often (in four of the equations); the numbers 2, 4, 6, 8 appear in three of the equations; and the numbers 1, 3, 7, 9 appear in two of the equations.

Let's arrange some of our equations in a circle with the most common number, 5, at the center:



That summarizes four of our equations. With a little rearrangement, we can get the other four:



Now, starting at any of the positions NW, NE, SW, SE and taking three numbers clockwise, we get our other four equations for 15.

The arrangement will be more familiar if we put it in a square instead of a circle:

8	1	6
3	5	7
4	9	2

Magic squares, again! You should recognize the resulting arrangement as a 3x3 magic square, with the magic number being 15.

The unfamiliar game that I described to you can thus be transformed to a very familiar game: Tic-Tac-Toe (on a magic square). Moves in one game correspond exactly to moves in the other; winning conditions in one game correspond to winning conditions in the other. To use a mathematical term, the games are *isomorphic*. It follows that strategy for the one game is the same as strategy for the other. In particular, Player 1 can force a draw if she plays well; Player 2 can only win if Player 1 plays badly.

Remainders (May 2016)

Dr. Theta wants to divide his class into equal groups. When he tries to divide his students into 5 groups, there are 2 students remaining without a group. He then tries to divide the students into 7 groups, but this leaves 3 students without a group. When he tries to divide the students into 9 groups, there are 4 students remaining. What is the smallest possible number of students in Dr. Theta's class?

Recall that in my original solution to the Remainders (b) problem, the problem was reduced to finding a number *N* which gives a remainder of 4 when divided by 9 and a remainder of 17 when divided by 35. The answer was found by systematically plotting the numbers 1, 2, 3, 4, … on a plane where the x-coordinate corresponds to the remainder when divided by 9 and the y-coordinate corresponds to the remainder when divided by 35:

```
y
34 |                139 104  69  34
33 |            138 103  68  33
32 |        137 102  67  32
31 |    136 101  66  31
30 |135 100  65  30
29 | 99  64  29                     134
28 | 63  28                     133  98
27 | 27                     132  97  62
26 |                    131  96  61  26
25 |                130  95  60  25
24 |            129  94  59  24
23 |        128  93  58  23
22 |    127  92  57  22
21 |126  91  56  21
20 | 90  55  20                     125
19 | 54  19                     124  89
18 | 18                     123  88  53
17 |                157 122  87  52  17
16 |            156 121  86  51  16
15 |        155 120  85  50  15
14 |    154 119  84  49  14
13 |153 118  83  48  13
12 |117  82  47  12                 152
11 | 81  46  11                 151 116
10 | 45  10                 150 115  80
 9 |  9                 149 114  79  44
 8 |                148 113  78  43   8
 7 |            147 112  77  42   7
 6 |        146 111  76  41   6
 5 |    145 110  75  40   5
 4 |144 109  74  39   4
 3 |108  73  38   3                 143
 2 | 72  37   2                 142 107
 1 | 36   1                 141 106  71
 0 |  0                 140 105  70  35
   +-----------------------------------
 x    0   1   2   3   4   5   6   7   8
```

We stopped after filling in the number at position (4,17), namely 157, which was the solution to the problem.

I asked you to think of a way to shorten the very long and elaborate process I used for solving this problem by concentrating only on row 17 in the table: that is, on finding numbers which give remainder 17 when divided by 35. To start off, obviously the number 17 is in that row; we just have to figure out where 17 sits with respect to division by 9, which is easy: 17 = 9 x 1 + 8. So we know that

```
17|                                    17
  +-----------------------------------
     0    1    2    3    4    5    6    7    8
```

Now, to stay in the same row, we just have to add 35 to the number we started with. 17 + 35 = 52 is another number in the 17 row. Dividing by 9, we get a remainder of 7.

```
17|                               52   17
  +-----------------------------------
     0    1    2    3    4    5    6    7    8
```

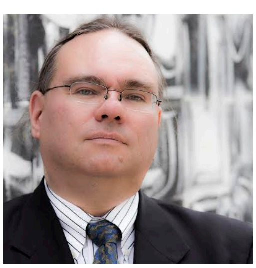
We could add 35 again and divide by 9 again to find the new remainder. However, there's a way of accelerating each of our next steps. Notice that adding 35 always results in subtracting 1 from the remainder when you divide by 9. (You can prove that statement using the division equation and a little bit of simple algebra; the key observation is that 35 is 1 less than a multiple of 9.) So we can quickly fill in the rest of the 17 line in the table:

```
17|                157 122  87   52   17
  +-----------------------------------
     0    1    2    3    4    5    6    7    8
```

We only have to go as far as item 4 to answer the question, but you could easily keep going. At each step we add 35 to our number and move one position to the left.

You should apply that idea to the additional question I gave: Find the smallest number which gives remainder 3 when divided by 4, remainder 1 when divided by 5, and remainder 5 when divided by 7. Here is the answer: numbers which have remainder 3 when divided by 4 and remainder 1 when divided by 5 are of the form 11 + 20n. Numbers of that form which also have remainder 5 when divided by 7 are of the form 131 + 140n. The smallest positive number with all the properties I requested is 131.

You can use those ideas as part of a trick to figure out someone's age given minimal information. Ask someone to tell you the remainder when they divide their age by 4, then the remainder when they divide their age by 5, and finally the remainder when they divide their age by 7. Assuming that they are younger than 140, you can use the process we worked out to figure out their age, which may be surprising to them. See if you can figure out a formula to do the calculation more quickly. Try a similar question with division by 3, 5, and 7 instead of 4, 5, and 7 (but then you'll have to assume your mark is younger than 105).



Edward Doolittle is Associate Professor of Mathematics at First Nations University of Canada. He is Mohawk from Six Nations in southern Ontario. He earned his PhD in pure mathematics at the University of Toronto in 1997. Among his many interests in mathematics are mathematical problem solving, applications of mathematics, and Indigenous mathematics and math education. He is also a champion pi-day debater at the University of Regina's annual pi day, taking the side of the other transcendental number, e.

SUM Conference 2016

It's almost here!

Join us on **November 4-5th** at the Circle Drive Alliance Church in Saskatoon for this year's Saskatchewan Understands Mathematics (SUM) Conference, two days packed with professional learning opportunities for educators teaching in Grades K-12! Registration includes lunch on Friday and a 2-year SMTS membership.

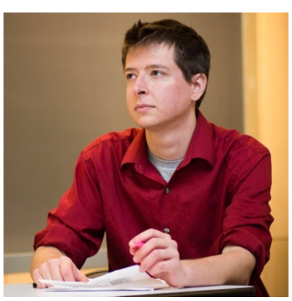
Curious about this year's keynote speakers?

Grace Kelemanik

Grace Kelemanik (@GraceKelemanik) works as a mathematics consultant to districts and schools grappling with issues related to quality implementation of the Common Core State Standards. She is particularly concerned with engaging special populations, including English Language Learners and students with learning disabilities, in the mathematical thinking and reasoning embodied in the eight Common Core standards for mathematical practice. Kelemanik is a secondary mathematics Clinical Teacher Educator for the Boston Teacher Residency Program, a four-year teacher education program based in the Boston Public School district that combines a year-long teacher residency in a school with three years of aligned new teacher support. Prior to BTR, Grace was a project director at Education Development Center (EDC). She was lead teacher of mathematics at City on a Hill Public Charter School in Boston where she also served as a mentor to teaching fellows and ran a support program for new teachers. Grace is co-author of the book, *Routines for Reasoning*, about instructional routines that develop mathematical practices.

Learn more about Grace Kelemanik's work and her upcoming SUM session in the August edition of Spotlight on the Profession (*The Variable* Volume 1, Issue 5).

Max Ray-Riek



Max Ray-Riek (@maxmathforum) works at The Math Forum, NCTM, and is the author of the book *Powerful Problem Solving*. Max is a former secondary mathematics teacher who has presented at regional and national conferences on fostering problem solving and communication and valuing student thinking.

Learn more about Max Ray-Riek's work and his upcoming SUM session in this month's Spotlight on the Profession column; see page 21.

Website: http://mathforum.org/blogs/max/

For more information about SUM and to register, visit our website at http://smts.ca/sum-conference/. We hope to see you there!

Saskatchewan Math Photo Challenge
September: Curves

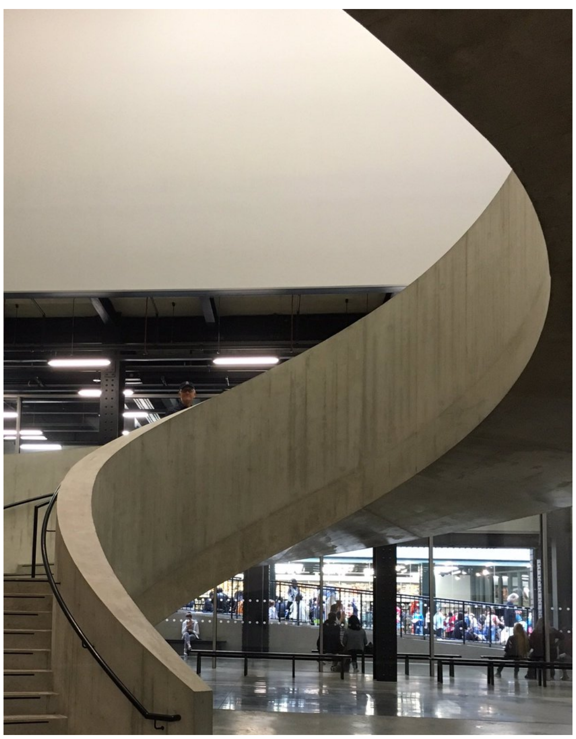
> What if I told you that you don't have to sail across an ocean or fly into space to discover the wonders of the world? They are right here, intertwined with our present reality.
>
> —E. Frankel, *Love & Math*

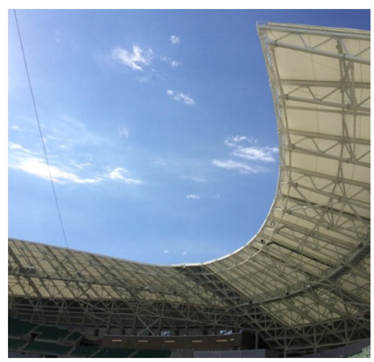
Inspired by the 2016 Math Photo Challenge (see the official website or the #mathphoto16 hashtag on Twitter), which revealed—through hundreds of submitted photos—the (mathematical) wonders all around us, we are continuing the fun with our very own Saskatchewan Math Photo Challenge!

Every month, we will choose a mathematical theme for you to explore in your photos. Keep your eyes peeled as you work and play, take photos of what you find, and share them on Twitter or Instagram using the #skmathphoto and current theme hashtags. See all photos submitted to the challenge on our website. At the end of the month, we will feature a few of our favorites here, in The Variable. Participation is *not* limited to math teachers, so encourage your friends, family, and students to play along!

