The Variable

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Mathematical Modeling in the High School Curriculum
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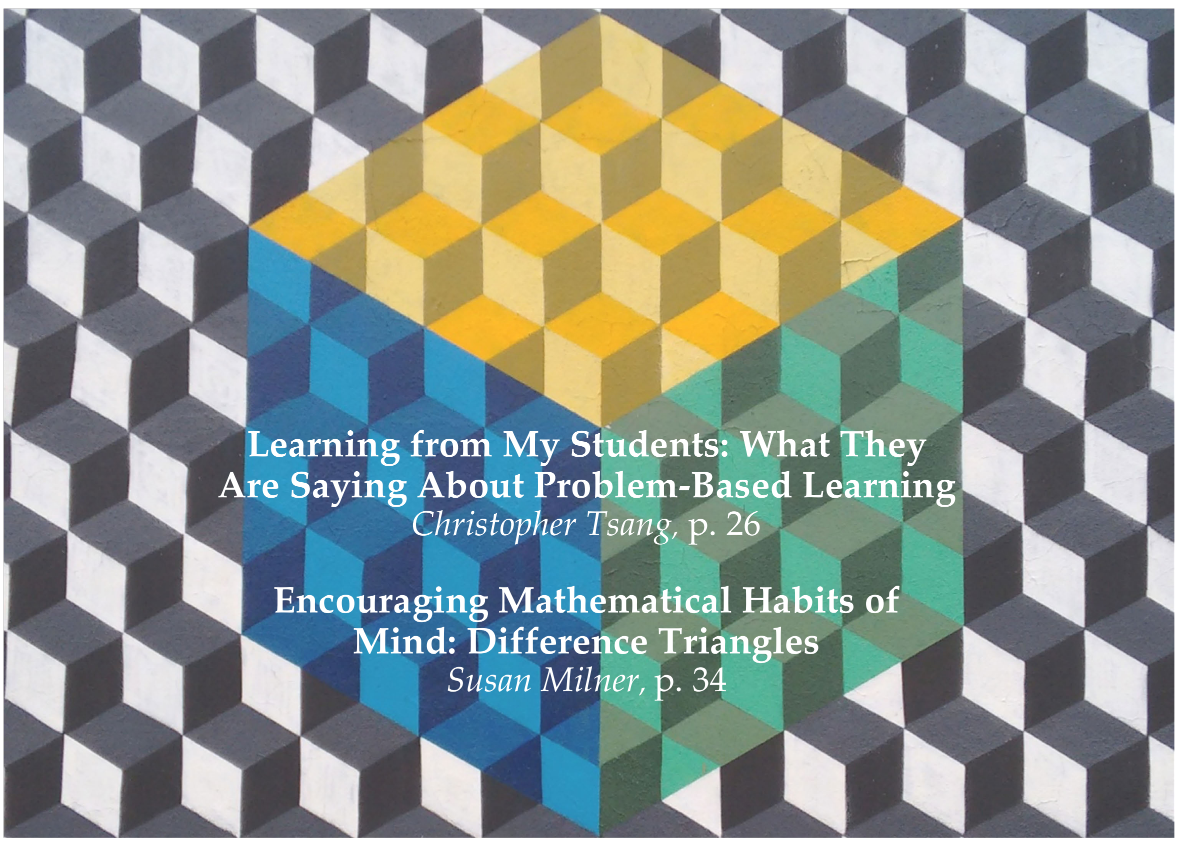

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Saskatchewan Mathematics Teachers' Society

AFFILIATE
NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS


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The Variable

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Notice to Contributors

The Variable welcomes a variety of submissions for consideration from all members of the mathematics education community in Canada and beyond, including classroom teachers, consultants, teacher educators, researchers, and students of all ages, although we encourage Saskatchewan teachers of mathematics as our main contributors. Submitted material is assessed for interest, presentation, and suitability for the target audience.

Please submit articles by email to thevariable@smts.ca in Microsoft Word format. Articles should be limited to 3000 words or less; authors should also include a photo and a short biographical statement of 75 words or less. Accepted contributions are subject to revision. Editors reserve the right to edit manuscripts for clarity, brevity, grammar, and style.


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SELU
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Leading Together
The SMTS, SELU and SPDU presents ...
#SUM2017
Who: K-12 teachers, coaches, consultants, coordinators, superintendents and directors
Where: TCU Place, Saskatoon
When: October 23-24, 2017
Cost: $315 (early registration) | $375 (regular)
Keynote Presenters
Lisa Lunney Borden, St. Francis Xavier University
Steve Leinwand, American Institutes for Research
Featured Presenters
Kristin Gray, Cape Henlopen School District, Illustrative Mathematics
Glenn Waddell, The University of Reno, Desmos Fellow
Egan Chernoff, University of Saskatchewan
Gale Russell, University of Regina
NCTM
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Message from the President

Happy Spring, everyone! Can you just humour me and pretend with me that it really *is* spring, despite the fact that there is probably a very high probability of at least one more cold snap, if not two? (I'm sure that somewhere here, there's a great lesson related to probability and Saskatchewan weather patterns, so thank me later for this greeting…)

This time of year is a flurry of activity for the SMTS. Spring is when strategic adjustments happen for the next school year, as well as much of the work that goes into our events. Among other initiatives, this includes Leading Together: Saskatchewan Understands Math (SUM) Conference 2017. Our keynotes and featured speaker session descriptions are starting to roll in, and I'm already worried about how I'm going to choose sessions. While it feels a bit strange not to be gearing up for Math Challenge this year, I hope you and your students are able to take advantage of the multitude of other contests and camps available to K-12 students who are enthusiastic about mathematics. These are highlighted in our new column, *Tangents* (see p. 40).

Although we're working hard for you during this time of year, we would also love your input. Do you have a great idea that you think would be beneficial to either teachers or students in the province? Consider applying for our grant! The SMTS awards $1000 annually to support initiatives that complement the teaching and learning of mathematics by students or teachers in Saskatchewan. Or, perhaps there is an opportunity for some partnership work. The SMTS is a small but mighty crew, and if you also have a small but mighty crew, we would love to combine resources to support Saskatchewan teachers.

Since this is the time of year that school divisions are also beginning their strategic planning for next year, it's a great time to remind your math and leadership teams that next year's SUM Conference is structured to support teachers *and* leadership. The SMTS is striving to support divisions as they put the Education Sector Strategic Plan (ESSP) into action, across multiple ESSP goals.

I also encourage you to consider putting forward your own session proposal for SUM 2017! While the deadline isn't until May 30, it's definitely time to start considering what your topic might be, or to talk to that colleague who you know would have a great session but needs some extra encouragement (bi-weekly for the next few months, perhaps). Your expertise and passion are what makes SUM great, year after year.

Until next time, stay mathy, friends!

Michelle Naidu

Problems to Ponder

Welcome to the March/April edition of Problems to Ponder! This collection of problems has been curated by Michael Pruner, president of the British Columbia Association of Mathematics Teachers (BCAMT). The tasks are released on a weekly basis through the BCAMT listserv, and are also shared via Twitter (@BCAMT) and on the BCAMT website.

Have an interesting solution? Send it to thevariable@smts.ca for publication in a future issue of The Variable.

British Columbia Association of Mathematics Teachers

I am calling these problems 'competency tasks' because they seem to fit quite nicely with the curricular competencies in the British Columbia revised curriculum. They are non-content based, so that all students should be able to get started and investigate by drawing pictures, making guesses, or asking questions. When possible, extensions will be provided so that you can keep your students in flow during the activity. Although they may not fit under a specific topic for your course, the richness of the mathematics comes out when students explain their thinking or show creativity in their solution strategies.

I think it would be fun and more valuable for everyone if we shared our experiences with the tasks. Take pictures of students' work and share how the tasks worked with your class through the BCAMT listserv so that others may learn from your experiences.

I hope you and your class have fun with these tasks.

Michael Pruner

Intermediate and Secondary Tasks (Grades 5-12)

Palindromes

A palindrome is a number reading the same backward as forward. Consider a two-digit number: for example, 84. 84 is not a palindrome, so reverse the digits and add it to the original number: 84 + 48 = 132. This is still not a palindrome, so try it again: 132 + 231 = 363. 363 is a palindrome, so 84 can be called a depth 2 palindrome. Find the depth of all two-digit numbers.

Extensions: What about 3-digit numbers? What about the depth for the second time becoming a palindrome? What happens when you shade a Hundreds Chart according to the numbers' depth?

Source: Heinz, H. (2010). *Palindromes*. Retrieved from http://www.magic-squares.net/palindromes.htm

Marching Band

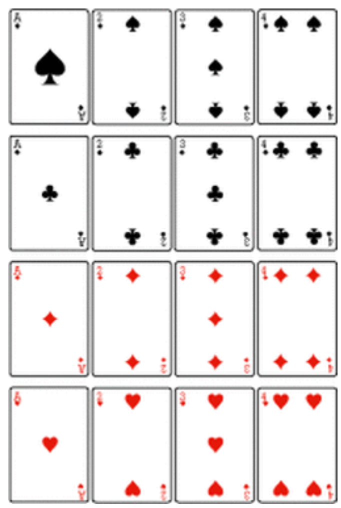
Students in a marching band want to line up for their performance. The problem is that when they line up in 2's, there is 1 student left over. When they line up in 3's, there are 2 left students over. When they line up in 4's, there are 3 students left over. When they line up in 5's, there are 4 students left over. When they line up in 6's, there are 5 students left over. When they line up in 7's, there are no students left over. How many students are there?

Extensions: What if there are over 200 students in the band? What if there are 6 left over when lined up in 7's?

Source: John Grant McLoughlin

Game of 22

Arrange four rows of cards from ace to four as shown below:



Two players alternately choose a card and add it to the common total. The winner is the player who makes 22 or who forces the other player to go beyond 22. What is a winning strategy?

Source: The 22 game. (n.d.). Retrieved from http://www.mathfair.com/the-22-game.html

Primary Tasks (Grades K-4)

Five Cubes

Using exactly five interlocking cubes, make as many shapes as you can so that all five cubes are touching the table. How many different shapes can you make?

Source: Spring 2011 problem set. (2011). *Vector, 52*(1), 59-60.

Subtraction Graph

Two play this game. Use two dice and markers.

Roll the dice and subtract the lower number from the higher. Put a marker in the first empty square above your answer. A player wins when they place a marker such that it is the first to reach the top of the graph.

0	1	2	3	4	5

Source: The Surrey School District

Box of Chocolates

Erica doesn't like odd numbers, so the box of chocolates shown to the left meets with her approval. The problem is that she has to remove six chocolates from the box in such a way that she leaves an even number of chocolates in each row and each column. How might she do this?