This month's theme was…

#skmathphoto #curves





Featuring photos by Martha Barrett, Diana Sproat, Sarah Thompson, and Ilona Vashchyshyn

Reflections

Reflections *is a monthly column for teachers, by teachers on topics of interest to mathematics educators: book/resource reviews, reflections on classroom experiences and professional development opportunities, and more. If you are interested in sharing your perspective with mathematics educators in the province (and beyond), consider contributing to this column! Contact us at thevariable@smts.ca.*

A Summer of Math: Waterloo Math Conference Reflections

Amanda Culver

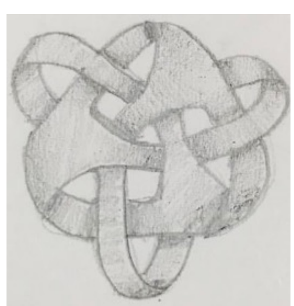
Most math enthusiasts have heard of the University of Waterloo—it's *the* math place to be in Canada. Last month, I was lucky enough to attend—for the second time in three years—their annual Math Teachers' Conference.

The conference consists of two and a half days of sessions, where participants are fed breakfast, lunch, and dinner and can stay in campus residence (free of charge, with registration). Sessions are led by both professors and teachers, and topics vary widely, from drawing Metamobius surfaces, to applications of mathematics, to problem solving, and more.

As the University of Waterloo—or more specifically, the CEMC (Centre for Education in Mathematics and Computing)—is known for its large offering of math contests and problems, we started off the conference with a session on problem solving. What was great was that we got to work on the problems ourselves, which is what I loved about the conference two years ago and what drew me (and two of my colleagues) back this year.

On the first day, Jason Van Rooyen led a session called *Integrating Problem-Solving in Grades 9 and 10* and J.P. Pretti led *Algorithmic Problem Solving*, and both sessions provided some great take-home problems that I'll definitely be using with students. I was also introduced to Binary Sudoku, which is quite a fun challenge (head to the following site for instructions and puzzles: http://www.binarypuzzle.com/index.php).

The afternoon session took a more artistic turn, when Ted Gibbons taught us how to draw some Metamobius surfaces. As I am venturing into teaching some arts education courses this year, I might have to include this topic in a lesson. I became obsessed and drew about a dozen. Check out my favourite:



Day Two was just as great. Five professors from the Department of Mathematics talked to us about their various fields, giving us examples of why math is important and why one should obtain a Bachelor of Mathematics. I found this of particular interest, as students never stop asking "Why do we need this?" in math class. Did you know that to compress files, the cosine function is used? Almost everything around us is based on some field of mathematics, such as Pure Mathematics (the math that happened *before* people said, "hey, that's useful!"), Actuarial Sciences and Statistics (how much money should we save now so that we can have a long retirement?), Combinatorics (the basis of GPS systems and finding the shortest distance from A to B), and Computer Science (image compression and modification, which is very useful in health sciences, among other fields).

Ian Vanderburgh, who is this years' recipient of the Canadian Mathematical Society's Excellence in Teaching Award, led a great session on geometry, which is unfortunately not a huge part of our Canadian math curriculum. We played with some interesting geometrical problems, including some involving circle geometry (which *is* in the Saskatchewan Grade 9 curriculum). An interesting point he made was that although we do so much teaching, we rarely get time to sit down and do some math ourselves. So he provided us with some really tough questions that pushed our thinking. Here's one that we worked on:

> "Although we do so much teaching, we rarely get time to sit down and do some math ourselves."

- *N* lines are drawn. No two lines are parallel. No three lines are concurrent (that is, they won't share an intersection point). How many regions are formed when *N* lines are drawn?

Next, David Hagen showed us how he flipped his classroom. The general idea is that students do the learning at their own pace at home, via teacher-made videos, and then work on practice questions in the classroom. I've attended a few sessions on the flipped classroom, but what made this one unique was that he actually showed us how he films! It definitely gave me some inspiration to flip some lessons in my own classroom.

The last session I got to attend was Carmen Bruni's *Patterns and Sequences*. The problems he had us work on were ones that could be adapted for a variety of classes, from Grades 7 to 12. Here's one that I particularly enjoyed (you can find some of the answers at the end of this post):

- A slime number is a number that, when "sliced," leaves behind prime numbers. You can "slice" before or after the number and anywhere in between. For example, 23 is slime because 2/3 gives 2 and 3, both of which are prime; also, 23/ and /23 gives 23, which is also prime.
  a) Find the first three even slime numbers.
  b) Find the first three square slime numbers.
  c) Find the first three cubic slime numbers.
  d) Find two consecutive slime numbers. Find three consecutive slime numbers. Can you find an arbitrarily large sequence of slime numbers?
  e) A number is super-slime if no matter how you slice the number, all of the slices are prime (e.g., 23 $\rightarrow$ 2/3, /23, 23/). Show that there are only finitely many such numbers and find them.

Overall, the conference was wonderful, yet again, and I am so glad I was able to spend almost 72 hours with math teachers and math enthusiasts from around the world. Next year, the conference will also make its way to Winnipeg, making the trip more accessible to us prairie teachers. In the meantime, be sure to check out the CEMC website, where you can find a vast collection of past and upcoming contests, fully-developed courses (they are working to include courses from Grades 7-12, and already have some Grade 12 material available, free of charge), and, my personal favourite, their Problem of the Week (which will also be offered in French for the 2016-2017 school year!).

Answers to the slime problem:

a) 2, 22, 32
b) 25, 225, 289
c) 27, 343, 729
d) Two consecutive slime numbers: (2,3), (22, 23), (32, 33); Three consecutive slime numbers: (331, 332, 333); No, an arbitrarily large sequence does not exist, as you can't find a sequence of 4 or 5 consecutive slime numbers

Amanda Culver has been a French and mathematics secondary teacher within the province of Saskatchewan for four years. She aims to make her classroom a safe and supportive space to be and to learn mathematics. Amanda's closet is full of math t-shirts, and she got a "pi" tattoo on Ultimate Pi Day. Needless to say, she loves math!

Spotlight on the Profession

In conversation with Max Ray-Riek

In this monthly column, we speak with a notable member of the mathematics education community about their work and their perspectives on the teaching and learning of mathematics. This month, we had the pleasure of speaking with Grace Kelemanik, who will be presenting at this year's Saskatchewan Understands Mathematics (SUM) Conference in Saskatoon.

Max Ray-Riek works at The Math Forum at NCTM and is the author of the book Powerful Problem Solving. *He is a former secondary mathematics teacher who has presented at regional and national conferences on fostering problem solving and communication and valuing student thinking.*

I would like to start off by asking you a bit about your background and your interest in mathematics. Was it a subject that you always enjoyed, or did something – or someone – hook you along the way? What drew you to teaching secondary mathematics rather than, say, research in mathematics?

I knew I wanted to be a teacher for as long as I can remember – I was one of those kids who played school with my stuffed animals and pretended I was teaching all the time. My mom taught pre-school and kindergarten, and when I was old enough to read chapter books I devoured books that told teaching stories, like *You Can't Say You Can't Play* or *Wally's Stories* by Vivian Gussin Paley. But I didn't think I wanted to be a *math* teacher until my sophomore year of college (not that my decision to become a math teacher surprised any of my own math teachers, like Lois Burke, @lbburke on Twitter – she was my Algebra II teacher and now a cherished colleague!).

> "One of the most exciting problems facing the world today is how to teach math in a way that builds on students sharing their ideas."

I had started off as a discouraged math student, fearing long packets of arithmetic problems that I was neither fast nor accurate with. I was lucky enough to have a 5th grade teacher, Ms. Allen, recognize that I enjoyed puzzles, problem solving, and thinking outside the box, and she invited me to try out some Math Olympiad problems. Even though I couldn't solve a single one on my first try, she invited me to share the approach I'd used to start thinking about one of the problems, and that was when I realized that I could have math ideas that mattered to other people. From then on I was interested in math, and enjoyed doing math and talking about math thinking with other people. When I got to college, I realized that this was actually one of the most exciting areas to teach in, because there's so much being figured out right now about how math can be taught as a dynamic, engaging subject where everyone has unique math ideas that matter.

Reading math education research by people like Alan Schoenfeld, Jo Boaler, Ana Sfard, Paul Cobb, Jean Lave, and others helped me see that one of the most exciting problems facing the world today is how to teach math in a way that builds on students sharing their ideas.

These days, although you still visit schools and teachers to observe, co-teach and model lessons, you have transitioned from the high school classroom to work and blog at the Math Forum at NCTM. What sparked this transition, and what does this position involve?

I've been a part of the Math Forum since high school, when I asked a question of Dr. Math. In college I served as a volunteer "math mentor," writing back to elementary students who solved our Problems of the Week, and then eventually as a research assistant on some of the grants that the Forum had received to study how to help college students like me improve as math mentors and learn about what kinds of feedback helped students reflect and revise. So I knew and loved the Math Forum's community, classroom resources, and staff before I got to work there. As much as I loved teaching high school math classes, I knew I wanted to work for and with the Math Forum as soon as I could – and I only had to wait 2 years before they had a job opening I could apply for! The reason I wanted to join them was because I am passionate about the subjects of problem solving and building math learning experiences out of students' individual problem solving methods. I wanted to spend time every day getting better at recognizing multiple methods to solve problems, recognizing students' novice attempts to use different methods, and honing my skills at responding to students in ways that supported them in persevering, reflecting, and learning. And then I wanted to spend time practising these skills with other teachers: by doing problems together, by coaching, by creating online environments where we can practise in a less fast-paced setting than the classroom, in PD workshops – whatever it takes!

One of your roles at the Math Forum is writing support materials for the Problems of the Week. (The Problems of the Week, published on a two-week cycle to give time for reflection and revision, are challenging mathematics problems for Kindergarten to calculus students, ranging from applications of mathematics in daily life to improbable scenarios that nevertheless spark interesting questions.) In your opinion, what makes a mathematics problem a "good" problem? Do the criteria change in relation to students' age and skill levels?

I think a good problem is one that, first and foremost, the students solving the problem understand – meaning that they understand the context the problem is set in (whether "real-world," imagined, or mathematical), they understand what is being asked, and they can appreciate why it's a reasonable question to have about this situation. My colleague Annie often says, "students can't answer a question they haven't asked."

Now, having said that, I don't mean that problems always have to come from familiar contexts – there are lots of ways to help students get into problems and make sense of them. For example, there's a PoW I really love about tagging salmon at a salmon hatchery. Some students might have never heard of salmon, or they might know it only as a food and not realize it's also a live fish. Even if they can picture it as a fish, they might not understand the wildlife management context or be able to picture the act of tagging or understand why someone would do it (or care how fast it can be done). To me, that doesn't mean we shouldn't use the problem, just that we shouldn't assume that it automatically makes sense to students. Can we work with students to find YouTube videos of salmon being tagged at a fish hatchery? Is there a hatchery nearby we could take a field trip to, or one further away that we could Skype with a worker at? Could we read more about fish hatcheries and tagging and make a movie in our mind of what's happening? Could we act out the problem using manipulatives until we understand it? Could we treat the problem as a *scenario* by leaving out the question and having students generate the questions they want to answer about the story themselves?

The other qualities a good problem has, besides being understood by the solvers after they have put some effort into understanding it, are that it is:

- Able to be tackled with a variety of methods;
- Worth talking about after you have an answer to compare methods, to look for generalizations, to analyze errors, etc.; and
- Connected to important mathematical ideas, so that it can be extended and/or made use of later.

In 2013, you published the book Powerful Problem Solving *(in collaboration with Math Forum education staff Annie Fetter, Steve Weimar, Richard Tchen, Suzanne Alejandre, Tracey Perzan, Ellen Clay, and Valerie Klein), and have spoken about fostering problem solving at many regional and national conferences in the United States. In your view, what does it mean to be a "powerful problem solver," and why is this an important skill for students to develop? Without spoiling the punchline of the book, what are some of the suggestions you offer readers in your book to foster students' problem-solving skills?*

There are many reasons we want students to be good problem solvers. However, I tend not to focus on the idea of problem solving as a life-long skill or important for 21st century careers. I tend to focus on problem solving as an inherent part of learning math. For example, think about how we teach students to solve equations in Algebra class. If we aren't careful, it can become a series of steps that students feel they need to memorize. They can solve problems like 2x + 5 = 12 but struggle with 12 = 5 + 2y because they've focused on superficial features of the steps rather than on making sense of them. At its heart, though, solving equations is about finding simpler, equivalent forms of the same relationship until a solution becomes obvious. Whether the problem is written as 2x + 5 = 12 or 12 = 5 + 2y, good algebraists look at the situation and ask, "What's making this hard to see the answer? Oh, it's because there are two things happening to the unspecified quantity. How can I turn this into a simpler equation without changing the relationships?" Then they rewrite the problems as 2x = 7 or 7 = 2y by thinking about the relationships in the problem.

> "What we've noticed is that getting good at guess and check is like learning to crawl before you learn to walk. While some people do skip this stage, it's really much better, developmentally, to go through it."

Or consider when we're teaching students to solve word problems, like "There are some ostriches and some llamas. Altogether, the animals have 42 heads and 114 legs. How many are ostriches and how many are llamas?" Too often, I hear teachers insist that solving a problem like this by the guess-and-check method is a "baby" approach and that students have to learn to use more algebraic approaches if they're going to be successful. What we've noticed is that getting good at guess and check is like learning to crawl before you learn to walk.

While some people do skip the guess and check stage, it's really much better, developmentally, to go through it. You see, figuring out how to guess and check your way to a solution to the llama and ostrich problem means:

- Figuring out what is unknown in the problem
- Deciding whether you need to guess for both of the unknown quantities or whether guessing a number of llamas will let you figure out a number of ostriches based on your guess
- Figuring out what calculations you can do with the guessed quantities
- Figuring out all of the constraints in the problem so you can check your guess
- Repeating the guesses

If students were to write equations to solve this problem, they would need to:

- Figure out what is unknown in the problem
- Decide whether they need a variable for both of the unknown quantities or whether assigning a variable for the number of llamas will let them write an expression for the number of ostriches based on the number of llamas
- Figure out what calculations they can do with the variables
- Figure out all of the constraints in the problem so they can set their expressions equal to something
- Solve the equations

Becoming a good problem solver using guess-and-check means doing 80% of the work needed to solve a problem algebraically. Students tend to naturally gravitate towards using more and more symbolic notation in their guess-and-check work as they get more confident in their problem solving abilities, and often end up making a very smooth transition to algebraic problem solving the longer we've let them choose to use guess and check.

So to me, being a good learner of math requires us to use habits of mind that we develop through problem solving, such as identifying unknowns, likely calculations, and constraints; or looking for simpler ways to write the same relationships; or looking for patterns in numbers or repeated calculations. The more we can emphasize and support students in becoming aware of and developing these thinking skills, the easier our job of teaching content can be.

> "Being a good learner of math requires us to use habits of mind that we develop through problem solving. The more we can emphasize and support students in becoming aware of and developing these thinking skills, the easier our job of teaching content can be."

In *Powerful Problem Solving*, the way that we emphasize building these thinking skills is through specific activities that draw students' attention to a problem solving strategy or habit of mind, and then having them learn from their own work and each other's work ways that they could get even better at that strategy or skill. A lot of our activities emphasize listening to peers share their thinking and then reflecting and revising based on those conversations. We also offer a lot of support for teachers and students to learn questions that help get us started in our thinking, such as:

- What do you notice?
- What do you wonder?
- What is an answer that would definitely be wrong? Why?
- Is there another way to say/write the same thing?
- What has to be organized?
- What must be true? What might be true? What can't be true?
- What makes this problem hard?

Do you feel that there is a conflict between problem solving and procedural fluency? How might one support the other?

> "Procedures without meaning have to be memorized separately and kept separate, even as you learn more and more algorithms. Therefore, they are prone to all sorts of bizarre errors."

Quite the opposite! Problem solving is the bridge from conceptual understanding (knowing what a problem is asking and how to go about thinking about it) to procedural fluency (being able to get the answer almost without thinking about it). Procedures without meaning have to be memorized separately and kept separate, even as you learn more and more algorithms. Therefore, they are prone to all sorts of bizarre errors: Think, for example, of students who used to be able to add 25 + 1.2 and get 26.2, but when they learn about the algorithm for multiplying decimals in which you line up the numbers, not the decimals, and just move the decimal at the end, now suddenly add 25 + 1.2 and get 3.7 – and don't see why that's wrong! Or consider students who could add -3 + -10 and get -13 until they learned that "two negatives make a positive," after which they are confident that -3 + -10 = 13.

These algorithms are competing with each other because students were focused on memorizing what to do when a problem has decimals or integers in it. They didn't learn tools for reasoning quickly about integers and decimals, didn't focus on estimation and reasonable answers first, didn't build up from their prior knowledge of place value, addition, multiplication, and opposites.

Imagine instead if students had been given the opportunity to learn to multiply decimals by building on their understandings of place value, multiplication, and fractions; if they'd started with problems like 20 * 2.5 and reasoned in contexts such as adding up 20 payments of $2.50 each, or scaling up a 20" picture two and a half times. Imagine that students had offered methods as diverse as:

- reasoning that 2 groups of 2.5 is 5, so 20 * 2.5 is the same as 10 * 5
- reasoning that 20 groups of 2 is 40 and 20 groups of ½ is 10, so the total is 50
- reasoning that 10 groups of 2.5 is 25, and so 20 groups of 2.5 is 50
- reasoning that doubling 20 is 40, and ½ of 20 is 10, so 2.5 times 20 is 50
- repeatedly adding 2.5 twenty times
- drawing a picture using arrays or rectangles that shows 2.5 by 20

Teachers could then scaffold students to share, learn from, and get good at these methods, particularly ones that relate to general algorithms such as partial products. They could engage students in solving more, and more difficult, decimal multiplication problems, until students had made sense of decimal multiplication, connected it to their ideas of place value and multiplication, and ultimately developed fluent strategies.

The connection from doing multiplication in a meaningful context to having procedures that make sense and don't interfere with addition procedures is to have built up to them through students' informal methods. For teachers of younger students or students learning arithmetic, books like *Children's Mathematics* (http://www.heinemann.com/products/E05287.aspx) and *Intentional Talk* (https://www.stenhouse.com/content/intentional-talk) have been helpful in laying out how that actually works.

More recently, you've been discussing the role of online communities in professional growth. The online math teacher community – referred to on Twitter as the MathTwitterBlogosphere, or MTBoS (see https://exploremtbos.wordpress.com/ for more information) – certainly has exploded in the past few years, with hundreds of teachers and teacher educators choosing to share ideas and to discuss issues in mathematics education online with colleagues around the world.

In your view, why do you feel that mathematics teachers are drawn to these (ever-growing) communities, and what are the benefits of becoming involved? What first steps would you suggest to teachers who would like to join the online conversation?

On the flip side, during your Ignite talk at the latest NCTM Annual Meeting and Exposition (https://youtu.be/j28BHpIzFzg), you argued that although blogs can be a great source of ideas and provide a great space for discussion and reflection, teachers shouldn't rely solely on them in lesson planning. Why would this be a problem, in your view?

Twitter and blogs have been incredibly useful for math teachers in lots of ways. A huge one is providing a sense of a community of people who are friends because we are all in this messy, complicated, wonderful project of being math teachers together. Online community gives everyone a chance to be their best selves and to be seen as their best selves – we get to frame who we are, and see new possibilities for who we can be, reflected back in online interactions, in a way that I think is different from what we can get with face-to-face interactions with colleagues and friends. With a face-to-face colleague, we have to deal together with the daily realities of teaching. They know us not just as, for example, someone who has awesome ideas about organizing the physical space of the classroom, but also as someone whose room is sometimes a little too noisy… there's friction there. With an online community, though, we get to focus on our passions and our strengths and be involved in a community who shares those passions and strengths. That doesn't mean we don't share and get support for our weaknesses, but when we're online, we're more likely to be in the position of asking for help and sharing on purpose, rather than being chastised by a colleague for doing something that disrupts their needs.

"With an online community, we get to focus on our passions and our strengths and be involved in a community who shares those passions and strengths."

Another huge way Twitter and blogs are useful is by connecting teachers to resources we can trust. As we come to trust other online colleagues, we eagerly check out or save for later anything they share about the topics we teach. Teachers learn together about content, share their best lessons, share student thinking and collaborate to adjust based on student thinking – all kinds of different collaborations are possible, and lots of those collaborations lead to activities, lessons, or whole sequences of lessons that teachers can use in class.

If you're just thinking about how to join online communities of teachers, here's what I always tell people:

- It's okay just to "lurk" – make a Twitter account and start following some math teachers so that you have some activity in your "timeline." You can go to my page, http://twitter.com/maxmathforum, and look at who I'm following to find over 2,000 math teachers on Twitter. Alternatively, go to http://twitter.com and enter one of these "hashtags" in the search field to find people tweeting in your grade band or subject area:
    - Elementary: #elemmathchat, #TCMchat
    - Middle School: #msmathchat
    - Students with Disabilities: #swdmathchat
    - Secondary: #GeometryChat, #AlgebraChat, #CalcChat, #PreCalcChat
    - General: #MTBoS, #mathchat
- You don't have to Tweet to be part of the community. And "liking" or "re-tweeting" is a great way to show that you're here and you care about what's being said, even when you don't feel like writing your own tweets.
- If you do decide to Tweet, try joining a conversation. Hit the "reply" button on Twitter to respond to a tweet – the person you reply to, even if they don't follow you, will see your message and will probably write back to you. It's not too hard to find the time to write 140 characters back, so Twitter often works better than email for getting a reply. Note that this doesn't necessarily apply to super famous people like Jo Boaler ;)
- It's also okay to just read blogs and not start one of your own. Commenting on other people's blogs is a great way to, again, show that you value their thinking and are learning from the community, as well as maybe even putting your own ideas out there.
- Communities are made up of lots of roles: appreciaters, users, question-askers, question-answerers, sharers, creators, etc. Embrace your current comfort zone and know that online, we can shift roles really easily as we find things that play to our strengths and spark our passions!

Now, on to the strengths and pitfalls of "community-curated" lessons.

The benefit of these "community-curated" lessons is that we know who came up with them, why they were invented, and how the lesson was designed to meet its goals. That makes lessons we get from our online friends highly likely to go pretty well – we're more likely to adjust them to match our own goals in useful ways, and/or more likely to implement them faithfully because we understand not just the hows and whats, but also the whys of the lesson.

> "The trouble with teaching students entirely from lessons we pull from a wide range of sources all over the Internet is that our lessons can lack cohesion."

However, the trouble with teaching students entirely from lessons we pull from a wide range of sources all over the Internet is that our lessons can lack cohesion.

Designing a coherent curriculum is time- and money-intensive – great curricula are written by teams of researchers and teachers who spend years crafting, piloting, reviewing, and revising. While individual lessons from the Internet are often great, sometimes they rely on prior knowledge your students don't have, or leave out concepts that are relied on later in your curriculum but weren't a big emphasis in your friend's curriculum.