Source: Kordemsky, B. A. (1992). *The Moscow puzzles: 359 mathematical recreations.* New York, NY: Dover Publications, Inc. (Original work published 1972)

Michael Pruner is the current president of the British Columbia Association of Mathematics Teachers (BCAMT) and a full-time mathematics teacher at Windsor Secondary School in North Vancouver. He teaches using the Thinking Classroom model where students work collaboratively on tasks to develop both their mathematical competencies and their understanding of the course content.

Reflections

Reflections *is a monthly column for teachers, by teachers on topics of interest to mathematics educators: reflections on classroom experiences, professional development opportunities, resource reviews, and more. If you are interested in sharing your own ideas with mathematics educators in the province (and beyond), consider contributing to this column! Contact us at thevariable@smts.ca.*

Foundations 20 Introduction Tasks: Linear Inequalities and Statistical Reasoning

Sharon Harvey

I've taught Foundations of Mathematics 20 a few times, and each time I have found myself struggling with the disconnect between topics. At times, it feels as though the course is a dumping ground for concepts students should know, but that didn't really fit anywhere else. And I notice that students struggle to remember concepts from the beginning of the course (that we do not use again after the unit) when it comes time to prepare for the final exam.

So I decided to look for opening tasks for units that would help introduce the main topic, perhaps shake up a little prior knowledge, and create memorable experiences that I could relate to when reviewing for the final exam. Collaborating with Amanda Culver and Andrea Klassen, we came up with activities that we felt would work well. Today, I'm going to share two of my favorites with you.

Unit: Linear Inequalities
Activity: Zombie Apocalypse Now
Time: 1 class

Purpose:
Students use the Zombie apocalypse scenario to review graphing linear functions using a table of values and the slope-point method. Each linear function has a direction (NESW) associated with it, which I use later to introduce the idea of inequalities.

Materials:
- double-sided activity sheet (one per person; see following pages)

Procedure:

1. Pair students (I randomly pair them using Triptico).

2. Give each student a handout and have them move into their pairs. In the story, I always use the names of 6 students in the story, 5 of which are killed by the zombies and 1 whom will survive and save the class—they find this amusing.

3. Tell them that the safe house and other cabins need to be identified and submitted by the end of class. (I have them hand in their worksheet and quickly check to see how it went and to know what to remind them of, such as what to do when the y-intercept is off the graph, in our lesson the next day.)

Zombie Apocolypse Now

The zombies are after us! Thankfully, Mrs. Harvey has been anticipating this for some time, and has set up six safe houses in Saskatchewan. She sends out Carter, Stefan, Kaelyn, Abby, Turner, and Dakota to check on the safety and infection status of the houses. Stefan, Kaelyn, Abby, Turner, and Dakota do not report back. It can only be assumed that the houses they went to are compromised and they are dead. Carter is our only hope. We know that he is at Spruce Cabin.

But dang it, Mrs. Harvey doesn't remember the code names for the cabins, so Carter has to give directions to the group to join him. Carter realizes that the zombies can listen in so he can't give traditional directions without the zombies finidng the house. He does know, as anyone who knows anything about zombies knows, that the virus prevents them from understanding basic math principles like Linear Functions. In order to make it to the safe house and prevent Zombies from finding it, Carter relays the following messages about where the others were headed.

Message #1 – Stefan was headed to Oak Cabin North of $5y = -2x + 70$.

Message # 2 – Kaelyn was sent to Cedar Cabin South of $3y – x = -39$.

Message # 3 – Abby checked out Maple Cabin North of $2x + 3y = 3$.

Message # 4 – Turner sprinted towards Fir Cabin East of $-7x + y + 7 = 0$.

Message # 5 – Dakota reluctantly hiked to Pine Cabin West of $y = 12x + 72$

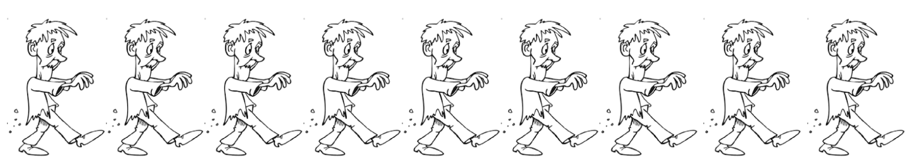
At first, you panic! But then Mrs. Harvey reminds you that you can just use what you already know.

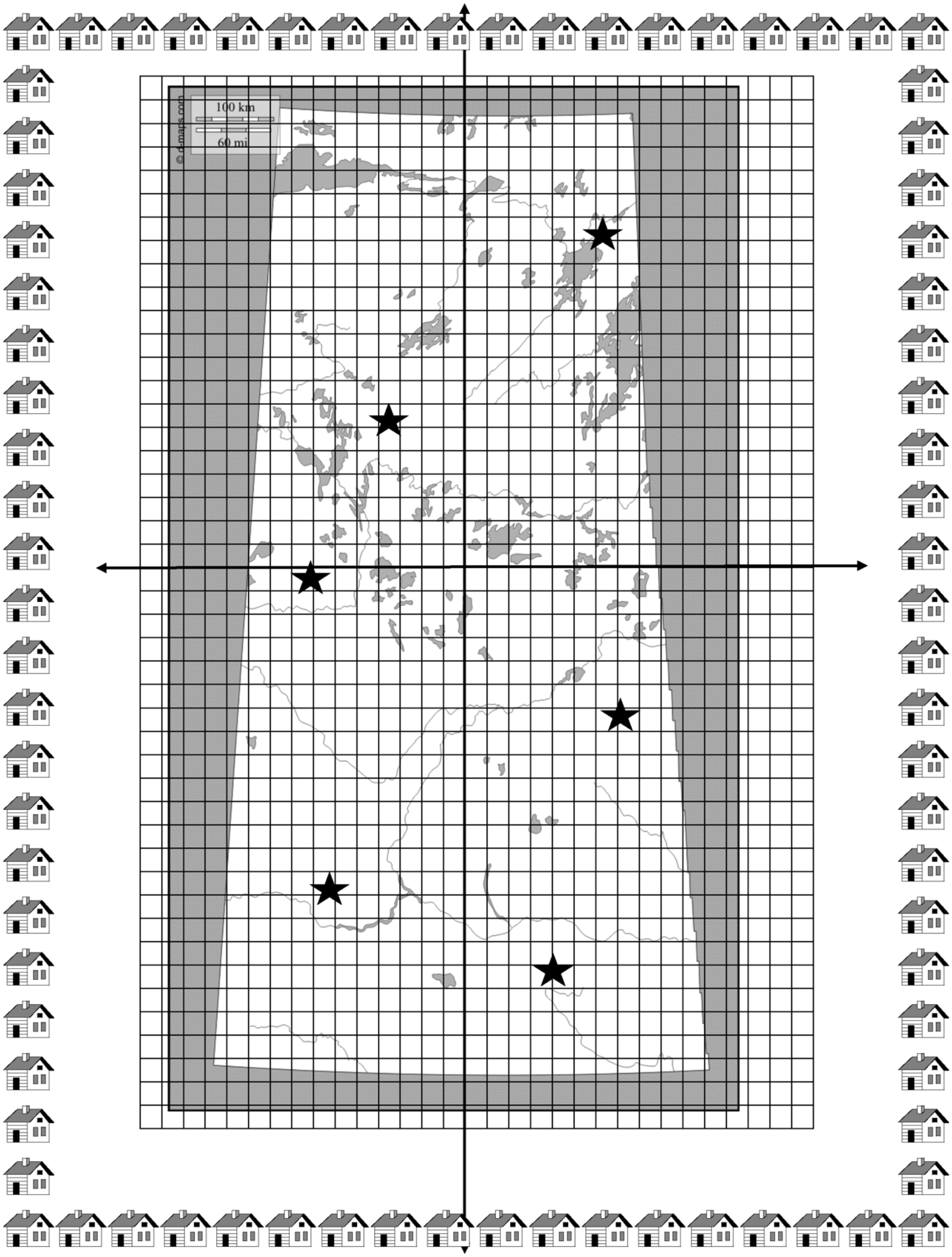
$y = mx + b$ b = y-intecept m = slope

or

make a table of values

Using the map attached, Identify each of the cabin by code name and find the safe house!




100 km
60 mi

Follow-up:
The next day, I project the same map and go through the first set of directions with them. I ask how they knew to cross out the cabin above the line—they say because that's north. Then, I ask about how we tell directions in math: If I wanted them to go above the line, what symbol would that be? How would they know they were right? We agree on which cabin is the safe house, then move on to the Cartesian plane and using mathematical symbols for inequalities.

Unit: Statistical Reasoning
Activity: Skittles Lab
Time: 1 class

Purpose:
Students access prior knowledge related to data management and statistics. This introduction activity allows for discussion about how to use a set of data to make predictions and what precautions need to be taken.

Materials:
- 1 bag of Skittles for each group of 3 or 4
- 1 extra bag of Skittles
- mini whiteboards
- markers
- erasers
- napkins

Procedure:

1. Give each group of three or four students a bag of skittles, a napkin, a whiteboard, a marker, and an eraser.

2. Ask each group to open the Skittles, pour them onto the napkin, then record as much data as possible about their bag of Skittles onto their whiteboard (typically, all of the groups will count the number of each colour, the unique/not round/deformed Skittles, and Skittles missing the 'S' marking; some groups will go further and calculate the weight of a Skittle, the number of calories per Skittle, etc.).

3. Collect the data from each group and put it onto the classroom whiteboard. I collect the numbers of the different colours and the number of "unique" Skittles only. It looks similar to the following table:

	Yellow	Red	Orange	Purple	Green	Unique
Group 1	15	22	16	12	15	1
Group 2	18	21	17	11	19	0
Group 3	12	16	24	8	27	2
Group 4	24	25	25	19	10	1
Group 5	16	24	18	15	17	3
Group 6	17	28	25	16	16	1
Group 7	18	21	19	14	23	2
Group 8	18	20	21	12	24	4

4. Let the students eat the Skittles.

5. Ask them to use the data to predict how many Skittles of each color and in total will be in the last unopened bag, and how many unique Skittles there will be. Tell them that they need to be ready to defend their predictions.

6. Discuss the predictions. This is where the fun really begins. In all the times that I have done this activity, there have been groups that use the mean, the median, and the mode to make their predictions. There is also always a group that just guesses. Of course, they don't often use these words, but then I ask for the "math words" that are associated with their methods and they usually come up with the vocabulary. This part of the task generally opens up a discussion about outliers. (For example, should they be using that 24 count in yellow if they're finding the average?) Often, we also take time to chat about the difference between the range and the interval.

7. Reveal the numbers.

8. On their boards, ask students to give each prediction a +/- score. So, if there were 17 yellow Skittles in the last bag and they predicted 16, that would be a -1 (they under-guessed by 1). If they predicted 18, that would be a +3. If they predicted the exact number of Skittles, they would score a 0.