I don't think the situation is hopeless, though. First of all, I think that teacher teams can do more work together to make sense of the curriculums they already have and make sure they understand the hows, whats, and whys before they replace one lesson with another. And I think that blogs and Twitter can help with that work – working with a team to dive in and understand curriculum is a very fun project that should totally be live blogged/Tweeted for everyone to learn from. Secondly, I think curriculum writers (and the various systems that fund them, from grants and public funding to private publishing companies) can do a lot more to make their process open and transparent, sharing more about the *whys* during the process of making decisions about the *whats* and *hows* of their curriculum.

Our readers will likely be aware that you will in Saskatoon this November to present as a keynote speaker at our very own Saskatchewan Understands Math (SUM) Conference. (We can't wait!) We don't want to spoil the surprise, but could you give our readers some insight into what you will be discussing during your sessions?

I always focus on two things:

- Doing math ourselves, as teachers, to help us see and appreciate more connections – connections to students' informal methods, connections between different strategies and methods, and connections to other areas of mathematics.
- Looking at student work to help us hone our skills of anticipating students' thinking, seeing connections in student work, posing purposeful questions, orchestrating discussions, etc.

So please come prepared to do some math (and hopefully try out a new strategy that you wouldn't normally use, like Guess and Check or Draw a Picture or Act It Out or Make a Mathematical Model…), to look at examples of student thinking about that math problem, and then to talk about how doing math and looking at student thinking can be a part of your planning routine, whether you do your planning online or in-person with your colleagues.

Thank you, Max, for taking the time to have this interview! We are eagerly looking forward to continuing the conversation at SUM 2016 in November.

Ilona Vashchyshyn

A Flipped Exploration: Using Desmos and Geometer's Sketchpad for Student Exploration Outside the Classroom [3]

Daniel Woelders

With increasing access to technology and more user-friendly programs, more teachers are moving from in-class lectures to video recorded lectures to be viewed as homework, alleviating class time for student work. The "flipped classroom," as it is known, moves traditional lectures to the home and brings homework to the classroom. However, after having spent some time observing, experimenting, and analyzing the flipped classroom, however, I have become increasingly concerned about how math teachers are using this strategy. Despite some perceived benefits, which include reducing the amount of class time devoted to lecturing and increasing student repetition, there are some hidden drawbacks that may be even more disastrous than those associated with traditional lectures. Having used the "flipped" approach to learning in my own classroom, I am convinced that it is capable of both enhancing and inhibiting student learning—and that how the teacher complements at-home lessons is what may make all the difference.

One of the problems I have found when teachers flip their classrooms is that they simply replace a traditional medium for delivering lectures with a technological one, and relocate high-volume repetition to the classroom. For many teachers, this is the whole point of flipping their class. However, much ineffective learning and negative attitudes from students arise from this type of traditional environment: the type of environment where a student sits in a class and watches someone else solve the problems, then goes home and spends an hour doing a worksheet or textbook questions. In a similar fashion, today's modern-traditionalists are using the idea of a flipped classroom to have the student go home, watch someone solve the problems, then come back to school and spend an hour working on a worksheet. By relying on either approach, we minimize the impact on student learning, and worse, we may perpetuate students' negative perceptions and attitudes towards mathematics. To quote Paul Lockhart (2009), "Routinization, memorization, and soul-crushing repetition are not the goals of math education."

> "In a subject where 'doing' is fundamental, lectures and flipped lessons can destroy a student's opportunity to engage with the true nature of the subject."

In a subject where "doing" is fundamental, lectures and flipped lessons can destroy a student's opportunity to engage with the true nature of the subject. Don't get me wrong, there is content knowledge that needs to be communicated and perhaps this could be the role of flipping, but the experiences that you offer students in the classroom will have the greatest impact on their learning. The arguments against a lecture-based or a traditional flipped math classroom are the same arguments I would present against having students take notes in a math class:

[3] A prior version of this article was published on July 12, 2014 on Daniel's blog, *Mathematical Paradigms* (https://danielwoelders.wordpress.com/). Reprinted with permission.

The Problems with Traditionalist Flipping

1. It does not require students to think. Why would they think when a teacher has so nicely organized, defined, and synthesized all of the information for them?
2. It positions the teacher as the source of information and the students as the recipients, rather than creators of knowledge. In most classrooms, this is how the teacher establishes a clear hierarchy. However, some disastrous effects result from creating an atmosphere where students always expect to receive information. Not only will students cease to explore the material, they will also stop asking questions, because it is assumed that the answers to all of their questions are found somewhere on the board or in the videos.
3. It does not promote dialogue. This is similar to the previous point, but we can add that when students become non-participants in their learning, the teacher receives little feedback for assessing the learning process.

> "By relying on apprenticeship through observation, our rapidly paced, closed, procedural lessons encourage the development of shallow, procedural knowledge."

Again, I'm not saying flipping the classroom is an approach that should never be used in a math class, but it is essential that the teacher make effective use of his or her time with the students when they are in class. This means addressing some of the problems listed above and ensuring that students have the opportunity to inquire, explore, and construct their understanding. My concern is that some math teachers are using flipped classrooms simply as a modern façade of a traditional approach. By relying on apprenticeship through observation and employing a traditional approach to teaching in our math classes, our rapidly paced, closed, procedural lessons encourage the development of shallow, procedural knowledge. Worse, such experiences shape the identities of the learners, encouraging a particular view of mathematics and their learning. In particular, they suggest to students that mathematics is all about remembering rules and procedures. Consequently, students develop a reliance on their memory to solve problems; they scour problems for cues, in a similar way that their textbooks do, rather than interpret situations mathematically. The impact extends outside of the school walls and into the students' real life application of mathematics. However, because the real world does not cue thinking, students abandon school mathematics and create their own strategies for solving problems—the same strategies that could be employed in the classroom.

> "My desire is to see flipping being used more dynamically. I don't want students to become passive learners—I want them to interact and play with math as they explore the concepts."

I am not opposed to flipping the classroom. In fact, I spent two days at a CanFlip Conference in B.C. However, my desire is to see flipping being used more dynamically. I don't want students to become passive learners—I want them to interact and play with math as they explore the concepts. This Piagetian type of learning (learning without being explicitly taught) is the reason that I have begun exploring various ways in which we can make a flipped classroom come to life with the use of some highly effective technological tools that are accessible to all students and teachers.

In particular, I introduced my own students to two programs popular in math education: Desmos and Geometer's Sketchpad. The purpose of this "flipped exploration" was to have students familiarize themselves with the graphical transformations and translations of functions in general form (i.e., the effect of a, b, c, and d in $af(b(x - c)) + d$), which would serve as the foundation for the conjectures they would then make for transformations and translations of sine and cosine functions. There were 16 students in this particular Grade 11 class who were enrolled in the International Baccalaureate standard level math course; the students were nearing the end of their first year in the program. At this stage in the course, they had already been exposed to composite functions, along with the graphical transformations of exponential, log, quadratic, and reciprocal functions. Students had also been made aware of the basic f(x) = sin(x) and cos(x) graphs. In order to make this a true flipped exploration, students were simply given a link that was posted to the class website (hosted by Edmodo) and asked to follow the prompts given in the task.

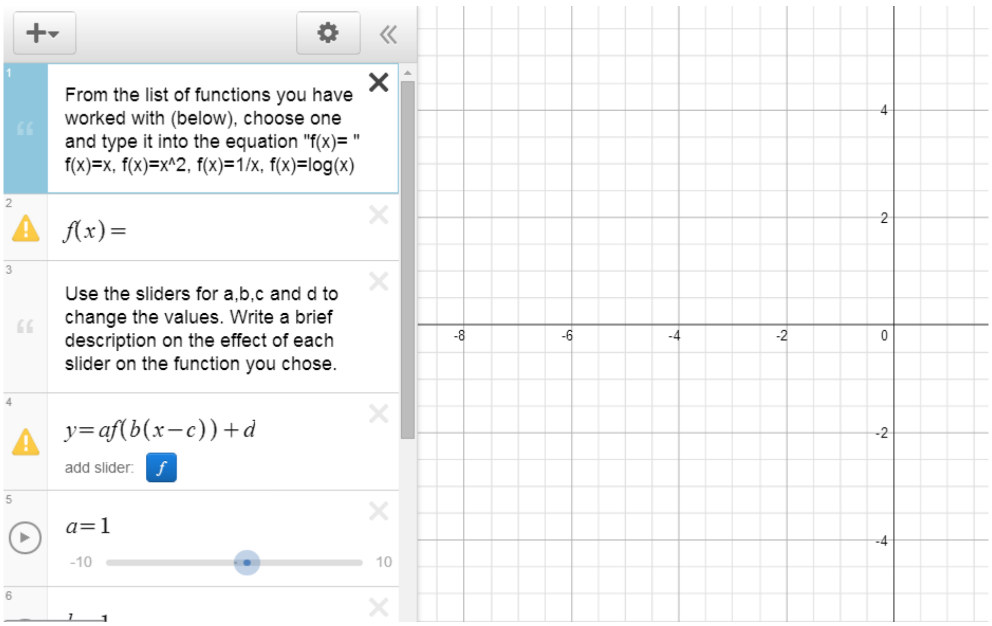


Figure 1 – Assignment constructed in Desmos

After clicking the link, students were led to a page created on Desmos (Fig. 1) and asked to create their own function. Due to their familiarity with linear, quadratic, and exponential functions, it was expected that students would opt to use one of these for the assignment. Once the function was created, students were asked to play with the sliders and determine their impact on the graph. To collect student feedback, I set up an assignment to be submitted on Edmodo; students were to write a brief description of the effect of each slider. As a way to connect the ideas to the current topic of study, the students were asked to predict and the summarize the sliders' effect on $f(x) = a\sin(b(x - c)) + d$. When the students returned to class, I had them attempt a short quiz in which they were given the graphs of two functions

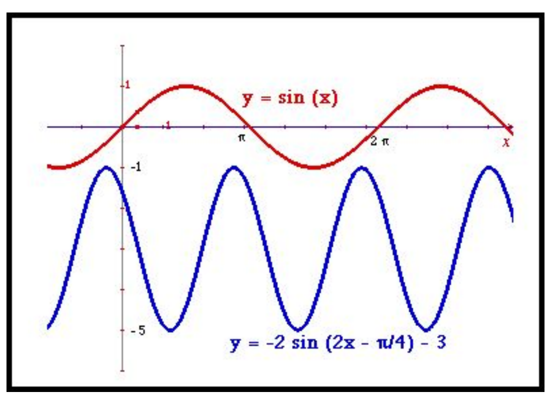


Figure 2 – Graphic attached to question posed in class

(Fig. 2) and asked to estimate the approximate values of a, b, c and d in the transformed graph.

My hopes were lofty. In the best-case scenario, I hoped that students would be able to communicate how the values of a, b, c and d alter functions and that they would build an understanding of transformations that would be applicable to all functions, instead of only to specific ones. In the worst-case scenario, I hoped that the students would at least familiarize themselves with Desmos and view it as a useful tool in searching for patterns and analyzing graphs.

Despite my ambitious expectations, I realized that, as with any new task, there would be constraints on what the students would be able to accomplish. I expected that some students would either forget or not bother to spend time doing the task, and that this would affect their understanding and participation the following day. Second, the nature of the task was novel to students and not presented in a format typical for this class. As Ruthven, Hennessy, and Deaney (2008) suggest, the "crafting of lessons around familiar activity formats and their supporting classroom routines helps make them flow smoothly in a focused, predictable, and fluid way." This certainly was not a task that had become routine. Thus, I expected that some students would have trouble navigating themselves between the two different websites (Desmos and Edmodo) and would not bother to complete the reflection piece that would provide valuable information about what understandings they were able to derive and where any misunderstandings occurred.

As predicted, these problems did occur, and the night was spent troubleshooting as students encountered various problems along the way. Many problems stemmed from students either deleting or altering the original instructions or the "f(x)" in Desmos, or shifting the graph in such a way that the function was not be visible. Students were encouraged to simply re-launch the link if any problems occurred, and this seemed to extinguish most of the fires.

When analyzing student submissions via Edmodo, I was not surprised to see a diversity of responses. The responses presented below are among the most interesting feedback received.

How does the value of 'a' affect your graph?
Student 1: "It flips the graph upside down"
Student 2: "It makes the graph skinnier"

How does the value of 'b' affect your graph?
Student 3: "It makes it go backwards"
Student 4: "It makes the graph wider or skinnier"
Student 5: "a and b do the same things…"
Student 6: "when a and b are zero, the graph is flat"

How does the value of 'c' affect your graph?
Student 7: "The graph moves whatever way c moves"
Student 8: "moves it right or left"

How does the value of 'd' affect your graph?
Student 9: "d moves the graphing up and down"

It was during this analysis that I noticed many gaps in students' understanding. For example, it seemed evident that although students recognized that *a* and *b* caused the graph to get "wider" or "thinner," they were unable to identify the difference between a vertical and horizontal expansion. This was reflected by the student who wrote that "a and b do the same things." This was not something to be particularly concerned about, since the students also failed to make any connection between the "stretch factor" and the value of *a* and "b." In other words, they could identify that the graph was thinner when *a* was 10 but they would not have been able to predict *how thin* it would become. Perhaps the most valuable piece of information they derived from controlling *a* and *b* was that "it flips the graph upside down" and "when a and b are zero, the graph is flat." Of course, the prediction that a graph would be flat when *b* is zero is rather dependent on the type of function, but would likely have been true for a student who only experimented with the functions suggested (if they hadn't experimented with the log and reciprocal functions).

In analysing the students' understanding of the effects of *c* and *d*, it appeared that most students understood their relationships with vertical and horizontal shifts, respectively. However, it became apparent that students were slightly confused that a positive value following *x* would result in a shift to the left. In other words, many students would predict that a possible equation for a quadratic graph shifted to the *right* by *c* units would be written as "$f(x) = (x + c)^2$". The effect of *d* seemed to be well understood following the task.

Students were also asked to extend their understanding to sine and cosine functions. At this point in the course, students already understood terms such as "amplitude," "period," and "shift," and so the following questions were intended to get some feedback as to whether the students had connected the ideas that had they learned to the task and could extend their understanding to trigonometric functions.

> **How is amplitude, period and shift of a sine function affected by the values of *a*, *b*, *c* and *d*?**
> *Student 1:* "increasing 'a' will increase the amplitude"
> *Student 2:* "making 'a' negative will make the amplitude negative"
> *Student 3:* "The amplitude will be the same as 'a'"
> *Student 4:* "increasing b makes the period shorter"
> *Student 5:* "b controls how many periods you see. When b is 10, there are 10 periods on one side of the y-axis
> *Student 6:* "c and d shift it side to side and up and down"

Once again, there was a strong indication that students understood the relationship between *a* and the amplitude and *b* and the period (e.g., "increasing 'a' will increase the amplitude" and "increasing b makes the period shorter"). However, there was still a gap between the given values of *a* and *b* and the actual scale factors of the graph. It seemed that students were reluctant to go much further than a general description of the transformation. The concern, of course, is that the students may be forming strongly held ideas based on conjectures that have not been tested. This appears to be the case for student 5, who believed that the value of *b* will determine how many periods are right of the y-axis. This may be true with a particular window size, but this student had failed to adjust the window size to determine the validity of the conjecture. It was evident that students had built some misconceptions that would need to be worked out in class. More than this, it was clear that students were not being pushed to analyse the exact effect of each value and to look for patterns in the numbers and measurements.

> "I wanted students to have control over the graph. This, to me, is fundamental to the use of technology in the classroom and is what separates the flipped lecture from the flipped exploration."

While this task gave students control over the values of a, b, c, and d and allowed them to observe their effects on the graph, I also wanted students to have control over the graph and see the effect on a, b, c and d. This, to me, is fundamental to the use of technology in the classroom and is what separates the flipped lecture from the flipped exploration. By giving students the opportunity to manipulate both the graph and the function, we allow the student to program the computer. In a video lecture, the opposite is occurring—the computer is programming the student. To this end, I created a task on Geometer's Sketchpad (Fig. 3). In this task, students were asked to stretch out the amplitude and the period, noting the changes to the equation. Although this task was given in class, students had very few instructions besides the ones given in the task itself (see Fig. 3). This time, the results were much more encouraging, with all students identifying exactly how a and amplitude are related. After encouraging students to set their period to easily identifiable values and then to record the value of b, a few groups were able to derive the numerical relationship between b and the period.

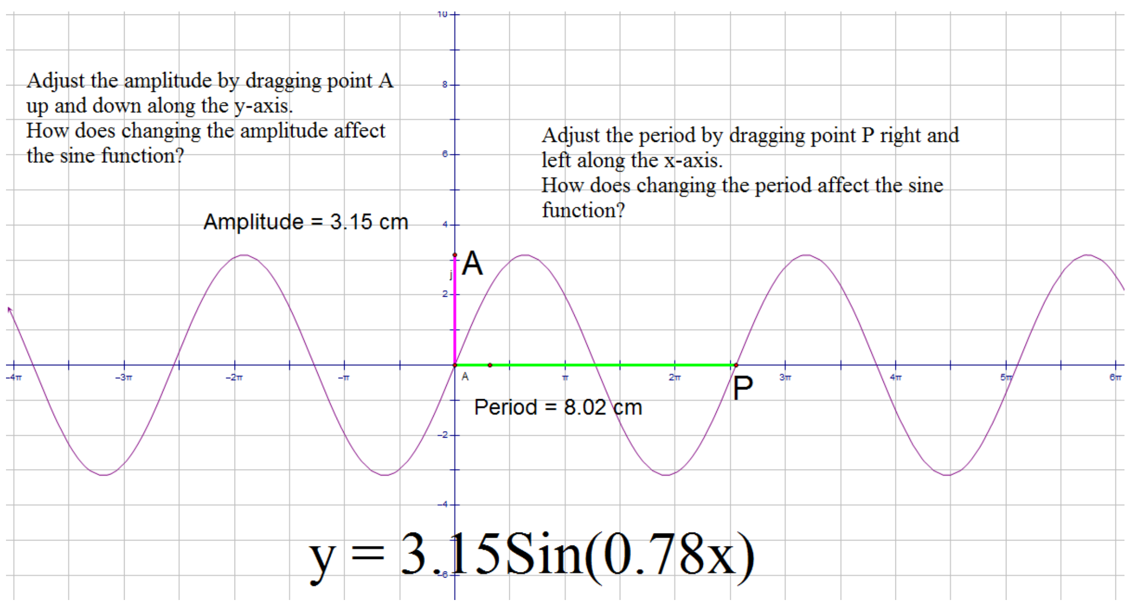


Figure 3 – Window that students viewed after opening the assigned sketch on Geometer's Sketchpad

While these exercises were a useful way to explore and experiment with various tools for me as a teacher, it is questionable whether they provided as much benefit to the students. It is arguable that the initial task of manipulating the values of a, b, c, and d to see their effect on a graph was simply replacing what a student could do with pen and paper, or perhaps with a simpler tool than the computer, the graphing calculator. Viewing the task in this way, some might see it as falling into the category of "substitution" in the SAMR (Substitution, Augmentation, Modification, Redefinition) model developed by Dr. Ruben Puentedura. It may be argued that this is true. However, Goldenberg (2000) challenges teachers to evaluate certain criteria when considering the use of technology for a particular task. In reference to these criteria, he suggests that "a well-designed lesson has a central idea and focuses students' attention on it, without distraction by extraneous ideas or procedural details" (p. 3). Without technology, one of the procedural details for this lesson would be having the student use a *t*-table and compute various values in order to plot the graph. However, this lesson is not about evaluating inputs and outputs. Accordingly, Desmos reduces the investment made on computation and focuses the students to the task of describing changes to the graph. It is true that this task *could* be replicated without Desmos, but the program allows students to interact with the central ideas more quickly and more easily. With this in mind, it is evident that in fact, Desmos "augments" (according to the SAMR model) the task. As they experiment with the sliders, Desmos also helps students make conjectures about the relations (Laborde, 2001). Thus, this tool is what Pea (1985) considers a "visual amplifier" when observing properties of the graph.

The advantage of using Desmos for this activity does not boil down to the question of whether or not the task had a positive impact on student learning. Rather, the question is whether it had *as much* impact as another task would have. The order of using Desmos and then Geometer's Sketchpad was chosen in response to student feedback, with Sketchpad used to compliment the first task. However, upon reflection, I wonder whether the order should be reversed in the future. By reversing the order, students would be able to manipulate the graph and construct conjectures that could then be tested in Desmos. This, in addition to directed questions, would steer students down a foreseeable path and perhaps focus their attention a bit more on the stretch factors *a* and *b*, which appeared to be major stumbling blocks for students. This foreseeable path may also help eliminate one of the biggest constraints that I had failed to recognize about doing this activity as a "flipped" lesson: that I could not predict where students would take this task. I could not see what students were seeing, and thus I could not direct them to appropriate tasks.

> "By assigning tasks where students are asked to look for patterns, we are creating an environment of active learners who acquire a new image of themselves as mathematicians and as programmers, roles previously viewed as being reserved for the intellectually gifted."

While this flipped class/technology experiment may have failed on many levels, it is often from failure that much useful information is revealed. What surfaces are the criteria for determining what kinds of lessons could be accomplished independently and which tasks should be done in the context of the classroom. Prior to this activity, it was assumed that a flipped lesson was simply a video lecture or a set of notes for students to read. This is certainly a common approach among some teachers, but likely only because it is a way to guarantee that one's students receive exactly what the teacher intends them to receive. For teachers who instead seek to help students develop conjectures, create contexts for classroom discussion, and who are open to a diversity of observations, a flipped exploration may also be an appropriate tool, provided that the teacher anticipates misconceptions that may arise. Tasks developed in this fashion lend themselves to using the technology as a way to test conjectures and develop proofs.

Perhaps one of the most important aspects of this task is its effect on the culture of student learning. By assigning tasks where students are asked to look for patterns, we are creating an environment of active learners. More than this, we are allowing students the opportunity to play with programming tools, and thus play the role of "programmers." Like Papert, who reflects on the use of the programming language Logo in *Mindstorms* (1980), I would argue that students participating in such tasks are acquiring a new image of themselves as mathematicians and as programmers, roles previously viewed as being reserved for the intellectually gifted. This not only increases self-confidence, but also gives students control over the mathematics. As Papert (1980, p. 19) asserts,

> The child, even at preschool ages, is in control: The child programs the computer. And in teaching the computer how to think, children embark on an exploration about how they themselves think. The experience can be heady: Thinking about thinking turns the child into an epistemologist, an experience not even shared by most adults.