9. Compile the results into the chart on the board (I just erase their old data and fill the chart with +/- numbers).

10. Ask what they notice and wonder. What colour were they best at predicting, and how do they know?

Follow-up:
The next day, we have a formal lesson on the concepts of measures of central tendency, range, outliers, and dispersion. Throughout the unit, I am able to reference parts of the Skittles activity, and when it comes time for final review, I add Skittles next to the title and it helps them to remember the relevant concepts. I also start with this unit, so it's a great chance to chat with students and establish a collaborative atmosphere in the room.

If you have any questions about the above activities, please contact me at derrick.sharon@gmail.com. And stay tuned for future editions, where I will share more introduction activities for Foundations of Mathematics 20.

Sharon Harvey has been a teacher within the Saskatoon Public School Division for eight years. She has taught all secondary levels of mathematics, as well as within the resource program. She strives to create an inclusive and safe environment for her students.

Spotlight on the Profession

In conversation with Dr. Alayne Armstrong

In this monthly column, we speak with a notable member of the mathematics education community about their work and their perspectives on the teaching and learning of mathematics. This month, we had the pleasure of speaking with Dr. Alayne Armstrong.

Alayne Armstrong joined the Faculty of Education of the University of Regina in July 2016 as an Assistant Professor in Mathematics Education. She completed her PhD in 2013 in the Department of Curriculum and Pedagogy at the University of British Columbia (UBC), where she was a SSHRC Doctoral Scholar. Her Masters and Bachelor of Education degrees were also obtained from UBC, and she has additional degrees from the University of Manitoba and Queen's University. Prior to joining the University of Regina, Alayne was a classroom teacher in the Coquitlam School District in the Lower Mainland of British Columbia, and she also taught undergraduate education courses in math methods and inquiry at UBC. She is currently getting to know Regina and enjoying the friendly people, the pelicans and muskrats, and the big blue sky.

First things first, thank you for taking the time for this conversation!

As you had spent 19 years teaching in the K-12 public school system prior to joining the faculty at the University of Regina in the summer of 2016, your research undoubtedly draws from a wealth of experience in the classroom. Whose work influenced you during your time as a teacher? Then, which gap in the research, or which classroom experiences, urged you to transition into the domain of educational research?

I taught a variety of subjects while I was in the school system, including drama, home economics, and art, so there were many different influences over the years. In terms of teaching mathematics, the work of John Van de Walle (e.g., Van De Walle, John A. & Folk, S. [2005/2007]) had a big impact on me in terms of showing that it was valuable to try a wide range of activities to help students grasp mathematical concepts, including ones that I'd normally use in other subject areas. Along that vein, the work of Gary Tsuruda (e.g., Tsuruda, 1994), an educator (now retired) and math consultant from California, helped me to see connections between students' mathematical thinking and the act of writing in problem solving.

> "I was intrigued by how non-linear my adolescent students' learning was—that they seemed to move forwards, backwards, forwards again, even sideways at times."

I remember being intrigued by how non-linear my adolescent students' learning was—that they seemed to move forwards, backwards, forwards again, even sideways at times—and I initially applied for graduate studies at UBC with the intent of researching ways teachers could most effectively support this kind of learning. Then I became interested in how this learning manifested itself in small group dynamics.

Among the more general themes in your work, group work in the mathematics classroom and the notion of authorship have emerged as central (e.g., Armstrong, 2005, 2013, 2015). In particular, you call for a shift in authority in the mathematics classroom from external sources (e.g., a textbook) to students. As you write,

> A knowledge-making community counters the role of textbook as the authority in the classroom. Responsibility passes to the students to break away from playing the role of empty vessels waiting to be filled with facts and formulae, and instead to make meaning of mathematics for themselves. (Armstrong, 2015, p. 4)

In this sense, does your work align with the theory of constructivism, or does the focus on groups of students, rather than individuals, necessitate a different lens?

The quote you've selected does suggest some version of constructivism, although I've always found constructivism, in the classical sense, to be such a lonely concept – the learner seems so isolated from the world around her. My work in general would probably fit better under the umbrella of enactivism, because what learners do affects, and are affected by, the groups (or systems) in which they are embedded.

In your work, you have referenced the distinction between the notions of cooperative, collaborative, and collective groups (Armstrong, 2015). How do you distinguish between these types of group action? Does the notion of authorship change depending on the particular character a group takes on at a particular point in time?

Groups can be defined in different ways—for instance, by the type of task they've been assigned, or by the length of time members stay together as a group. I've been looking at groups in terms of how cohesive their behaviour is. The more the interests and actions of the members align, the more the groups can be considered as learning agents in their own right. "Collections" have the least social cohesion—for instance, a group of people who happen to be waiting for the same elevator—while "collectives," like a jazz trio that is jamming together, have the most.

> "Teachers and students have the power not only to choose what texts to use as references, but also to consider themselves as authors in the mathematics that they do."

Who is deemed to have authorship can be a complicated thing. There is the notion of author/ity that Povey, Burton, Angier, and Boylan (1999) describe, where the presence of the "/" in the altered term "author/ity" points to the presence of a person, an author, who perhaps wields the power that the unaltered term "authority" conveys. Following this view, mathematics texts are recognized as having been authored by someone; they weren't just handed down from the heavens somehow! Teachers and students have the power not only to choose what mathematics texts to use as references, but also to consider themselves as authors in the mathematics that they themselves do, just as they would in other subject areas (for instance, if they were writing up science reports or drafting stories).

How is the notion of collectivity related to that of flow (discussed in Armstrong, 2008), and is this a sustainable state for a group? What role might a teacher play in sustaining cohesive activity within a group of students working on a mathematical task?

Drawing on R. Keith Sawyer's work (e.g., Sawyer, 2003), group flow can be considered a level of peak performance for a collective, and it occurs when the collective's behaviour is at its most synchronous and cohesive and is acting as if it were of "one mind." I don't believe it is sustainable for very long, just as a flow state for an individual person only lasts for a limited amount of time. And that's probably a good thing—it takes a lot of energy and effort to retain that high level of focus. As well, one of the benefits of being in a group comes through the diversity of its members and the variety of ideas and actions that are available; in a flow state, convergence is key, so that kind of diversity isn't in play in those moments.

> "Being in a state of flow is very enjoyable, so the more flow experiences a student can have during mathematical activities, the better!"

Still, being in a state of flow is very enjoyable, so the more flow experiences a student can have during mathematical activities, the better! In terms of a teacher's role in encouraging group flow, a lot of what is in the literature about effective group work, in terms of balancing structure and challenge, would be applicable: the establishment of effective group routines; flexible leadership roles within the group so that various students can move in and out of these roles, depending on the activities; a safe and accepting environment where all group members feel comfortable that they can both share their ideas and have them fairly considered, etc.

In your view, (how) can collaboration and collectivity in the mathematics classroom coexist with the culture of individual accountability in our current school system?

The short answer might be that they can coexist in mathematics classes in the same way that they co-exist in classes in other subject areas. For some activities, collaboration is very beneficial—diversity of ideas, immediate feedback, explaining one's thinking, and questioning others about their ideas; for other tasks, individual work may be most effective so a student can go deep within the self to build understanding.

The long answer might be that there are a number of pressures related to school mathematics, particularly at the secondary level, that make the idea of collaboration suspect:

- the pressure of limited time, the amount of curriculum that must be "covered," and the perception that collaboration simply takes too long;
- the perception of mathematics being a black or white subject, and that sharing ideas and possible solutions paths is a waste of time because your final answer is either right or it's wrong;
- the definition of expertise in mathematics, where student learning comes only from listening to the teacher/expert rather than working with peers to explore and wrestle with concepts;
- the difficulty of grading group performance. Who gets the marks? How does the teacher know which group members did the work and who actually understands?

While it is a challenge to assess group performance, perhaps we also need to acknowledge that it's equally artificial to set the boundaries that enable us to assess individual performance. I remember an experience at an educator forum where I was part of a group working on a probability problem, and I was feeling quite pleased during our session that I was really "getting it." Then we took a break for lunch. While we were walking down the hallway, someone in our group mentioned something about the problem and another person wondered aloud if she really had understood what we had been doing. Then I started wondering if, now that I was away from the support that the group situation had provided, I had really "gotten it" either.

> "While it is a challenge to assess group performance, it's equally artificial to set the boundaries that enable us to assess individual performance."

About a month later, I was at my school during nutrition break, and a teaching colleague asked me a probability question that one of his students had asked him. To my surprise, the question was related to that probability problem from the previous month, and to my greater surprise I was able to answer it in a manner that was clear and coherent enough that he was able to go back to his student with a satisfactory explanation. So, when would it have been most appropriate to evaluate my level of understanding—while I was working with the forum group, or individually during my lunch break (so the issue of group versus individual)? As well, there are boundaries to assessment related to time: Should I have been evaluated the day of the group session, or later, after I had been away from the problem for a month? Should I only have had one shot at being evaluated on my understanding of a certain concept, or should I have had another chance to be evaluated once I've had more time to process it?

In terms of accountability, group or individual, the issues of assessment and evaluation are complex. It has been interesting to see how the changes in assessment and evaluation practices in more recent years (formative and summative assessment, standards-based grading, etc.) have been affecting teaching practices in mathematics classrooms.

Thank you, Dr. Armstrong, for taking the time for this conversation. We look forward to your upcoming work and to continuing the discussion in the future!

Ilona Vashchyshyn

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Mathematical Modeling in the High School Curriculum [1]

Maria L. Hernández, Rachel Levy, Mathew D. Felton-Koestler, and Rose Mary Zbiek

Mathematical modeling—using mathematical approaches to understand and make decisions about real-world phenomena—"can be used to motivate curricular requirements and can highlight the importance and relevance of mathematics in answering important questions. It can also help students gain transferable skills, such as habits of mind that are pervasive across subject matter" (GAIMME 2016, p. 8). Although teachers recognize the value of engaging their students in mathematical modeling, few have had opportunities to experience modeling, and many teachers feel unsure of how to teach it.

> "Although teachers recognize the value of engaging their students in mathematical modeling, few have had opportunities to experience modeling, and many teachers feel unsure of how to teach it."

In 2015, mathematics leaders and instructors from the Society for Industrial and Applied Mathematics (SIAM) and the Consortium for Mathematics and Its Applications (COMAP), with input from NCTM, came together to write the *Guidelines for Assessment and Instruction in Mathematical Modeling Education* (GAIMME) report as a resource for teachers who want to incorporate the practice of mathematical modeling in their classrooms. The GAIMME report, which can be downloaded for free (http://www.siam.org/reports/gaimme.php), provides insight into the modeling process, what mathematical modeling looks like across the grades, the role of the teacher, and how to assess students' modeling processes.

The GAIMME report provides a sense of what mathematical modeling is and is not, as well as practical advice on how to teach modeling through the grade levels. It is consistent with but not limited to how the Common Core (CCSSI 2010) includes modeling as one of the Standards for Mathematical Practice that are meant to span kindergarten through grade 12. The GAIMME authors demonstrate the importance of mathematical modeling and how and why it should be an essential part of every student's mathematics experience throughout their education. This article, written by members of a joint committee between NCTM and SIAM, serves as an introduction to the GAIMME report and offers information on how to incorporate modeling into the high school mathematics classroom.

What is Mathematical Modeling?

Mathematical modeling has been defined in many ways. The authors of the GAIMME report define it as "a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena" (GAIMME 2016, p. 8). Of particular importance is the emphasis on modeling as a *process*. Modeling is iterative and involves frequent revision. Moreover, modeling involves messy, open-ended problems that require students to make genuine choices about how to approach problems mathematically, what assumptions to make, and how to determine the effectiveness of the approach used.

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Components of a Modeling Process

Identify the problem: We identify something in the real world we want to know, do, or understand. The result is a question in the real world.

Make assumptions and identify variables: We select "objects" that seem important in the real-world question and identify relations between them. We decide what we will keep and what we will ignore about the objects and their interrelations. The result is an idealized version of the original question.

Do the math: We translate the idealized version into mathematical terms and obtain a mathematical formulation of the idealized questions. This formulation is the model. We do the math to see what insights and results we get.

Analyze and assess the solution: We consider: Does it address the problem? Does it make sense when translated back into the real world? Are the results practical, the answers reasonable, and the consequences acceptable?

Iterate: We iterate the process as needed to refine and extend our model.

Implement the model: For real world, practical applications, we report our results to others and implement the solution.

Figure 1: *The figure shows the components of the modeling process. (Excerpts from GAIMME 2016, pp. 12-13.)*

The GAIMME report breaks the modeling process into six components, described in Figure 1. Most articles represent these steps (or some variation of them) as a cycle, but the GAIMME report intentionally uses the representation shown in Figure 2, which "reflects the fact that in practice a modeler often bounces back and forth through the various stages" (GAIMME 2016, p. 13).

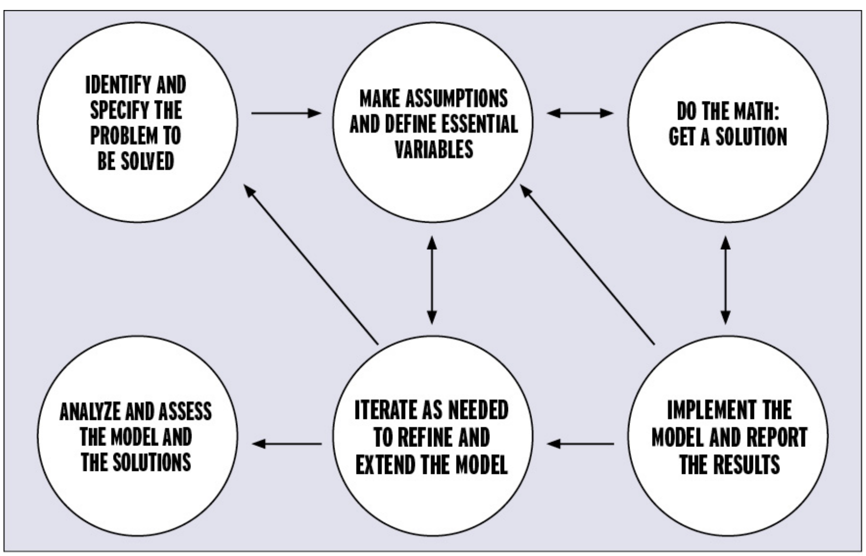

Figure 2: *The diagram illustrates the mathematical modeling process (GAIMME 2016, p. 31). Copyright © 2016 Consortium for Mathematics and Its Applications and Society for Industrial and Applied Mathematics. Reprinted with permission. All rights reserved.*

In the high school curriculum, mathematical modeling can be implemented in any course and the nature of this implementation can vary depending on the learning goals. Embedded into a subject-centered course, modeling problems provide opportunities for students to develop mathematical approaches that apply tools from that course to problems that matter to the students. They can also form the basis of a stand-alone course in modeling, which could reinforce mathematical concepts from current or previous courses. High school students also have opportunities to engage in mathematical modeling competitions, such as Moody's Mega Math Challenge (m3challenge.siam.org) or High School Mathematical Contest in Modeling (HIMCM)® (comap.com/highschool/contests/himcm/). Preparing for these can provide focus for math clubs and summer programs.

An Example: Driving for Gas

In the Driving for Gas example from the GAIMME report, we demonstrate how high school students can engage in the modeling process through a problem situated in a context that is familiar to them. The problem can be used as a classroom activity or as a practice problem for students preparing for a modeling competition. This problem can be a good first step for students who are not familiar with solving open-ended modeling problems. Figure 3 illustrates how students engage in the six

Identify the problem	The teacher selects the real-world problem. The Driving for Gas problem is situated in a context that many students are familiar with or have an interest in. It is open (at the beginning, middle, and end) and requires students to make decisions before they can tackle it mathematically.
Make assumptions and identify variables	Students consider a number of real-world factors. They may need to narrow the focus of the question to make the problem more tractable. For example, they might focus on a specific route that they travel regularly and create a solution based on that specific case. Some questions to consider might be these: What's my car's gas mileage? They may want to limit their focus to two cars with different gas mileage. How far off the route is the gas station with the cheaper gas? How much gas will we be purchasing?
Do the math	Students research the cost of gas at the gas station on the route and one location (with substantially cheaper gas) off the route. They make calculations for the specific type(s) of cars they decided to consider. They calculate the total cost of buying gas at each station. The choice of mathematical approach is ideally left to the student but may be constrained by the teacher to the techniques in a course or unit.
Analyze and assess the solution	Throughout the process, students consider if their assumptions and strategy make sense in the real-life context of the problem. Students give mathematical arguments explaining why some answers are unreasonable or not useful and support their own answer as reasonable or useful. For example, if the distance to the cheaper gas is 25 miles away, is it reasonable to think you would drive that far to get cheaper gas? They attend to the accuracy that is appropriate for the answer, such as rounding to the nearest mile or gallon of gas.
Iterate	Depending on how satisfied students are with their final product, they may go back and modify their approach. In the refinement of their model, students may consider what is meant by "cost efficient" in the original problem statement. They may have taken into account only the dollar cost of the gas and may not have considered such factors as the cost of their time or the cost to the environment of driving farther for gas.
Implement the model	One of the goals of creating mathematical models is to answer a question posed in a real-life context. After students have engaged in the process of creating a model, the model would be used to decide whether or not they should drive the extra miles to buy the cheaper gas.

Figure 3: *The figure shows the components of mathematical modeling in the Driving for Gas problem, as discussed in the GAIMME (2016) report.*

components of the modeling process for this task.

Most drivers have a 'usual' region in which they do most of their driving. However, gas prices may vary widely so that gas may be substantially cheaper somewhere other than within that usual region. Would it be more economical to go to a station outside the usual region to buy gas? Thus, the general question we wish to address is, "How might we determine which gas station is the most cost-efficient?" (GAIMME 2016, p. 181).

The GAIMME report provides a comprehensive narrative explaining how to enact the Driving for Gas problem (GAIMME 2016, Appendix C). The narrative includes various levels of scaffolding, ideas for formative assessment, and extensions, with each component of the modeling process addressed explicitly.

The Teachers' Role

Teaching modeling is challenging, especially for teachers who are new to the process. Figure 4 illustrates a framework for thinking about the role of the teacher in facilitating student modeling (Carlson et al. 2016). The dark squares in the center of the figure show a simplified version of the modeling process that students engage in: posing questions, building solutions, and validating conclusions. Surrounding this center are teachers' actions.

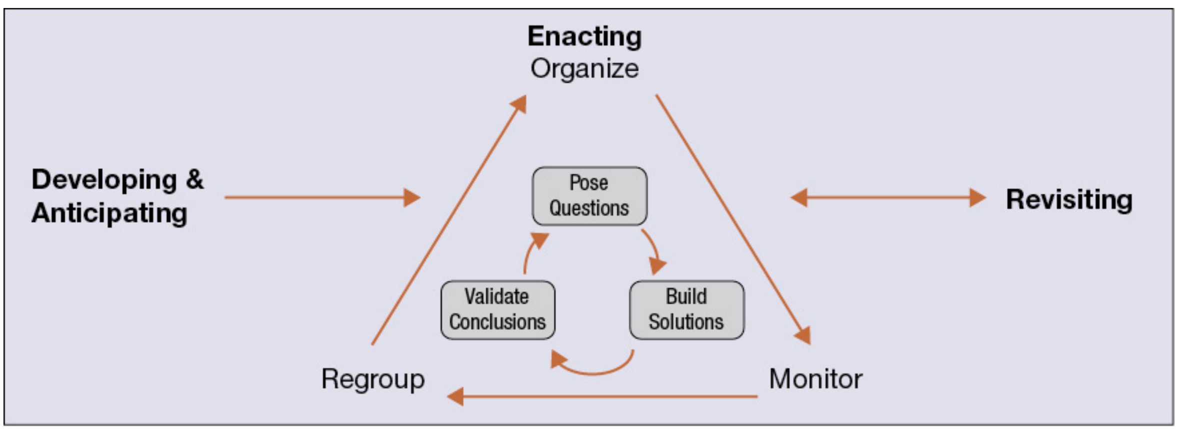

Figure 4: *The diagram illustrates the framework for teachers' roles in modeling (Carlson et al. 2016, p. 122).*

Teaching modeling begins by selecting or developing a task. There is a growing body of task resources for teachers to consider, such as the GAIMME report, NCTM's *Mathematical Modeling and Modeling Mathematics* (Hirsch 2016), COMAP's Math Models (www.mathmodels.org), and Moody's Mega Math Challenge (www.m3challenge.siam.org). In creating or selecting a task, the most important question to consider is whether the task requires students to make decisions about how to approach the problem mathematically. Teachers also consider how familiar students are with the context and the mathematical concepts they hope students will use on the task. They anticipate how the task might play out. Teachers might ask themselves the following questions:

- What kinds of questions will the students have about the context?
- What additional information will they need or want?

- How will they get that additional information?
- What assumptions will they make as they begin to build their models?
- How can I help students feel comfortable about making assumptions?
- What kinds of problem-solving strategies are students likely to use?
- How do I want to balance small-group and whole-class discussions?
- At what point in the modeling process are students likely to get stuck?
- What kinds of strategies can I use to intervene without taking over the modeling process?
- What tools will students use to analyze their solutions and assess their models?

After developing and anticipating, the teacher moves into enacting the modeling task. Just as modeling is an iterative process for the students, supporting student modeling is an iterative process for the teacher. Teachers must first organize the presentation of the task. This involves introducing the context and the problem and allowing students time to ask clarifying questions to ensure they understand what they are being asked to do. Depending on their students' familiarity with modeling and the teachers' instructional goals, teachers may ask students to brainstorm initial ideas on how to approach the task and to discuss the need to use mathematics to analyze the problem.