The effect on classroom culture and development of students' identities as learners are valuable enough for teachers to consider integrating explorations in their flipped classrooms.

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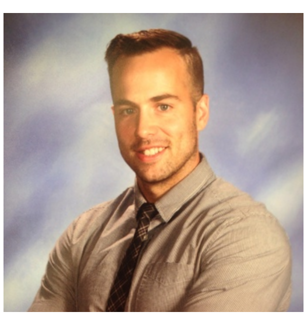

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Mirrors & Windows Into Student Noticing [4]

Higinio Dominguez

In many classrooms, students solve problems posed by others—teachers, textbooks, and test materials. These problems typically describe a contrived situation followed by a question about an unknown that students are expected to resolve. Unsurprisingly, many students avoid reading these problems for meaning and instead engage in a suspension of sense making (Schoenfeld 1991) characterized by rule-following behavior (Boaler 1998) and keyword searches (English 2009). Problems in everyday situations, however, do not come preformulated. Instead, these problems and the reasoning that they instantiate develop simultaneously as problem solvers informally question the situation (Why is this happening?) and begin to formulate conjectures and possible pathways for solving these problems (How can I begin to solve this problem?). For example, posing an equal-sharing problem exclusively on an equal-amount-per-sharer basis ignores how children may notice fairness experientially, including who is hungrier, who prefers to eat less, or who likes the shared product the most.

"In everyday situations, problems do not predate the person-world interaction; rather, problems are defined and redefined by problem solvers."

In everyday situations, problems do not predate the person-world interaction; rather, problems are defined and redefined by problem solvers. In fact, many situations in daily life are not necessarily problems. Consider the following example.

> Luis had some candies. His sister gave him 7 more candies.

After all, ending up with more candies is hardly a problem for children. Therefore, an agreement between students and teacher regarding what constitutes a "problem" seems to be foundational for fostering student problem posing.

A windows-and-mirrors framework for investigating student noticing

Word problem difficulties have been investigated primarily by looking at what students do or do not do, including looking at student errors (Newman 1977) or students' failure to use linguistic knowledge (Greeno 1985). For English language learners (ELLs), evidence points to vocabulary and other language-related skills that have not yet been acquired, which often leads to suggestions about simplifying the vocabulary and linguistic structures of word problems (Abedi 2001). Alternatively, good problem solvers are those who use schemata—meaning structures that exist in the mind (Fischbein 1999; Hershkovitz and Nesher 2003). These explanations, although valid, leave the mathematical problem formulation untouched, suggesting that students, not the problems, need to change. Gutiérrez (2002) has questioned why students must adapt to school mathematics and not the other way around. Silver (1994) also noted that "students are rarely, if ever, given opportunities to pose in some public way their own mathematics problems" (p. 19).

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I suggest using a windows-and-mirrors framework for encouraging students to be problem posers. Like a window, a problem should be an opportunity for students and teachers to look out for what makes sense to solve the problem. Simultaneously, like a mirror, a problem should be an opportunity for students and teachers to look into what students notice as relevant for solving a problem. Is the window too wide, too open or, on the contrary, too narrow; or is it just right? For the same matter, does the mirror allow students to recognize in the situation something that they know and that can be useful for solving a problem? Readjusting the window and/or the mirror—as it is recognized in word problems—signifies reformulating a problem. Expert problem solvers spend considerable time formulating and reformulating problems (Silver and Marshall 1989). This reformulating is important because asking students to solve preformulated problems has shown that students do not make sensible connections of mathematics to situations (Silver and Shapiro 1992). When such experiences repeat every day, students develop self-perceptions that they are not good at math.

> "Like a window, a problem should be an opportunity to look out for what makes sense to solve the problem. Simultaneously, like a mirror, a problem should be an opportunity to look into what students notice as relevant for solving a problem."

Blas, who used to be bad in math

Working with a first-grade bilingual Latino student who used to think of himself as not good in math reminded me of the importance of problem posing in racially and socio-economically diverse classrooms. I met Blas in a school that offers 100 percent reduced-price or free lunch to a large population of Latino/a and African American students. Their parents are predominantly working class, and many have been unemployed for a long time. In an interview administered to forty-three elementary students in this school, Blas, like many other students, identified himself as not being good in math. To challenge this self-perception and the premises on which it may have been constructed, I gave Blas the following separate, change-unknown problem.

> Juan had 16 toys, but he lost some. Now he has 9 toys left.

To invite Blas into problem posing, I asked, "What's the problem here?"

He replied, "That he lost some toys."

"Right!" I said, adding, "It's a problem when people lose things, isn't it?"

Blas reciprocated with a possible scenario about how Juan could have lost his toys, as he conjectured, "Maybe he was playing with his toys in his room and some ended up under the bed," suggesting that "he would have to clean his room, because some toys may be under the bed, and some may be behind his dresser or something."

Silver (1994) explained that—

> problem formulation represents a kind of problem-posing process because the solver transforms a given statement of a problem into a new version that becomes the focus of solving… If the source of the original problem is outside the solver, the

> problem posing occurs as the given problem is reformulated and "personalized" through the process of reformulation. (pp. 19–20)

Clearly, Blas was not noticing—at least not yet—the mathematics embedded in a problematic situation but, instead, the problematic situation embedded in the mathematics that I wanted him to learn. In talking about how it is problematic to lose something, Blas and I opened a window that was just right to see the mathematics, and in so doing, he recognized his own experiences with losing toys.

> "In talking about how it is problematic to lose something, Blas and I opened a window that was just right to see the mathematics, and in so doing, he recognized his own experiences with losing toys."

Blas successfully solved this problem by first creating a set of sixteen counters. He then separated a subset, saying, "He lost some!" thus demonstrating understanding of the concept of an unknown. Next Blas turned to count the known set, explaining that he wanted that set to have nine. Seeing that this set had only seven, he moved two counters from the unknown set to make nine. Finally he counted those that Juan had lost and got seven.

I asked, "How come you said earlier that you were not good in math?"

Blas did not say a word. Instead, he looked down slightly with a smile on his face that combined modesty, joy, discovery, and pride.

Inviting students to problem pose

Several strategies—all research-based—can support all students in problem posing, particularly those whose perspectives do not figure in classroom instruction. For example, problem-solving interviews have been used as tools for understanding a child's mathematical reasoning. Teachers can recalibrate this tool to develop cognitive empathy with children—particularly racially and socioeconomically diverse students—and learn more about what matters in their world (Dominguez 2011). We could enter the child's mind (Ginsburg 1997), but we could also look for ways to enter the child's world of experiences, funds of knowledge, or simply what matters in the child's world by deliberately inviting students to pose problems.

Problem posing can also be promoted with problems describing situations in which contexts are unfinished, thus creating opportunities for students to engage meaning and interpretation (Carraher and Schliemann 2002). Finally, research on children's mathematical thinking (Carpenter et al. 1999) highlights how children attend to action words in problems (e.g., give, lose, share), presenting opportunities to promote problem posing by inviting students to customize these action words. Following are four practical problem-posing strategies.

1. Let students specify some quantities.

If students are familiar with the number of items included in a set (e.g., crayons, gum, cards), leaving that quantity unspecified can serve as a mirror into what students know.

For example, I used the following problem with two groups of bilingual Latino/a third graders (Dominguez 2011):

> The cafeteria cook needs to make scrambled eggs for breakfast for 270 children. How many egg cartons does the cook need to open?

Students initially reacted, "But we don't know how many eggs are in a carton!"

When I asked, "Really?" they remembered having seen various sizes at the grocery store, such as 12, 24, or even 36 eggs per carton. Using this knowledge, they began problem posing by arguing that if the school had 270 children, using the largest carton would make more sense. Students also considered how many eggs per breakfast, with some using only one egg, and others using two eggs. This problem scenario invited these young problem solvers to use what they knew about egg cartons and about how many eggs they like to eat for breakfast. But the problem also prompted students to open multiple mathematical windows as they engaged in various negotiations that were saturated with meanings and interpretations. For example, some students remarked, "If we use 12, it's going to take longer to divide 270 by 12; so let's use a bigger carton."

Others said, "If the cook opens smaller cartons, it's going to take longer to make the breakfast; and the line at the cafeteria is going to be longer, and kids would have to wait forever!"

2. Let students frame problem questions.

A group of practicing teachers (grades 1, 2, and 4) and preservice teachers (K–grade 5), who were interested in eliciting student noticing, replaced the question at the end of a problem with open-ended questions similar to those I had used with Blas. Consider the following task.

> Juan had 16 toys, but he lost some. Now he has 9 toys.

Instead of asking, How many toys does Juan have left? teachers asked, What's the problem here? thus eliciting the children's personal—therefore meaningful—interpretations of the problem situation. Like Blas, many students confidently said, "That he lost some toys!" As they continued asking open-ended questions, teachers listened hermeneutically—without judging or evaluating (Davis 1996)—to children's problem posing.

Students often changed the problem situation. For example, one student grabbed two pencils to animate his own story:

> OK, let's say this [holding one pencil] is a good guy who had sixteen dollars in his wallet, and this one [using another pencil] is a bad guy who stole some dollars from him. Now he [waving the pencil representing the good guy] needs to find out how many dollars the bad guy stole!

"Instead of asking, How many toys does Juan have left? teachers asked, What's the problem here? thus eliciting the children's personal—therefore meaningful—interpretations of the problem situation."

Sometimes the "mirror" in these problems reflected so many details that students focused more on the details than on the mathematics. A gentle turn toward a "window" so that students could see what the teachers wanted them to learn was all it took to refocus most students. For example, the teachers asked, “Instead of Juan cleaning his room to find the toys, do you think you could help Juan solve his problem with just math?” Teachers validated the idea of cleaning the room, but at that point, asking children to use math was a challenge that these problem posers were prepared to take on, because they had transformed the problem from “He lost some toys” to “How many toys did he lose?”

None of these students had received instruction on separate, change-unknown problems (e.g., 16 – ? = 9). More important, many of them had self-identified as not good in math. However, nearly all of them solved the problem using meaningful math strategies that stemmed from equally meaningful problem posing. In addition, students are more likely to problem pose when they interpret a situation as problematic, for example, when they lose something. Certain situations, however, may not be perceived as problematic at all. Consider this example:

> Diego has fifty books, and he wants to donate thirteen to the public library.

In this case, such action words as donate (or share, give, etc.) are all voluntary actions that students may not perceive as problematic. Open-ended questions are still possible for a task like this; for example, “What do you think is going to happen after Diego donates thirteen books to the library?” Such open-ended questions reflect an interest in eliciting how the child thinks about situations that others have considered as “problematic.”

3. Promote problem posing at various points.

Many students are not used to—and often resist—problem posing. Like Blas, these students have developed a view that being good in math means quickly getting right answers, and they tend to see problem posing as interfering with this view. The aforementioned preservice teachers found that children across K–grade 5 often did not respond well to their invitations to problem pose. However, teachers also noticed that those who problem posed reported their solutions with more confidence than those who did not, locating the evidence for this claim in the students’ voice intonation. For example, unlike their peers who reported answers with raising intonation indicating doubt, problem posers reported their answers without such voice inflexion.

> “Many students are not used to problem posing. These students have developed a view that being good in math means quickly getting right answers, and they tend to see problem posing as interfering with this view.”