Next, as students begin working, teachers monitor their work by taking note of the strategies used, the assumptions students are making, the mathematical opportunities that arise, and the places where students are getting stuck. As students progress, the teacher can occasionally regroup by bringing the whole class together. This regrouping can be used for a variety of purposes, depending on where students are in the modeling process. A teacher can use regrouping to address misconceptions, answer clarifying questions, provide small suggestions, and have students share their progress and receive peer feedback. More frequent regrouping may lead to greater similarity in approaches across groups, while less whole-class communication can lead to a greater range of mathematical models.

As students draw closer to their final solutions, teachers should encourage them to analyze their solutions and assess their models. This stage can include asking students to make sense of the answers in the context of the problem, to find ways to measure the accuracy of their solutions, or to explore how their solutions change if they vary their assumptions.

The teacher's role in revisiting can serve to support students in several ways. The teacher might summarize the major mathematical ideas that students used in their solutions. This step can also serve as an opportunity to unpack and discuss the modeling process itself. Teachers might ask students to reflect on the modeling process and comment on strategies that helped them succeed in finding their solutions. Revisiting is an ideal opportunity for discussing how the problem could be changed or extended and whether the students' solutions are still viable in these new situations.

The authors of the GAIMME report suggest the following guiding principles to aid teachers as they consider how to introduce mathematical modeling into their classrooms:

- "Start small
- Scaffold initial experiences with leading questions and class discussion
- Use common, everyday experiences to motivate the use of mathematics

- Use bite-sized modeling scenarios that require only one or two components of a full modeling cycle
- Share your goals and instructional practices with parents and administrators" (GAIMME 2016, p. 59)

Developing tasks that draw on student interests gives teachers an ideal opportunity for integrating students' lives into the curriculum. For example, some high school students may be interested in social justice or sustainability issues. By having a say in choosing their topics of study in a math classroom, students can become more invested in their own learning while seeing the relevance of mathematics in their lives.

Assessment Questions

Before thinking about assessing students as modelers, teachers may consider the learning goals for their students. These goals can be focused on how students implement the modeling process (or components of the process), how they communicate their findings, or how they work as team members. The GAIMME report includes suggestions for learning goals and means for assessing each of those goals. Figure 5 offers some guiding questions for teachers who wonder, "How can I tell if my students are engaged in the modeling process?"

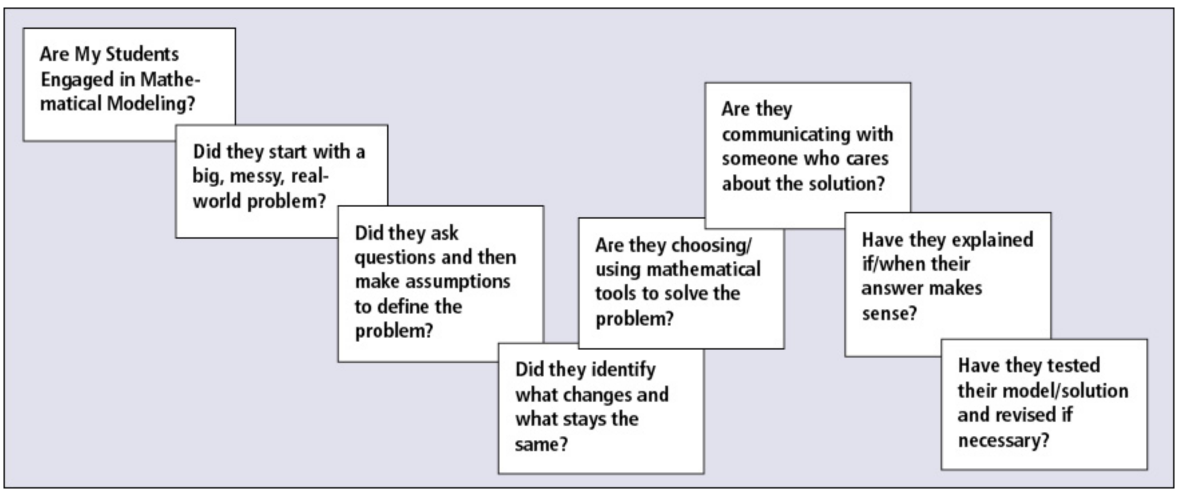

Figure 5: *The figure provides questions to assess whether students are modeling (NCTM 2016, p. 256).*

As students are given more opportunities to engage in the math modeling process, we hope that they become better mathematical modelers. The open-ended nature of the tasks can present a significant challenge when grading student work. The solutions can vary because students are free to make different assumptions and choose from a variety of tools to create models and solve problems. The main idea to keep in mind is that "assessment should focus on the process and not on the product or pieces only" (GAIMME 2016, p. 47).

The GAIMME report provides questions designed for formative assessment of student engagement in math modeling for each component of the modeling process. (See Figure 6 for samples.)

MODELING COMPONENT	QUESTIONS ABOUT YOUR MODEL AND HOW YOU MADE IT	MODELING-RELATED VOCABULARY TO BUILD
DEFINING THE PROBLEM	What is the specific problem your model is going to solve? (My model will tell you . . .)	specific, focus
MAKING ASSUMPTIONS	What have you assumed in order to solve the problem? Why did you make these choices?	assumption, assumed
DEFINING VARIABLES	Where did you find the numbers that you used in your model?	resources, citations
GETTING A SOLUTION	What pictures, diagrams or graphs might help people understand your information, model, and results?	diagram, graph, labels
ANALYSIS AND MODEL ASSESSSMENT	How do you know you have a good/useful model? Why does your model make sense?	testing, validation
REPORTING RESULTS	What are the 5 most important things for your audience/ client to understand about your model and/or solution?	client, audience

Figure 6: *The table shows a portion of GAIMME's "Modeling Assessment Rubric." (Excerpt from GAIMME 2016, p. 197; adapted from Rachel Levy, IMMERSION program.)*

Students engaged in mathematical modeling tasks can share their work in a variety of ways. Rubrics offered in the GAIMME report are designed to articulate expectations and standards for success on products that range from informal presentations of approaches and solutions to posters, or more formal written reports. It is important to remember that the goal of the sharing is for students to communicate their mathematical thinking in a clear fashion, making sense of both the mathematics and the context. As mathematician Henry O. Pollak notes,

> Mathematicians are in the habit of dividing the world into two parts: mathematics and everything else, sometimes called the 'real world'. People often tend to see these two as independent of one another—nothing could be further from the truth. When you use mathematics to understand a situation in the real world, and then perhaps use it to take action or even to predict the future, both the real world situation and the ensuing mathematics are taken seriously. (GAIMME 2016, p. 95)

Students "Remember the Math"

The ideas in this article and in the GAIMME report help to answer the questions "What is mathematical modeling?" and "How can we begin to incorporate mathematical modeling in high school classrooms?" Students who have engaged in the modeling process appreciate the opportunity to use their own ideas in creating a mathematical solution to a real-world problem and have experiences that help them regardless of what college or career path they follow. When asked to reflect on her experience during a math-modeling task, one precalculus student wrote,

> It [modeling] helps me remember the math, because then I have some kind of example that can help me think through a problem logically and relate it to something that I know about outside of the classroom. I feel like I can apply this method to a lot of things outside of math, like sciences and literature and history.

Modeling is not for science only—it transcends disciplines and affords tools for students to engage with real problems in their community and in society. If you have not yet tried modeling with your students, we hope the GAIMME report will help you "Start big. Start small. Just start" (GAIMME 2016, p. 92).

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Learning from My Students: What They Are Saying About Problem-Based Learning

Christopher Tsang

It's the first day of school, and the first time I get to meet and work with the two groups of Grade 9 students. Inspired by Peter Liljedahl's thinking classroom model (see Liljedahl, 2016), I assign each student a number and form random groups as class begins. In this model, the students are placed in visibly random groups to work collaboratively on problem-solving tasks using vertical non-permanent surfaces. My purpose as a teacher is to facilitate and encourage mathematical discourse. As such, I instruct the students to introduce themselves to their group members, after which a problem is posed:

> There is 1 gorilla, 2 monkeys, and 3 ape trainers who want to cross a river, but there is only 1 boat and it can only fit at most 2 at a time. The trainers can paddle and so can the gorilla, who has been trained to paddle. However, the trainers will be attacked at any point they are outnumbered by the apes. How can all the apes and trainers cross the river safely? (P. Liljedahl, personal communication, September 2013)

Students begin talking about the problem at their tables, and I offer manipulatives for them to work through the problem. The room is noisy, but full of positive energy. There is a sense of excitement when a group of students think they have solved the problem… until I point out misconceptions in their solution. And yet, they are not deterred from jumping right back in. When groups appear to have exhausted their strategies, I give them a small nudge, a clue, or a direction to reignite their process. Students continue for almost 45 minutes until one group shouts with excitement after having found a solution, which in turn motivates others to refocus their attention, now knowing that the problem is solvable. Soon, all groups catch onto the solution, and in the end, we talk as a class about their strategies.

> "A group of students think they have solved the problem... until I point out misconceptions in their solution. And yet, they are not deterred from jumping right back in."

These kinds of problem-solving tasks were interspersed throughout the term in the two classes, alongside the teaching of curricular content. While some tasks involved the use of manipulatives, most tasks involved groups working collaboratively on vertical surfaces, such as the problem below:

> Consider the following pattern of 5 whole numbers, where each number is the sum of the previous two numbers:
>
> 3, 12, 15, 27, 42
>
> I want the 5th number to be 100. Find all the whole seed numbers that will make this so (3 and 12 are the seed numbers in the above sequence). (P. Liljedahl, personal communication, September 2013)

The common theme among all of the chosen problems was that they gave students an opportunity to engage in problem solving through collaboration and communication, which is one of the mathematical processes named in the British Columbia mathematics curriculum (Western and Northern Canadian Protocol, 2008). This provided motivation for me to incorporate these problem-solving tasks regularly in my classroom, but I felt guilty about taking curricular time away from students if these tasks were 'just for fun.' To justify the allocation of valuable class time on these tasks, as well as providing a more balanced assessment of their mathematical skills and understanding, I decided to include term portfolios for students to highlight 'portfolio problems' of their choice and the creative processes used in solving them.

> "The implementation of problem-based learning requires a safe environment where students can express their thinking and where mistakes are viewed as learning opportunities."

Perhaps motivated by its success in the medical field, educational reform is trending towards problem-based learning. According to Erickson, students in a problem-based learning setting are expected to "solve problems or make sense of mathematical situations for which no-well defined routines or procedures exist" (1999, p. 516). Its implementation requires a safe environment where students can express their thinking and where mistakes are viewed as learning opportunities. The classroom must also have an atmosphere of mutual respect, where clarification and justification are expected (Erickson, 1999).

However, the truth is that with limited resources, I feel ill-prepared for a full implementation of problem-based learning as a medium to meet the prescribed learning outcomes (PLOs). As a consequence, I refer to these non-curricular problem-solving tasks as portfolio problems, liberating me of curricular associations. They are not intended to cover specific PLOs, although from time to time they might. I have also incorporated curricular problems in collaborative group work, but these are excluded from portfolios since curricular topics often have a very specific outcome that may be revealed to students if they simply look ahead in the current unit in their textbooks.

A Reflection on Student Responses

In an empirical study, researchers Capon and Kuhn observed that "students who experienced problem-based instruction more often were able to integrate newly acquired concepts with existing knowledge structures that had been activated" (2004, p. 74). In other words, students were better prepared to make connections between new concepts with ones that had been learned previously. Although a literature review (Wilder, 2014) reported that current research lacks the significant and rigorous evidence that problem-based learning is superior to traditional methods of instruction in terms of content knowledge, there is evidence that problem-based learning develops a wide range of desired skills, such as communication, collaboration, decision-making, problem-solving, and critical-thinking skills, in addition to encouraging self-directed learning.

However, although research suggests that there are benefits to problem-based learning, I wanted to know directly from my students what they thought about these non-curricular portfolio problems. What were the students getting out of the portfolio problems? Did the students perceive benefits similar to those that the research on problem-based learning suggests? What benefits did they perceive outside of their curricular content knowledge?

To answer these questions, I gathered qualitative data from my two Grade 9 classes in the form of journal entries. The journal entries were collected at irregular intervals throughout the course, and commenced at the start of the school year. Students' age varied between 1 and 15 years of age. The students were situated in a classroom with ample whiteboard and window space for a class of 30 students to work. Desks were stationed as work groups that ranged from pods of 4 desks to pods of 8.

The students were grouped randomly at the beginning of the year and eventually transitioned to self-appointed groups of 3-4 students for each collaborative task. Most problems involved vertical surfaces for collaborative group work, but some problems were aided by manipulatives, such as linking cubes or playing cards. The problems chosen were intended to be challenging, yet provide multiple access points for a class of students with mixed abilities. Throughout their collaborative problem-solving work, I observed and moderated their progress as needed. When groups had appeared to have achieved a solution to their problems, the class debriefed by highlighting and sharing student work from their whiteboards. All classes were involved in the development of a common assessment rubric for their problem-solving portfolio (see Figure 1). At the end of each term, students compiled a collection of at least three problems they wished to highlight as evidence of their use of mathematical processes.

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Portfolio Rubric

• **Process**: Student places too much emphasis on the final solution, and not the problem solving process. • **Mathematical Validity**: the logic strategies used in solving the problem is flawed and was unable to achieve a result. • **Organisation**: student is unclear in communicating his/her ideas. • **Thoughtfulness**: little effort was made in developing ideas, or does not communicate reason(s) for his/her choice of portfolio problems.	• **Process**: Student describes problem solving strategies from the start to the final solution, but some aspects of the process is unclear/omitted. • **Mathematical Validity**: the logic strategies used in solving the problem is mostly effective and gives a final result. Mathematical errors exist, but not all errors were identified by the student. • **Organisation**: student somewhat clearly communicates his/her ideas *using words and/or diagrams.* • **Thoughtfulness**: student demonstrates effort but ideas could be developed further. Student also communicates reason(s) for his/her choice of portfolio problems.	• **Process**: Student clearly describes problem solving strategies from the start to the final solution. • **Mathematical Validity**: the logic strategies used in solving the problem is effective and gives a final result. There may be mathematical errors, but student identifies and corrects these errors. • **Organisation**: student clearly communicates his/her ideas *using words and/or diagrams.* • **Thoughtfulness**: student clearly demonstrates effort (fully developed ideas), and clearly communicates reason(s) for his/her choice of portfolio problems.

Figure 1

The question "What is different from completing portfolios and/or portfolio problems in contrast to textbook problems?" was posed in the second term of the school year. At this time, students had completed a full cycle of collaborative, non-curricular problem-based learning along with portfolio submission and assessment.

Below is an analysis of student responses (names have been changed to protect their identity) that provided interesting insights into perceived outcomes of the portfolio problems.

Janice and John

Janice is a strong mathematics student who has a modified schedule, being a member of the Sports and Performing Arts program in the school. She describes her previous experience in mathematics as "easy," and would be qualified to be enrolled in the honours mathematics course if her schedule permitted it. John, on the other hand, was observed to be a quiet participant in problem-solving tasks. He does not appear to have developed close social connections with his classmates. In a few instances, he appeared not to be participating in the problem-solving tasks, but further questioning revealed that he was thinking and had insight that would be beneficial to the group.

Janice: I think working on portfolio problems really teaches us how to cooperatively work on a problem with people who we don't necessarily know very well. In textbook questions, I feel like I have less self-motivation to actually complete the problems than I do during portfolio problems. Portfolio problems also teach me how to apply math skills I've learned to "real life" situations, whereas textbook problems, the concept seems sort of forced on us. It's also refreshing to be up and writing it out rather than confined to your own notebook. I think that the most important thing that I've learned during the portfolio problems is how to communicate my thoughts and ideas to others in a way that others will understand.

John: I believe that portfolio problems are much more important to your education compared to textbook work. This is because the portfolio questions better simulate the types of problems that you will encounter at your job, and in life in general.

People spend a much larger percentage of their life dealing with physical pandemics rather than pen and paper procedures. At your job, you are never going to have to complete a math worksheet, but you will have portfolio simulated problems very often.

In conclusion, I believe that portfolio problems are much more beneficial [*sic*] than textbook work for your preparation [*sic*] for the general future.

As Janice's response suggests, she appears to have embraced the collaborative problem-based learning environment, citing increased engagement, learning to cooperate and communicate with peers, and the opportunity to apply her mathematical knowledge to "real-life" problem-solving tasks.

Both Janice and John's responses referred to the authenticity of problem solving. Although the context of the problems posed had little to do with the real world, the students may be perceiving the open-ended nature, critical thinking, or the social experience of problem solving as being authentic. Even though it was difficult to observe John's level of engagement, his response indicates that even though he was relatively quiet during the problem-solving tasks, he was participating, albeit not collaboratively.

> "Students may be perceiving the open-ended nature, critical thinking, or the social experience of problem solving as being authentic."

Amy, Jackson, and Fiona

The following students are of average or above-average ability. All of these students had been observed as being actively engaged in the portfolio problems.

Amy: Portfolio problems involve more interaction, and lets [*sic*] you think in more detail. Textbook problems are more closed - you do them on your own, and there is typically with [*sic*] only one answer. Portfolio problems have different approaches, and multiple answers. Although, this can put up more of a challenge, as there might be multiple answers! I personally enjoy portfolio problems more, because of these factors.

Jackson: I am learning the fundamentals of math from the textbook problems because it doesn't get creative enough to make me think in different ways. Textbook questions are also not as modern as portfolio questions and that's why I like portfolio questions more because I can connect to it more. Portfolio questions are always fun because you can get as creative as you can get. You always get the full idea of the question because you can ask the "questioner" all the time.

Fiona: Textbook questions are so repetative [*sic*] and boring. They are routine and you are always testing a formula that we are learning in class. With portfolio problems, we go blind. We don't know what we are supposed to be using, wich [*sic*] makes the problem so much more interesting. I enjoy portfolio problems over textbook, but both are important to do.

When comparing textbook questions to portfolio questions, Amy and Jackson wrote that their enjoyment in working with portfolio problems stems from the open-endedness of the problems and the opportunity to think creatively. This contrasts with their view that textbook questions are closed-ended and repetitive.

> "Amy and Jackson wrote that their enjoyment in working with portfolio problems stems from the open-endedness of the problems and the opportunity to think creatively."

Jackson identifies one significant benefit of textbook work as addressing the fundamentals of mathematics. This might be expected, since mathematical texts provide a pedagogical structure that organizes each unit by topic while providing a linear sequence of concepts to be learned. One limitation of non-curricular portfolio problems is the lack of structure in mathematical content. The ill-structured nature of portfolio problems also affects my ability as a teacher to confidently cover all of the prescribed learning outcomes. Interestingly, Fiona wrote that the lack of structure in the portfolio problems is the reason she enjoys these problems more than textbook work.

Josh

Josh is a student identified as having a learning disability. His first journal entry appears to indicate that he perceived mathematics as being associated with procedures: In response to the question 'What is mathematics to you?', he wrote that "math is also just formulas that you need to solve." He also wrote that, based on his prior experiences, mathematics is not a subject of particular interest to him, and any enjoyment in the subject appears to have been loosely connected to understanding how to solve a problem.

Josh: I honestly like doing portfolio questions much better than textbook questions because it is group work. Since I have some trouble with math, I like interacting with others to see how they got their answer and I compare them to my answer that I got. I really like that much better. You can ask people for help and you collaborate with others when trying to do a problem. This makes math more fun.

When I am working out of a textbook at home by myself, it can get frustrating when I don't understand exactly what I am doing. Sure there are answers in the back of the book to check, but they are almost no use when you don't understand the question. I often have questions, so I do get frustrated often.

Portfolio questions can be more work than the homework because near the end of the term you must put together your actual portfolio and when you go back and trace your steps and process. Even though you have to do this, I still like being in groups.

Josh's perceived benefit of collaboration is different from Janice's. Being a student who has struggled when working individually, collaboration allows Josh to cross-reference his own thought processes with those of others in his group. While Janice benefits by learning how to communicate her ideas, Josh benefits from the opportunity to learn from peers. Both students expressed that the experience of working on mathematics in collaborative groups was positive.

> "While Janice benefits by learning how to communicate her ideas, Josh benefits from the opportunity to learn from peers."

Cheryl

The final analysis is from Cheryl, a student who teachers might identify as having below-average math ability. One of her earlier journal responses to the prompt 'Share your prior experiences with mathematics' indicated that she learns mathematics by memorizing procedures to solve problems:

Cheryl: Last year in math, I didn't do so well. The material was really hard for me. The area I have the most trouble with is word problems. I find it easier when I am just given an equation and told the steps to solve it. I really want to improve on my math skills this year and I'll try my best to study lots.

Here is Cheryl's response to the prompt 'What is different from completing portfolios and/or portfolio problems in contrast to textbook problems?', given later in the year:

Cheryl: Portfolio problems are always challenging and I think we should do those after all the teaching for the unit is done. Textbook questions help me a lot because they are simple and show me a simpler way of the big picture. They are straight forward and usually just put in equations so I prefer that.

Cheryl's suggestion to do portfolio problems at the end of the unit indicates that her perception of mathematics as a sequence of procedures has not changed. Even though the portfolio problems are not directly related to the curricular topics taught in class, she identified her difficulty in working through them as not being given the procedures to do so. Although Josh also struggles with mathematics, he approaches learning with a growth mindset and a willingness to create understanding for himself. In contrast, Cheryl approaches mathematics with a fixed mindset. Dweck, Walton, and Cohen describe students with "fixed mindset believe that their intellectual ability is a limited entity, and they then to worry about proving rather than improving" (2014, p. 5). To help students like Cheryl, intervention strategies may include promoting a strong supportive classroom community, offering hints of strategies that might help the student re-engage in problem solving, and re-emphasising the value of learning rather than the value of the final outcome.