Teachers can help students restore the mirrors and open the windows of mathematics. For example, I often ask students, “What if Juan had not sixteen but twenty-three toys and after losing some, he had nine left? Do you think your answer to the original problem could help you solve this new situation?”

For students who find the original problem challenging, the “what if” prompt could be “What if Juan had fifteen toys, but he loses some, and now he has ten?” Changing to easier number combinations can help students develop the habit of reformulating problems. Once students can solve these reformulated problems, I give them the original problems again, asking them to use their work and ideas while they are fresh from their problem-posing activity.

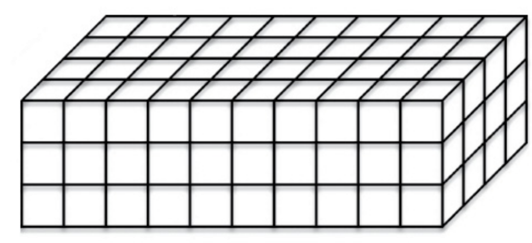
4. Invite students to interpret representations.

Often, well-intended representations of mathematical concepts (e.g., shaded parts in a circle for fractions) generate unintended student misinterpretations emerging from the inherent ambiguity of static representations. For instance, are students supposed to notice the shaded or unshaded parts for naming a fraction in a circular model? In my current work with teachers, I encourage them to invite students to scrutinize these representations before they begin the problem-solving process. For example, a third-grade bilingual Latina misinterpreted the intended three-dimensionality of a 4 × 3 × 10 rectangular prism on a benchmark test (Dominguez 2014). As a result, she counted only the faces of the one cubic centimeter on the three visible faces of the model, resulting in an incorrect answer that she found as part of the multiple choices on the test (see Fig. 1).

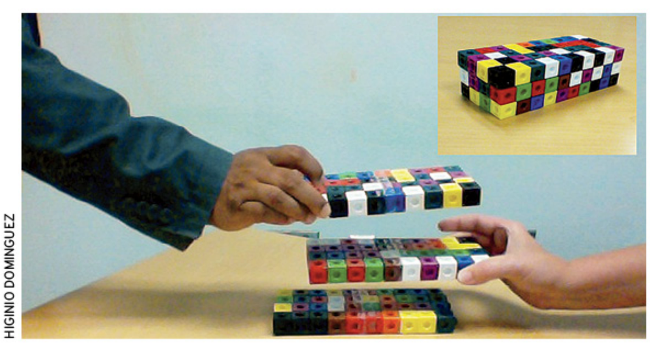
FIGURE 1

Reinterpreting the picture of a 3 × 4 × 10 rectangular prism on a test, a fourth-grade bilingual Latina corrected her answer for a volume problem. The initial partial surface area of eighty-two little squares (produced by counting only the squares on the visible faces) was revised and re-envisioned as the student first invented successive composite units of ten connected cubes and then the larger composite unit of forty connected cubes.

(a) El siguiente modelo está hecho con cubos de 1 centímetro.



(b) ¿Cuál es el volume del modelo?



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Refusing to believe that she could not correctly compute the volume, I asked her to use connected cubes to reconstruct the drawn model. This reformulation allowed her to shift her attention back and forth from one "window" (the model drawn on the test) to another "window" (the emerging model in front of our eyes) as she noticed the dimensions of length, width, and height on both models. This student also kept opening larger and larger "windows" as she invented two progressively complex composite measurement units. First she invented the composite unit of ten (ten connected cubes). She subsequently came up with a unit of forty by looking at three layers of forty cubic centimeters each.

Empowering all students through problem posing

Questions that can elicit student noticing by restoring experiential mirrors and opening mathematical windows include the following:

- What is the problem here?
- What do you know about this situation that is not mentioned in this task?
- To make this task easier for you, would you change the story or the numbers?

- What do you think it is going to happen next in this situation?
- If you do not think this is a problem, can you create a true problem either with a different situation or different numbers or both?
- What would the solution be if the situation involved different quantities?
- Would the previous solution support you in finding the new solution?
- Could we represent this given model in an easier way?

It is in this noticing that students can see reflections of what they already know and opportunities to see what else they can know as they continue learning mathematics.

TCM Twitter Chat

Did you miss the Twitter chat about this article? We've got you covered! Here's a recap of the conversation. At 9:00 PM EST on the second Wednesday of each month follow #TCMchat on Twitter for a new conversation about an article in the latest issue of *TCM*.

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Higinio Dominguez, Higinio@msu.edu, an assistant professor in the department of teacher education at Michigan State University in East Lansing, focuses his research on supporting teachers to elicit student noticing as a continuous, self-sustained source of ideas for developing common resources for teaching and learning mathematics.

Minecraft in the Math Class

Dean Vendramin

Minecraft has become a gaming phenomenon in the last few years. Kids (and adults) everywhere are digging holes, mining for precious metals and minerals, crafting a wide variety of objects, and, oh yeah, taking on monsters such as ‘creepers.’ This sandbox video game is like an electronic version of Lego, and here, only one’s imagination is the limit. The ability to modify and create the world is what makes the game unique, and also what makes it so popular.

The game itself is fairly easy to understand and play, but the journey that you then embark on can be as complicated as you make it. You can play in survival mode, where ‘bad guys’ come after you and you must collect and manufacture resources, or you can play in creative mode, where the world is your canvas and you are pre-equipped with all of the materials you need with no threat of ‘bad guys’ at your heels.

Adapting this game for educational purposes is natural, and there are many ways to infuse this tool into a variety of class situations and grades. Math is no exception, and there are many mathematical concepts that can be explored in this virtual world. My own experience with using Minecraft for educational purposes has mostly been in the context of Grade 9 math, exploring geometry and basic algebra curriculum objectives (although I have also had some experience implementing the game in the Workplace and Apprenticeship 20 course as well).

> “Instead of passively receiving information, learners can create, analyze, and draw conclusions for themselves in this setting. Concepts such as symmetry, area, and perimeter can be visualized through students’ own constructions.”

Geometry lends itself well to being explored in this medium. Instead of passively receiving information, learners can create, analyze, and draw conclusions for themselves in this setting. Concepts such as symmetry, area, and perimeter can be visualized through students’ own constructions. Concepts such as volume and surface area, which can be very difficult to conceptualize using two dimensional images, become easier to understand with the help of three-dimensional models that students can build in the game. One could even tackle concepts such as solving equations and basic algebra. To give you more ideas, I have created a publicly accessible presentation with videos, lesson outlines, resource links, and more: see http://bit.ly/1Td2d8R.

One way to explore the concept of surface area in Minecraft is to have students create structures and calculate their surface area. In one lesson that I developed, the first task was to create a shelter with a flat roof (students could make a door, but pretend that it is part of the blocks when calculating the total surface area). Each cubed block was assigned a length and width of one meter, and students were able to calculate that each square face was 1 m^2. Students then started to build their shelter and calculate the surface area (the bottom of the shelter was not included in the calculations). Students took screen shots (see Figure 1) and showed their surface area calculations. As students built their structures, they could see the area of all sides and 'walk around' their shelter to visualize its three dimensions. This concept can be hard to understand using traditional methods. This and other lessons / ideas can be accessed at http://bit.ly/1Td2d8R.

Figure 1. Surface area of a shelter
$2(3\times7) + 2(3\times6) + (6\times7) = 120\ m^2$

Another great application of this tool is integrating it into a Genius Hour project. In this setting, students explore their own ideas, solve their own problems, experience rigor and grit, and ultimately share their understandings and creations with others. Through the sharing process, the students reflect on concepts that have been reinforced or acquired and take pride and invest in the learning process. Over the years, my students have created some amazing projects as part of Genius Hour that have involved collaboration, problem solving, and critical thinking skills—all of which are important skills in the 21st century.

I have accessed Minecraft in the classroom in multiple ways. Typically, I have purchased (through Apple's Volume Purchasing Plan) Minecraft Pocket Edition and used the software on a class set of iPads. Many students also have had Minecraft on their own device that they brought to class, which aligns perfectly with the Bring Your Own Device movement (of which I am a supporter). Some students would do the assignment on their home computers or on gaming consoles. I have even had some students use different block building apps to create their projects. In the fall, I will be involved with my division's 'Connected Educator' program, and will therefore have 1-to-1 (1 device per student) access to laptops for my classes. Consequently, I will be looking into Minecraft Education Edition (http://education.minecraft.net/), which is offered for free. (It does require an Office 365 account, but all students have one in my division.) I am very excited to expand and 'build' upon the Minecraft experience in my classroom.

There have been some students in my classes who haven't played the game and are either uncomfortable, used to traditional ways, or apprehensive of the tool. I always encourage students to be open to new ideas, but nevertheless provide them with alternatives to show understandings of the concept. Still, most of my students have enjoyed exploring math concepts using this tool. There have been some 'learning' moments along the way, which have been good conversation starters about the concept of digital citizenship. There have also been some great moments with students who are normally quiet or seen as inattentive flat-out sing when an assignment involves something that they have a passion for, such as Minecraft. In my opinion, the risk of using the tool is worth the reward, and learning opportunities only seem to continue to grow where they might have stopped with traditional methods.

> "It might involve giving up some control in the classroom, but giving students the opportunity to create, explore, and play with math provides some amazing rewards."

I encourage educators to explore the opportunities that this tool offers. It might involve giving up some control in the classroom, learning from the students, and taking some calculated risks, but giving students the opportunity to create, explore, and play with math or other concepts provides some amazing rewards. If you are a little apprehensive or nervous about taking this plunge, there are many resources out there to get you started and to give you support. For instance, check out the Minecraft Education Edition website, these YouTube videos (http://bit.ly/29lrmtA), and these people on Twitter (@immersiveminds, @jpedrech, and @playcraftlearn) for ideas on how to use Minecraft in the classroom. Also, talk to your students—many of them will have ideas for projects.

If you have any questions, you can contact me at d.vendramin@rcsd.ca, follow me on Twitter at @vendi55, or check out my blog and eportfolio at deanvendramin.weebly.com.

Dean Vendramin has had a variety of teaching experiences over his 19-year teaching career. Currently, he is a Math/Science Education Leader at Archbishop M.C. O'Neill Catholic High School in Regina, Saskatchewan. He is passionate about technology integration and assessment practices. Dean has been recognized for his work throughout his career, having received among other awards a Certificate of Achievement from Prime Minister's Teaching Excellence Awards and an Innovative Math Teacher Award from the Saskatchewan Math Teachers' Society, and has been recognized by a number of technology companies for his work in integrating technology in the classroom. He is passionate about improving his craft, sharing ideas, and supporting the needs of those he serves. Contact Dean at d.vendramin@rcsd.ca, follow him on Twitter at @vendi55, or check out his blog and eportfolio at deanvendramin.weebly.com.

Intersections

In this monthly column, you'll find information about upcoming math education-related workshops, conferences, and other events. If travel is not an option at this time or if you prefer learning from the comfort of your own home, see the Online workshops and Continuous learning online sections. Some events fill up fast, so don't delay signing up!

For more information about a particular event or to register, follow the link provided below the description. If you know about an event that should be on our list, please contact us at thevariable@smts.ca.

Within Saskatchewan

Conferences

Saskatchewan Understands Math (SUM) Conference
November 4th – 5th, Saskatoon, SK
Presented by the SMTS

Our own annual conference! The Saskatchewan Understands Math (SUM) conference is for math educators teaching in K-12 who are interested in curriculum, incorporating technology, number sense, and problem solving. Join us for two days packed with learning opportunities, featuring keynote speakers Max Ray-Riek of the Math Forum at NCTM and Grace Kelemanik of the Boston Teacher Residency Program. Registration includes lunch on Friday and a two-year SMTS membership. See the poster on page 3, and head to our website for more information and to register.