Implications

The feedback from students on non-curricular problem-based learning has been overwhelmingly positive. Students referred to opportunities to communicate and collaborate, the open-ended nature of the problems, opportunities for creative thought, and the authentic processes emerging from the open-ended problems as reasons for enjoying working with portfolio problems. Only one student response expressed a preference for learning procedures before attempting portfolio problems.

With the trend towards problem-based learning, I consider this to be my gradual entry as I prepare for the upcoming changes in the teaching and learning of mathematics. Although the portfolio problems were juxtaposed with teaching particular curricular concepts, I also made deliberate efforts to pose problems directly related to curricular topics, typically as a "launching point" into these topics. By letting the students explore and discover the mathematics *before* I formally teach and introduce a new concept, I believe students are taking part in the authentic and creative processes involved in thinking through a problem. However, I do acknowledge that there is a contradiction in this message: namely, how can a student discover mathematics if there is already an intended outcome that I want them to achieve? I think this is an important tension to acknowledge and something to consider if you, too, are interested in experimenting with problem-based learning.

> "Without doubt, mathematical texts offer a convenient educational program for both teachers and students, but perhaps we should not be so strict in following their path."

Of course, as we move towards problem-based learning in mathematics, this is only one among many other questions to consider, which include: How would this approach change the way we interact with mathematical texts? What other resources could support this pedagogical approach to learning? Without a doubt, mathematical texts offer a convenient educational program for both teachers and students, but perhaps we should not be so strict in following their path. In the end, I strongly believe that pedagogy is a dynamic process, and the more we teachers experiment, the more we learn about our students through their mathematical discourse.

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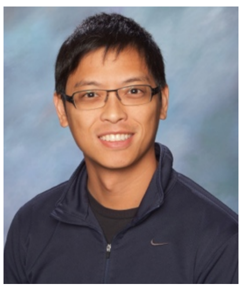


Christopher Tsang is currently and has been teaching secondary mathematics in the Vancouver school district since 2004. He is passionate about creating a collaborative and supportive learning environment where students share and learn about each other's understanding. Christopher holds a Masters degree in Secondary Mathematics Education from Simon Fraser University. His greatest gifts are his two beautiful children and a supportive and loving wife.

Encouraging Mathematical Habits of Mind: Puzzles and Games for the Classroom

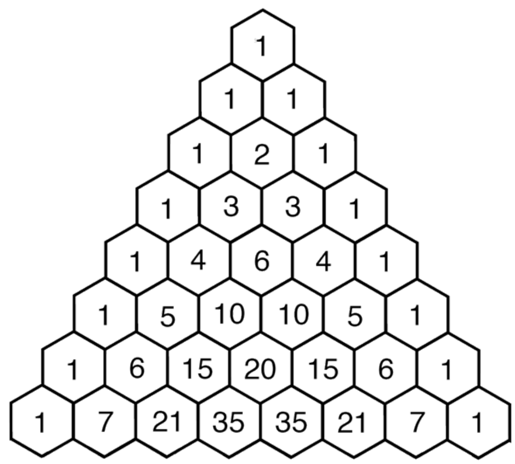
Difference Triangles

Susan Milner
Professor Emerita, Department of Mathematics & Statistics, University of the Fraser Valley, British Columbia
www.susansmathgames.ca

Here is a puzzle that can be played by anyone who has learned about subtraction. It can be introduced quickly and played when you have a few minutes to spare. Solving difference triangles requires a certain amount of trial and error, which can be less demanding than puzzles that rely on pure logic, so it is a good activity for times when students have been working hard and need to relax somewhat while still doing mathematics.

Many people are familiar with Pascal's triangle, which is based on *addition*. It has a long history, predating Pascal, and can be used for playing with number patterns, for working with combinations in probability, and for finding coefficients in the expansion of the binomial expression $(x+y)^n$. It is completely determined – creating the triangle is, frankly, not very exciting.

Subtraction, on the other hand, can be rather more entertaining! I like to make this triangle of numbers upside down, with the point at the bottom. The difference between each pair of numbers produces the number diagonally below the pair. To make a puzzle, we have one further rule: we may use each number only once.

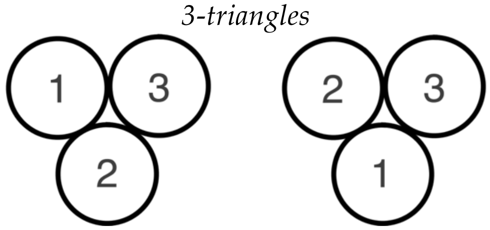


Pascal's triangle

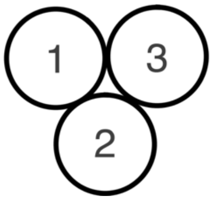
The edges of the triangle are all 1s. To find any other number, we just add the two numbers that are diagonally immediately above it.

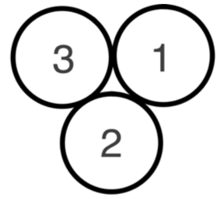
I usually introduce difference triangles by putting the two 3-triangles on the board, filling in one and asking what students notice. It's hard to notice much with such a limited example, so I explain the rules and ask for suggestions for the second 3-triangle.

3-triangles



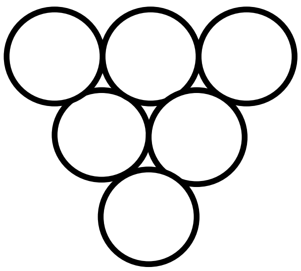
Is it possible to have more than two of these? This is a good place to point out that "difference" has no order: the difference between 2 and 3 is the same as the difference between 3 and 2.

For example, 

is the same difference triangle as 

Once everyone is clear on the rules of the game, I draw the 6-triangle and turn the students loose. It doesn't take long for even Grade 2s and 3s to start coming up with answers.

6-triangle



Here, we need to use the numbers 1-6, each number only once.

How many unique 6-triangles are possible? (Four, but don't tell the students ahead of time! Encourage them to find as many as they can.)

This may be all you have time for, if the activity has been shoehorned into the end of a class period. Further analysis and bigger triangles can be saved for another time when you have 10 minutes to spare, or you might suggest that students try to find a 10-triangle and report back to the class in a few days. (These can easily be found online, so if you want to be sure that students are doing their own thinking, it might be best to keep the puzzles as an in-class activity.)

Thinking about the puzzle
If you try solving a 6-triangle yourself, you'll find that, while some trial and error is necessary, you can cut down on the number of trials with a little reasoning. You can also do some exploration by changing the original rules. The following are great discussions to have with your students, once they've had a chance to play with the puzzle:

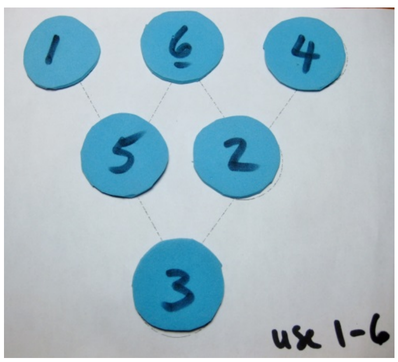
- Are there any numbers that *must* go in the top row?
- Are there any numbers that *cannot* go in the top row?
- Which *pairs* of numbers cannot go in the same row?
- Can you think of any other strategies?
- Could we make a puzzle using only even numbers? Only odd numbers? Only multiples of 5?
- What about if we started at the bottom point and worked our way up?
- What would happen if we started at the bottom and allowed ourselves to use any numbers we liked, as long as there was no repetition?

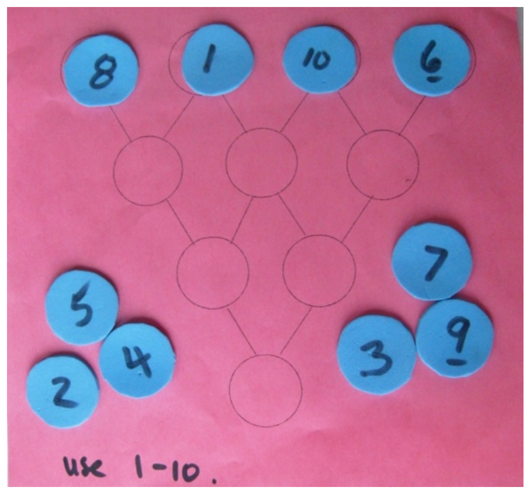
It is interesting to note that a solution is completely given simply by listing the numbers in the top row.

Finding all of the 6-triangles is quite straightforward, because there just aren't that many possible ways to fill in the top row. Finding 10-triangles is rather more difficult. Finding even one 15-triangle is very hard indeed!

Materials
All we really need for this activity is scrap paper, a pencil, and an eraser, but it can involve a lot of erasing. Personal whiteboards or slates might make the process more enjoyable for some students. If your classroom is set up with vertical surfaces, then the activity could be turned into a longer problem-solving session for small groups.

I really like to use manipulatives, which tend to bring the "game" aspect to the fore, making it very easy to try out possibilities. It takes a bit of time to print off the triangle templates[2] and to create the counters, but I find it worthwhile to make the counters as they can be used for a variety of other games. While mine are round, it would be quicker to cut squares, either out of cardstock or craft foam sheets.





And that's it for now. I'd love to hear from you about how your students responded to difference triangles! There is a "contact me" form on my website, www.susansmathgames.ca

Susan Milner taught post-secondary mathematics in British Columbia for 29 years. For 11 years, she organized the University of the Fraser Valley's secondary math contest – her favourite part was coming up with post-contest activities for the participants. In 2009 she started Math Mania evenings for local youngsters, parents, and teachers. This was so much fun that she devoted her sabbatical year to adapting math/logic puzzles and taking them into K-12 classrooms. Now retired and living in Nelson, BC, she is still busy travelling to classrooms and giving professional development workshops. In 2014 she was awarded the Pacific Institute for the Mathematical Sciences (PIMS) Education Prize.

[2] Here are templates for 6-triangles and 10-triangles. Alternatively, you can go to my website and look under the heading "Other games," then "Arithmetic & algebra".

Intersections

In this column, you'll find information about upcoming math education-related workshops, conferences, and other events. Some events fill up fast, so don't delay signing up!

For more information about a particular event or to register, follow the link provided below the description. If you know about an event that should be on our list, please contact us at thevariable@smts.ca.

Within Saskatchewan

Workshops

Using Tasks in Middle Years Mathematics
March 13, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

Using tasks in a middle years mathematics classroom can provide rich opportunities for differentiated learning and authentic assessment.

How do we choose tasks that meet both curricular outcomes and student needs? Tasks allow students to enter mathematics where they are at and extend their learning. In this workshop we will look at a variety of resources for finding good middle years tasks. We will also reflect and discuss what planning and teaching moves can assist in maximizing student learning through mathematics tasks.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/using-tasks-middle-years-mathematics

Structures for Differentiating Middle Years Mathematics
March 21, Tisdale, SK
Presented by the Saskatchewan Professional Development Unit

We know that assessing where students are at in mathematics is essential, but what do we do when we know what they don't know? What do we do when they DO know? Understanding does not change unless there is an instructional response to what we know from that assessment. The question we ask ourselves is how we might respond to individual needs without having to create completely individualized mathematics programs in our classrooms.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/structures-differentiating-middle-years-1

Beyond the Worksheet: Building Early Numeracy and Automaticity Through Exploration and Authentic Tasks
April 26, Saskatoon, SK
Presented by the Saskatchewan Professional Development Unit

This workshop will focus on the teaching and learning of early number concepts, including place value, addition, subtraction and early multiplication. Participants will consider ways to have their students explore these number concepts in hands-on, meaningful learning experiences that allow them to construct their own understanding. Along with ideas related to content and curriculum, participants will problem solve around structures and transitions amongst whole group, small group and individual learning experiences, allowing for authentic differentiation and rich classroom conversations.

See https://www.stf.sk.ca/professional-resources/professional-growth/events-calendar/beyond-worksheet-building-early-numerac-0

Conferences

Saskatchewan Understands Math (SUM) Conference

October 23-34, Saskatoon, SK
Presented by the Saskatchewan Mathematics Teachers' Society (SMTS), the Saskatchewan Educational Leadership Unit (SELU), and the Saskatchewan Professional Development Unit (SPDU)

This year, the Saskatchewan Mathematics Teachers' Society, the Saskatchewan Educational Leadership Unit and the Saskatchewan Professional Development Unit are partnering to co-ordinate a province-wide conference to explore and exchange ideas and practices about the teaching and learning of mathematics. The Saskatchewan Understands Math (SUM) conference is for mathematics educators teaching in Grades K-12 and all levels of educational leadership who support curriculum, instruction, number sense, problem-solving, culturally responsive teaching, and technology integration, and will bring together international and local facilitators to work in meaningful ways with participants in a variety of formats. This year, SUM is featuring keynote speakers Steve Leinwand of the American Institutes for Research and Lisa Lunney-Borden of St. Francis Xavier University. See the poster on page 3, and head to our website for more information.

Beyond Saskatchewan

NCTM Annual Meeting and Exposition
April 5-8, 2017, San Antonio, TX
Presented by the National Council of Teachers of Mathematics

Join more than 9,000 of your mathematics education peers at the premier math education event of the year! NCTM's Annual Meeting & Exposition is a great opportunity to expand both your local and national networks and can help you find the information you need to help prepare your pre-K–Grade 12 students for college and career success. Classroom teachers, administrators, math coaches, supervisors, college professors, and preservice teachers can all benefit from the sessions and learning at this event. Improve your knowledge and skills with high quality professional development and hands on activities; gain insights by connecting and sharing with like-minded educators; collect free activities that will keep students engaged and excited to learn; and learn from industry leaders and test the latest educational resources.

See http://www.nctm.org/Conferences-and-Professional-Development/Annual-Meeting-and-Exposition/

OAME Annual Conference: Now for Something Completely Different
May 11-13, Kinston, ON
Presented by the Ontario Association for Mathematics Education

This year's keynote speakers are Dan Meyer, well-known for his work integrating multimedia into an inquiry-based math curriculum, and Gail Vaz Oxlade, host of the Canadian television series *Til Debt Do Us Part*, *Princess* and, most recently, *Money Moron.* Featured speakers are George Gadanidis, Marian Small, Ruth Beatty, and Cathy Bruce.

See http://oame2017.weebly.com/; follow @oame2017 on Twitter for updates.

Whistler Conference
May 12, Whistler, BC
Presented by the BCAMT, myPITA, and the BCSSTA

The British Columbia Association of Mathematics Teachers (BCAMT), along with myPITA (The Provincial Intermediate and Middle Years Teachers' Association) and the BCSSTA (BC Social Studies Teachers Association), are proud to announce the 2017 Whistler Conference to be held May 12 in Whistler. This year's keynote speakers are Adrienne Gear, Amy Burvall, and Dr. Peter Liljedahl.

See http://www.bcamt.ca/2017-whistler-conference/

Online Workshops

Education Week Math Webinars
Presented by Education Week

Once a month, Education Weekly has a webinar focusing on math. They also host their previous webinars on this site. Previous webinars include Formative Assessment, Dynamic vs. Static Assessment, Productive Struggling and Differentiation.

See http://www.edweek.org/ew/marketplace/webinars/webinars.html

Did you know that the Saskatchewan Mathematics Teachers' Society is a **National Council of Teachers of Mathematics Affiliate**? When registering for an NCTM membership, be sure to support the SMTS by noting your affiliation during registration.

NCTM | AFFILIATE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Tangents

Extracurricular Opportunities for K-12 Students

This column highlights local and national extracurricular opportunities for K-12 students interested in mathematics, including collaborative, individual, online, and in-person challenges, contests, and camps. For dates, registration procedures and applications, and other information about the contests listed, please head to the contest websites, included below the descriptions. If we have missed an event that should be on our list, please contact us at thevariable@smts.ca.

If you are looking for contests available in other provinces, head to the Canadian Mathematical Society website (cms.math.ca/Competitions/othercanadian). The CMS also maintains a list of resources for students who are looking to build their problem-solving skills and succeed in competitions: see cms.math.ca/Competitions/problemsolving.

The following events are hosted locally:

University of Regina Regional Math Camp
March 11, 2017

The Math Camp is a full day event for students in Grades 1 through 12 who are interested in exploring the infinite frontier of mathematics beyond the school curriculum. Participants are guided by professors and students through a fun and enriching day. In a variety of grade appropriate sessions, students will explore mathematical topics in hands-on activities, games, puzzles, and more.

See https://www.uregina.ca/science/mathstat/community-outreach/mathcamp/

Extreme Math Challenge
March 18, 2017

The Extreme Math Challenge is a math competition for students in Grades 7 to 10 consisting of three rounds: individual, team, and team relay. The event will challenge students mathematically and encourage them to work together to solve math problems. Oh, and there will be prizes, too!

The contest will be held at Centennial Collegiate in Saskatoon. Register teams of 3-5 students by Tuesday, March 14 at 6 pm; pay $5 per student on the day of the competition to cover the cost of the pizza lunch. Contact MilnerC@spsd.sk.ca for more details.

Canadian Math Kangaroo Contest
March 26, 2017

The purpose of this competition is to introduce youngsters from Grade 1 to Grade 12 to math challenges in a fun and enjoyable way, thus inspiring their further interest and advancement in mathematics. The competition is held yearly in more than 40 Canadian cities. Students may choose to participate in English or in French.

Students in Saskatoon may write the contest at Walter Murray Collegiate; students in Regina may write the contest at the University of Regina. Contact Janet Christ at christj@spsd.sk.ca (Saskatoon) or Patrick Maidorn at patrick.maidorn@uregina.ca (Regina).

See https://kangaroo.math.ca/index.php?lang=en

Students may also compete in their schools in the following national contests:

Team Mathematics Contest
April 2017
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)
The Canadian Team Mathematics Contest (CTMC) is a fun and challenging competition for teams of 6 secondary school students in any combination of grades. One teacher and groups of six students participate at their own school over 3 consecutive hours.

See http://www.cemc.uwaterloo.ca/contests/ctmc.html

Caribou Mathematics Competition
April, May 2017

The Caribou Mathematics Competition is a worldwide online contest that is held six times throughout the school year. Each of these days, five contests are offered, one for each of the grade levels 3/4, 5/6, 7/8, 9/10 and 11/12 and each one in English, French and Persian. Available in English, French, and Persian.

See https://cariboutests.com/

Euclid Mathematics Contest
April 2017
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Euclid Mathematics Contest is an opportunity for students in their final year of secondary school and motivated students in lower grades to have fun and to develop their mathematical problem solving ability. Most of the problems are based on the mathematical curriculum up to and including the final year of secondary school. Written in April.

See http://www.cemc.uwaterloo.ca/contests/euclid.html

Fryer, Galois and Hypatia Mathematics Contests
April 2017
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Fryer, Galois and Hypatia Math Contests are an opportunity for students to write a full-solution contest. They are fun way to develop mathematical problem-solving skills through a written mathematical activity. For students in Grades 9 (Fryer), 10 (Galois) and 11 (Hypatia).

See http://www.cemc.uwaterloo.ca/contests/fgh.html

Gauss Mathematics Contests
May 2017
Presented by the Centre for Education in Mathematics and Computing (University of Waterloo)

The Gauss Contests are an opportunity for students to have fun and to develop their mathematical problem-solving ability. For all students in Grades 7 and 8 and interested students from lower grades.

See http://www.cemc.uwaterloo.ca/contests/gauss.html

Opti-Math
March 2017
Presented by the Groupe des responsables en mathématique au secondaire

A French-language mathematics challenge for secondary students who would like to exercise and develop their problem-solving skills.

See http://www.optimath.ca/index.html

Sun Life Financial Canadian Open Mathematics Challenge
November 2017
Presented by the Canadian Mathematical Society

A national mathematics competition open to any student with an interest in and grasp of high school math. The purpose of the COMC is to encourage students to explore, discover, and learn more about mathematics and problem solving. The competition serves to provide teachers with a unique student enrichment activity during the fall term. Available in English and French. Written in November.

Approximately the top 50 students from the COMC will be invited to write the Canadian Mathematical Olympiad (CMO). Students who excel in the CMO will have the opportunity to be selected as part of Math Team Canada — a small team of students who travel to compete in the International Mathematical Olympiad (IMO).

See https://cms.math.ca/COMC

Call for Contributions

Did you just teach a great lesson? Or maybe it didn't go as planned, but you learned something new about the complexities of teaching and learning mathematics. Maybe you just read a book or attended a workshop that gave you great ideas for presenting a topic your students have always found difficult, or that changed your perspective about some aspect of teaching. **Why not share your ideas with other teachers in the province—and beyond?**

The Variable is looking for a wide variety of contributions from all members of the mathematics education community in Canada and beyond, including classroom teachers, consultants and coordinators, teacher educators, and researchers. Consider sharing a favorite lesson, a reflection, an essay, a book review, or any other article or other work of interest to mathematics teachers in Saskatchewan. If accepted for publication, your piece will be shared, as part of this periodical, with a wide audience of mathematics educators in Saskatchewan and beyond.

We are also looking for student contributions in the form of artwork, stories, poems, interesting problem solutions, or articles. This is a great opportunity for students to share their work with an audience beyond that of their classroom and their school, and for teachers to recognize their students' efforts during their journey of learning mathematics.

All work will be published under a Creative Commons license. If you are interested in contributing your own or (with permission) your students' work, please contact us at thevariable@smts.ca.

We look forward to hearing from you!