Workshops

Crossing Curricula to Develop Better Learners
November 1st, Regina, SK
Presented by the Saskatchewan Professional Development Unit

Curricula often have common themes or have concepts that can work in unison. Rather than teaching subjects in isolation, framed by a bell schedule, cross-curricular instruction immerses students in a theme or topic. This workshop will look at effective and authentic cross-curricular teaching and how you can do it in your classroom.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/crossing-curricula-develop-better-0

Number Talks and Beyond: Building Math Communities Through Classroom Conversation
November 16th, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

Classroom discussion is a powerful tool for supporting student communication, sense-making and mathematical understanding. Curating productive math talk communities requires teachers to plan for and recognize opportunities in the live action of teaching. Come experience a variety of classroom numeracy routines including number talks, counting circles, quick images and more. Take math conversations to the next level by strengthening your skills as a facilitator of classroom discourse and student thinking.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/number-talks-and-beyond-building

Number Talks and Beyond: Building Math Communities Through Classroom Conversation
January 17th, Regina, SK
$110 (early bird), $150 (standard)
Presented by the Saskatchewan Professional Development Unit

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See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/number-talks-and-beyond-building

Technology Integration for Differentiation in Mathematics
January 19th, Saskatoon, SK
$110 (early bird), $150 (standard)
Presented by the Saskatchewan Professional Development Unit

Are you interested in using technology to help differentiate your mathematics classroom? Workshop participants will be introduced to various blended learning structures, then focus on the station rotation and flipped classroom models. Whether you have one device or a classroom of devices, these two classroom structures are beneficial to increasing student engagement and to providing opportunity for teachers to have individual and small group instruction. The idea of using technology to create differentiated opportunities through adaptive instructional websites and math and presentation-related apps will be explored and connected to curricular outcomes, student learning progressions and assessment.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/number-talks-and-beyond-building-0

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See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/technology-integration-differentiation

Early Learning With Block Play – Numeracy, Science, Literacy and So Much More!
January 25th, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

This is a one-day workshop for early learning educators from prekindergarten, kindergarten and Grade 1 to join together, discover and deepen their understandings around the many foundational skills that children develop during block play. Through concrete, hands-on activities, participants will experience and examine the many connections between block play and curricular outcomes, and the current research on the topic. Participants will have opportunity for reflection on their current practice, planning for block play and for creating a network of support.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/early-learning-block-play-numeracy-2

Extending Early Learning Block Play into Project-Based Inquiry
January 27th, Yorkton, SK
Presented by the Saskatchewan Professional Development Unit

This one-day workshop for early learning educators from prekindergarten, kindergarten and Grade 1 will deepen understanding around the foundational skills that children develop during block play and extend that understanding into project-based learning in early years. Through concrete, hands-on activities participants will experience and examine the many connections between block play, curricular outcomes and project-based inquiry in early years.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/extending-early-learning-block-play

Using Tasks in High School Mathematics
February 8th, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

Using tasks in a high school mathematics classroom can provide rich opportunities for differentiated learning and authentic assessment. How do we choose tasks that meet both curricular outcomes and student needs? Tasks allow students to enter mathematics where they are at and extend their learning. In this workshop we will look at a variety of resources for finding good high school tasks. We will also reflect and discuss what planning and teaching moves can assist in maximizing student learning through mathematics tasks.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/using-tasks-high-school-mathematics

Technology in Math Foundations and Pre-Calculus
February 9th, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

Technology is a tool that allows students to understand senior mathematics in a deeper way. This workshop is designed to have math foundations and pre-calculus teachers experience a variety of technology tools that allow students to represent and visualize mathematics concepts. Tools highlighted are useful for students to explore, learn, communicate, collaborate and practice, in order to enhance their understanding of mathematics in secondary mathematics.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/technology-math-foundations-and-pre

Beyond Saskatchewan

BCAMT Fall Conference 2016: Math is Social
October 21, Vancouver, AB
Presented by the British Columbia Association of Mathematics Teachers

The theme of this year's BCAMT Fall Conference is "Math is Social," chosen because there has been increasing attention paid to the social nature of learning (and teaching) in recent years, and a move away from teacher-centered and individual learning. This year's keynote speakers are Fawn Nguyen (UC-Santa Barbara Mathematics Project leadership team) and Richard Hosino (Quest University Canada).

See http://www.bcamt.ca/fall2016/

MCATA Fall Conference 2016: Opening Your Mathematical Mind
October 21-22, Canmore, AB
Presented by the Mathematics Council of the Alberta Teachers' Association

Come join the Mathematics Council of the Alberta Teachers' Association in celebrating their annual fall conference "Opening Your Mathematical Mind" at the Coast Canmore Hotel & Conference Centre, 511 Bow Valley Trail, Canmore, Alberta. Featuring Keynote Speakers Dr. Peter Liljedahl of Simon Fraser University and Dr. Ilana Horn of Vanderbilt University.

See https://event-wizard.com/OpeningYourMathmind/0/welcome/

55th Northwest Mathematics Conference
October 21-23, Yakima, WA

The Northwest Mathematics Conference is a collaborative conference held annually, alternating between Washington, Oregon, and British Columbia. The target audience of preK-16 math educators includes pre-service, active, and retired elementary, middle, and high school teachers, community college and university instructors, math coaches, staff development specialists, and special needs and ELL math teachers.

Approximately 1,000 participants will gather in Yakima for this year's two-day conference, which will kick off with a Maker's Fair and is centered around the theme "What is Next in Mathematics? WIN with Math." This year's keynote speaker is Michael Stevens (Vsauce1); featured speakers are Steve Leinwald, Ruth Parker, and Sandy Atkins. Five strands will be highlighted across the event: Early Numeracy – Setting the Foundation for the Future; the CCSS Standards for Mathematical Practice – Engaging Students in Learning; Post-Secondary Education – Preparing for Tomorrow; STEAM – Driving Innovation in Learning, and Assessment – Deepening Understanding.

See http://www.northwestmathconference2016.org/

NCTM Regional Conferences & Expositions 2016: Great Math at Your Doorstep
October 26-28, Phoenix, AZ
October 31 - November 2, Philadelphia, PA
Presented by the National Council of Teachers of Mathematics

Sharpen your skills, gain new techniques, and achieve your professional goals when you make your plans for this professional development opportunity today. Five focus strands and numerous featured speakers allow you to learn the most effective teaching practices within the topics most essential to your teaching. Whether you're a classroom teacher, math coach, administrator, math teacher educator, teacher-in-training, or math specialist, there's something for you at the NCTM Regional Conferences & Expositions.

Note: Did you know that the Saskatchewan Mathematics Teachers' Society is an NCTM Affiliate? When registering for an NCTM membership, be sure to support the SMTS by noting your affiliation during registration.

See http://www.nctm.org/Conferences-and-Professional-Development/Regional-Conferences-and-Expositions/

Innov8 Conference
November 16-18, St. Louis, MO
Presented by the National Council of Teachers of Mathematics

Join your peers at the inaugural Innov8 conference, November 16-18, in St. Louis, Missouri! This innovative and team-based professional development is centered around acquiring the necessary skills to provide high-quality mathematics education for learners of all abilities. Innov8 provides opportunities for attendees to receive hands-on experience implementing research-based mathematics education practices; connect with like-minded teachers facing similar problems of practice; collaborate to determine effective solutions to advance student learning; and return to the classroom, school, or district with an action plan and commitment to implement refreshed tools and techniques.

Note: Did you know that the Saskatchewan Mathematics Teachers' Society is an NCTM Affiliate? When registering for an NCTM membership, be sure to support the SMTS by noting your affiliation during registration.

See http://www.nctm.org/innov8/

Online Workshops

Math Daily 3
October 2 – October 29
Presented by the Daily CAFÉ

Learn how to help your students achieve mathematics mastery through the Math Daily 3 structure, which comprises Math by Myself, Math with Someone, and Math Writing. Allison Behne covers the underlying brain research, teaching, and learning motivators; classroom design; how to create focused lessons that develop student independence; organizing student data; and differentiated math instruction. Daily CAFE online seminars combine guided instruction with additional resources you explore on your own, and are perfect for those who prefer short bursts of information combined with independent learning.

See https://www.thedailycafe.com/workshops/10000

Continuous Learning Online

Education Week Math Webinars
Presented by Education Week

Once a month, Education Weekly has a webinar focusing on math. They also host their previous webinars on this site. Previous webinars include Formative Assessment, Dynamic vs. Static Assessment, Productive Struggling and Differentiation.

Past webinars: http://www.edweek.org/ew/webinars/math-webinars.html
Upcoming webinars:
http://www.edweek.org/ew/marketplace/webinars/webinars.html

SMTS Awards

Do you know a teacher who works tirelessly to use and promote sound mathematical pedagogy among peers, students, and parents? A teacher who uses innovative teaching strategies to excite and inspire their students in math class? An individual or group who has had a tremendous and meaningful impact on mathematics education in Saskatchewan? **Nominate them for a Saskatchewan Mathematics Teachers' Society Award**!

This year, the SMTS is offering three awards to recognize excellence in mathematics teaching and leadership in Saskatchewan.

Our first award, the **Master Teacher Award**, is awarded to an experienced K-12 Saskatchewan teacher of mathematics who has garnered a reputation for teaching excellence and promoting sound mathematical pedagogy among peers, students, and parents (this year's changes to the Master Teacher Award are detailed on our website). To accompany the Master Teacher Award is the **Teaching Innovation Award**, whose purpose is to recognize teachers who regularly employ non-traditional teaching strategies and in turn foster student engagement. Our third award is a **Service Award** for a person or group who has created a significant and positive impact on mathematics education in Saskatchewan. Note that the recipient of an SMTS award is *not* required to be an SMTS member.

Take a moment to read through the award descriptions and to nominate one of your colleagues on our website. We are also working to round out our history of past winners; if you know of a past winner who is not on our list, please email evan@smts.ca.

We look forward to hearing from you!

Call for Contributions

Did you just deliver a great lesson? Or maybe it didn't go as planned, but you learned something new about the complexities of teaching and learning mathematics. Maybe you just read a book or attended a workshop that gave you great ideas for presenting a topic your students have always found difficult, or that changed your perspective about some aspect of teaching. **Why not share your ideas with other teachers in the province – and beyond?**

The Variable is looking for a wide variety of contributions from all members of the mathematics education community in Canada and beyond, including classroom teachers, consultants and coordinators, teacher educators, and researchers. Consider sharing a favorite lesson plan, a reflection, an essay, a book review, or any other article or other work of interest to mathematics teachers in Saskatchewan. If accepted for publication, your piece will be shared, as part of this periodical, with a wide audience of mathematics teachers, consultants, and researchers across the province, as well as posted on our website.

We are also looking for student contributions, whether in the form of artwork, stories, poems, interesting problem solutions, or articles. This is a great opportunity for students to share their work with an audience beyond that of their classroom and their school, and for teachers to recognize their students' efforts during their journey of learning mathematics.

All work will be published under a Creative Commons license. If you are interested in contributing your own or (with permission) your students' work, please contact us at thevariable@smts.ca.

We look forward to hearing from you!